Automatic flight control systems mclean pdf

Lyapunov Scheme Parameter Adaptation Scheme Conclusions Notes References Appendices A Actuators and Sensors A. 1 A.2 A.3 A.4 A.5 A.6 A.7 B Stability Derivatives for Several Representative Modern Aircraft B. 1 B.2 C Introduction Actuator Use in AFCSs Actuators Sensors Accelerometers Angle of Attack Sensor References Nomenclature Aircraft Data Mathematical Models of Human Pilots C. 1 C.2 C.3 Introduction Classical Models References Preface This is an introductory textbook on automatic flight control systems (AFCSs) for undergraduate aeronautical engineers. It is hoped that the material and the manner of its presentation will increase the student's understanding of the basic problems of controlling an aircraft's flight, and enhance his ability to assess the solutions to the problems which are generally proposed. Not every method or theory of control which can be used for designing a flight controller is dealt with in this book; however, if a reader should find that some favourite technique or approach has been omitted, the fault lies entirely with the author upon whose judgement the selection depended. The method is not being impugned by its omission. Before understanding how an aircraft may be controlled automatically in flight it is essential to know how any aircraft will respond dynamically to a deliberate movement of its control surfaces, or to an encounter with unexpected and random disturbances of the air through which it is flying. A sound knowledge of an aircraft's dynamic response is necessary for the succesful design of any AFCS, but that knowledge is not sufficient. A knowledge of the quality of aircraft response, which can result in the aircraft's being considered by a pilot as satisfactory to fly, is also important. In this book the first six chapters are wholly concerned with material relevant to such important matters. There are now so many methods of designing control systems that it would require another book to deal with them alone. Instead, Chapters 7 and 8 have been included to provide a reasonably self-contained account of the most significant methods of designing linear control systems which find universal use in AFCSs. Emphasis has been placed upon what are spoken of as modern methods of control (to distinguish them from the classical methods): it is most unlikely that today's students would not consider the use of a computer in arriving at the required solution. Being firmly based upon time-domain methods, modern control theory, particularly the use of state equations, is a natural and effective technique for use with computer aided engineering and harmonizes with the mathematical description of the aircraft dynamics which are most completely, and conveniently, expressed in terms of a state and an output equation. The form involved leads naturally to the use of eigenvalues and eigenvectors which make consideration of the stability properties of the aircraft simple and straightforward. Since computers are to be used, the need for normalizing the dynamic equations can be dispensed with and the differential equations can be solved to find the aircraft's motion in real time. The slight cost to be borne for this convenience is that the stability derivatives of the aircraft which are used in the analysis are dimensional; xii Preface however, since the aircraft dynamics are in real time, the dynamics of the flight controller, the control surface actuators, and the motion sensors can also be dealt with in real time, thereby avoiding the need for cumbersome and unnecessary transformations. Since dimensional stability derivatives were to be used, the American system of notation for the aircraft equations of motions was adopted: most papers and much data throughout the world now use this system. Chapters 9 to 11 relate to particular modes of an AFCS, being concerned with stability augmentation systems, attitude and path coitrol systems. A particular AFCS may have some, or all, of these modes involved in its operation, some being active at all times in the flight, and others being switched in by the pilot only when required for a particular phase of flight. Although helicopter flight control systems do not differ in principle from those used with fixed wing aircraft, they are fitted for different purposes. Furthermore, both the dynamics and the means of controlling a helicopter's flight are radically different from fixed wing aircraft. Consequently, helicopter AFCSs are dealt with wholly in Chapter 13 to emphasize the distinctive stability and handling problems that their use is intended to overcome. Active control systems are dealt with in Chapter 12 and only a brief treatment is given to indicate how structural motion can be controlled simultaneously, for example, with controlling the aircraft's rigid body motion. Ride control and fuselage pointing are flight control modes dealt with in this chapter. In the thousands of commercial airliners, the tens of thousands of military aircraft, and the hundreds of thousands of general aviation aircraft which are flying throughout the world today, examples of the types of AFCS discussed in this book can easily be found. But most modern AFCSs are digital, and to account for this trend Chapter 14 has been added to deal solely with digital control methods. The consequences for the dynamic response of the closed-loop system of implementing a continuous control law in a digital fashion is emphasized. Results complementary to those in Chapters 9 to 11, obtained using wholly digital system analysis, are also shown. The final chapter deals briefly with the subject of adaptive flight control systems, and three appendices provide a summary of information relating to actuators, sensors, aircraft stability data, and human operators. In writing a textbook, ideas and techniques which have been used effectively and easily by the author over the years are discussed and presented, but the original source is often forgotten. If others find their work used here but unacknowledged, please be assured that it was unintentional and has occurred mostly as a result of a middle-aged memory rather than malice, for I am conscious of having had many masters in this subject. At the risk of offending many mentors, I wish to acknowledge here only the special help of three people, for the list of acknowledgements would be impossibly long otherwise. Two are American scholars: Professors Jack d'Azzo and Dino Houpis, of the United States Air Force Institute of Technology, in Dayton, Ohio. They are nonpareil as teachers of control and taught me in a too-short association the importance of the student and Preface xiii his needs. The other is my secretary, Liz Tedder, who now knows, to her lasting regret, more about automatic flight control systems than she ever wished to know. D. McLEAN Southampton Aircraft Flight Control 1.I INTRODUCTION Whatever form a vehicle may take, its value to its user depends on how effectively it can be made to proceed in the time allowed on a precisely controllable path between its point of departure and its intended destination. That is why, for instance, kites and balloons find only limited application in modern warfare. When the motion of any type of vehicle is being studied it is possible to generalize so that the vehicle can be regarded as being fully characterized by its velocity vector. The time integral of that vector is the path of the vehicle through space (McRuer et al., 1973). The velocity vector, which may be denoted as ;6, is affected by the position, x, of the vehicle in space by whatever kind of control, u, can be used, by any disturbance, 6 , and by time, t. Thus, the motion of the vehicle can be represented in the most general way by the vector differential equation: where f is some vector function. The means by which the path of any vehicle can be controlled vary widely, depending chiefly on the physical constraints which obtain. For example, everyone knows that a locomotive moves along the rails of the permanent way. It can be controlled only in its velocity; it cannot be steered, because its lateral direction is constrained by the contact of its wheel rims on the rails. Automobiles move over the surface of the earth, but with both speed and direction being controlled. Aircraft differ from locomotives and automobiles because they have six degrees of freedom: three associated with angular motion about the aircraft's centre of gravity and three associated with the translation of the centre of gravity.1 Because of this greater freedom of motion, aircraft control problems are usually more complicated than those of other vehicles. Those qualities of an aircraft which tend to make it resist any change of its velocity vector, either in its direction or its magnitude, or in both, are what constitutes its stability. The ease with which the velocity vector may be changed is related to the aircraft's quality of control. It is stability which makes possible the maintenance of a steady, unaccelerated flight path; aircraft manoeuvres are effected by control. Of itself, the path of any aircraft is never stable; aircraft have only neutral stability in heading. Without control, aircraft tend to fly in a constant turn. In order to fly a straight and level course continuously-controlling corrections must be made, either through the agency of a human pilot, or by means of an automatic 2 Aircraft Flight Control flight control system (AFCS). In aircraft, such AFCSs employ feedback control to achieve the following benefits: 1. 2. 3. The speed of response is better than from the aircraft without closed loop control. The accuracy in following commands is better. The system is capable of suppressing, to some degree, unwanted effects which have arisen as a result of disturbances affecting the aircraft's flight. However, under certain conditions such feedback control systems have a tendency to oscillate; the AFCS then has poor stability. Although the use of high values of gain in the feedback loops can assist in the achievement of fast and accurate dynamic response, their use is invariably inimical to good stability. Hence, designers of AFCSs are obliged to strike an acceptable, but delicate, balance between the requirements for stability and for control. The early aeronautical experimenters hoped to make flying easier by providing 'inherent' stability in their flying machines. What they tried to provide was a basic, self-restoring property of the airframe without the active use of any feedback. A number of them, such as Cayley, Langley and Lilienthal, discovered how to achieve longitudinal static stability with respect to the relative wind, e.g. by setting the incidence of the tailplane at some appropriate value. Those experimenters also discovered how to use wing dihedral to achieve lateral static stability. However, as aviation has developed, it has become increasingly evident that the motion of an aircraft designed to be inherently very stable, is particularly susceptible to being affected by atmospheric turbulence. This characteristic is less acceptable to pilots than poor static stability. It was the great achievement of the Wright brothers that they ignored the attainment of inherent stability in their aircraft, but concentrated instead on making it controllable in moderate weather conditions with average flying skill. So far in this introduction, the terms dynamic and static stability have been used without definition, their imprecise sense being left to the reader to determine from the text. There is, however, only one dynamic property - stability - which can be established by any of the theories of stability appropriate to the differential equations being considered. However, in aeronautical engineering, the two terms are still commonly used; they are given separate specifications for the flying qualities to be attained by any particular aircraft. When the term static stability is used, what is meant is that if a disturbance to an aircraft causes the resulting forces and moments acting on the aircraft to tend initially to return the aircraft to the kind of flight path for which its controls are set, the aircraft can be said to be statically stable. Some modern aircraft are not capable of stable equilibrium they are statically unstable. Essentially, the function of static stability is to recover the original speed of equilibrium flight. This does not mean that the initial flight path is resumed, nor is the new direction of motion necessarily the same as the old. If, as a result of a disturbance, the resulting forces and moments do not tend initially to restore the aircraft to its former equilibrium flight path, but leave it in its disturbed state, the aircraft is neutrally stable. If it tends initially to deviate 3 Control Surfaces further from its equilibrium flight path, it is statically unstable. When an aircraft is put in a state of equilibrium by the action' of the pilot adjusting the controls, it is said to be trimmed. If, as a result of a disturbance, the aircraft tends to return eventually to its equilibrium flight path, and remains at that position, for some time, the aircraft is said to be dynamically stable. Thus, dynamic stability governs how an aircraft recovers its equilibrium after a disturbance. It will be seen later how some aircraft may be statically stable, but are dynamically unstable, although aircraft which are statically unstable will be dynamically unstable. 1.2 CONTROL SURFACES Every aeronautical student knows that if a body is to be changed from its present state of motion then external forces, or moments, or both, must be applied to the body, and the resulting acceleration vector can be determined by applying Newton's Second Law of Motion. Every aircraft has control surfaces or other means which are used to generate the forces and moments required to produce the accelerations which cause the aircraft to be steered along its three-dimensional flight path to its specified destination. A conventional aircraft is represented in Figure 1.1. It is shown with the usual control surfaces, namely elevator, ailerons, and rudder. Such conventional aircraft have a fourth control, the change in thrust, which can be obtained from the engines. Many modern aircraft, particularly combat aircraft, have considerably more control surfaces, which produce additional control forces or moments. Some of these additional surfaces and motivators include horizontal and vertical canards, spoilers, variable cambered wings, reaction jets, differentially operating horizontal tails and movable fins. One characteristic of flight control is that the required motion often needs a number of control surfaces to be used simultaneously. It is shown later in this book that the use of a single control surface always produces other motion as well as the intended motion. When more than one control surface is deployed simultaneously, there often results Figure 1.1 Conventional aircraft. Aircraft Flight Control ading edge (LE) slats canard Vertical canard q 4 ' Figure 1.2 A proposed control configured vehicle. considerable coupling and interaction between motion variables. It is this physical situation which makes AFCS design both fascinating and difficult. When these extra surfaces are added to the aircraft configuration to achieve particular flight control functions, the aircraft is described as a 'control configured vehicle' (CCV). A sketch of a proposed CCV is illustrated in Figure 1.2 in which there are shown a number of extra and unconventional control surfaces. When such extra controls are provided it is not to be supposed that the pilot in the cockpit will have an equal number of extra levers, wheels, pedals, or whatever, to provide the appropriate commands. In a CCV such commands are obtained directly from an AFCS and the pilot has no direct control over the deployment of each individual surface. The AFCS involved in this activity are said to be active control technology systems. The surfaces are moved by actuators which are signalled electrically (flyby-wire) or by means of fibre optic paths (fly-by-light). But, in a conventional aircraft, the pilot has direct mechanical links to the surfaces, and how he commands the deflections, or changes, he requires from the controls is by means of what are called the primary flying controls. 1.3 PRIMARY FLYING CONTROLS In the UK, it is considered that what constitutes a flight control system is an arrangement of all those control elements which enable controlling forces and moments to be applied to the aircraft. These elements are considered to belong to three groups: pilot input elements, system output elements and intervening linkages and elements. The primary flying controls are part of the flight control system and are defined as the input elements moved directly by a human pilot to cause an Primary Flying Controls 5 operation of the control surfaces. The main primary flying controls are pitch control, roll control and yaw control. The use of these flight controls affects motion principally about the transverse, the longitudinal, and the normal axes respectively, although each may affect motion about the other axes. The use of thrust control via the throttle levers is also effective, but its use is primarily governed by considerations of engine management. Figure 1.3 represents the cockpit layout of a typical, twin engined, general aviation aircraft. The yoke is the primary flying control used for pitch and roll control. When the yoke is pulled towards, or pushed away from, the pilot the elevator is moved correspondingly. When the yoke is rotated to the left or the right, the ailerons of the aircraft are moved. Yaw control is effected by means of the pedals, which a pilot pushes left or right with his feet to move the rudder. In the kind of aircraft with the kind of cockpit illustrated here, the link between these primary flying controls and the control surfaces is by means of cables and pulleys. This means that the aerodynamic forces acting on the control surfaces have to be countered directly by the pilot. To maintain a control surface at a fixed position for any period of time means that the pilot must maintain the required counterforce, which can be very difficult and fatiguing to sustain. Consequently, all aircraft have trim wheels (see Figure 1.3) which the pilot adjusts until the command, which he has set initially on his primary flying control, is set on the control surface and the pilot is then relieved of the need to sustain the force. There are trim wheels for pitch, roll and yaw (which is sometimes referred to as 'nose trim'). Dual r.p.m. gauge Magnetic compass Dual manifold pressure gaug Dual exhaust gas temperature Fuel pressure WR fuel quantity Avionics cluster ropellors blade pitch controls Instrument landing system (I Pitch trim wheel Figure 1.3 Cockpit layout. Aircraft Flight Control .. ,. . , m ~ i e v a t o r l\, Aileron Figure 1.4 Control surface deflection conventions. In large transport aircraft, or fast military aircraft, the aerodynamic forces acting on the control surfaces are so large that it is impossible for any human pilot to supply or sustain the force required. Powered flying controls are then used. Usually the control surfaces are moved by means of mechanical linkages driven by electrohydraulic actuators. A number of aircraft use electrical actuators, but there are not many such types. The command signals to these electrohydraulic actuators are electrical voltages supplied from the controller of an AFCS, or directly from a suitable transducer on the primary flying control itself. By providing the pilot with power assistance, so that the only force he needs to produce is a tiny force, sufficient to move the transducer, it has been found necessary to provide artificial feel so that some force, representing what the aircraft is doing, is produced on the primary flying control. Such forces are cues to a pilot and are essential to his flying the aircraft successfully. The conventions adopted for the control surface deflections are shown in Figure 1.4. In the event of an electrical or hydraulic failure such a powered flying control system ceases to function, which would mean that the control surface could not be moved: the aircraft would therefore be out of control. To prevent this occurring, most civilian and military aircraft retain a direct, but parallel, mechanical connection from the primary flying control to the control surface which can be used in an emergency. When this is done the control system is said to have 'manual reversion'. Fly-by-wire (and fly-by-light) aircraft have essentially the same kind of flight control system, but are distinguished from conventional aircraft by having no manual reversion. To meet the emergency situation, when failures occur in the system, fly-by-wire (FBW) aircraft have flight control systems which are triplicated, sometimes quadruplicated, to meet this stringent reliability requirement. With FBW aircraft and CCVs it has been realized that there is no longer a direct relationship between the pilot's command and the deflection, or even the use, of a particular control surface. What the pilot of such aircraft is commanding from the AFCS is a particular manoeuvre. When this was understood, and when Flight Control Systems Figure 1.5 Side arm controller. the increased complexity of flying was taken into account, it was found that the provision of a yoke or a stick to introduce commands was unnecessary and inconvenient. Modern aircraft are being provided with side arm controllers (see Figure 1.5) which provide signals corresponding to the forces applied by the pilot. Generally, these controllers do not move a great deal, but respond to applied force. By using such controllers a great deal of cockpit area is made available for the growing number of avionics displays which modern aircraft require. 1.4 FLIGHT CONTROL SYSTEMS In addition to the control surfaces which are used for steering, every aircraft contains motion sensors which provide measures of changes in motion variables which occur as the aircraft responds to the pilot's commands or as it encounters some disturbance. The signals from these sensors can be used to provide the pilot with a visual display, or they can be used as feedback signals for the AFCS. Thus, the general structure of an AFCS can be represented as the block schematic of Figure 1.6. The purpose of the controller is to compare the commanded motion with the measured motion and, if any discrepancy exists, to generate, in accordance with the required control law, the command signals to the actuator to produce the control surface deflections which will result in the correct control force or moment being applied. This, in turn, causes the aircraft to respond appropriately so that the measured motion and commanded motion are finally in correspondence. How the required control law can be determined is one of the principal topics of this book. Whenever either the physical or abstract attributes of an aircraft, and its motion sensing and controlling elements, are considered in detail, their effects are so interrelated as almost to preclude discussion of any single aspect of the system, Aircraft Flight Control Pilot's direct command input Atmospheric disturbances controls actuators - dynamics Motion variables - I deflections - Flight controller (control law) sensors + Sensor noise Figure 1.6 General structure of an AFCS. without having to treat most of the other aspects at the same time. It is helpful, therefore, to define here, albeit somewhat broadly, the area of study upon which this book will concentrate. 1. 2. 3. The development of forces and moments for the purpose of establishing an equilibrium state of motion (operating point) for an aircraft, and for the purpose of restoring a disturbed aircraft to its equilibrium state, and regulating within specific limits the departure of the aircraft's response from the operating point, are regarded here as constituting flight control. Regulating the aircraft's response is frequently referred to as stabilization. Guidance is taken to mean the action of determining the course and speed to be followed by the aircraft, relative to some reference system. Flight control systems act as interfaces between the guidance systems and the aircraft being guided in that the flight control system receives, as inputs from the guidance systems, correction commands, and provides, as outputs, appropriate deflections of the necessary control surfaces to cause the required change in the motion of the aircraft (Draper, 1981). For this control action to be effective, the flight control system must ensure that the whole system has adequate stability. If an aircraft is to execute commands properly, in relation to earth coordinates, it must be provided with information about the aircraft's orientation so that right turn, left turn, up, down, roll left, roll right, for example, are related to the airborne geometrical reference. For about sixty years, it has been common practice to provide aircraft with Flight Control Systems 9 reference coordinates for control and stabilization by means of gyroscopic instruments. The bank and climb indicator, for example, effectively provides a horizontal reference plane, with an accuracy of a few degrees, and is as satisfactory today for the purposes of control as when it was first introduced. Similarly, the turn indicator, which shows the aircraft's turning left or right, to about the same accuracy, is also a gyroscopic instrument and the use of signals from both these devices, as feedback signals for an AFCS, is still effective and valid. However, the use of conventional gyroscopic instruments in aircraft has fundamental limitations which lie in the inherent accuracy of indication, which is to within a few degrees only, and also in the inherent drift rates, of about ten degrees per hour. Such instruments are unsuitable for present-day navigation, which requires that the accumulated error in distance for each hour of operation, after an inertial fix, be not greater than 1.5 km. An angle of one degree between local gravitational directions corresponds to a distance on the earth's surface of approximately 95 km. Consequently, special motion sensors, such as ring laser gyros, NMR gyros, strap-down, force-balance accelerometers, must be used in modern flight control systems. Because this book is concerned with control, rather than guidance, it is more convenient to represent the motion of aircraft in a system of coordinates which is fixed in the aircraft and moves with it. By doing this, the coordinate transformations generally required to obtain the aircraft's motion in some other coordinate system, such as a system fixed in the earth, can be avoided. When the origin of such a body-fixed system of coordinates is fixed at the centre of gravity of the aircraft, which is in an equilibrium (or trimmed) state of motion along a nominal flight path, then, when only small perturbations of the aircraft's motion about this equilibrium state are considered, the corresponding equations of motion can be linearized. Since many flight control problems are of very short duration (5-20 seconds), the coefficients of these equations of motion can be regarded as constant, so that transfer functions can sometimes be conveniently used to describe the dynamics of the aircraft. However, it must be remembered that a notable feature of an aircraft's dynamic response is how it changes markedly with forward speed, height, and the aircraft's mass. Some of the most difficult problems of flight control occurred with the introduction of jet propulsion, the consequent expansion of the flight envelope of such aircraft, and the resulting changes in configuration, most notable of which were the use of swept wings, of very short span and greatly increased wing loading, and the concentrated mass of the aircraft being distributed in a long and slender fuselage. In aircraft of about 1956 these changes led to marked deficiences in the damping of the classical modes of aircraft motion, namely the short period mode of the aircraft's longitudinal motion, and the Dutch roll mode of its lateral motion. Other unknown, coupled Aircraft Flight Control modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender fuselages meant greater flexibility of the aircraft structure, and the modes associated with this structural flexibility coupled with the rigid-body modes of the aircraft's motion, caused further problems. One of the first solutions to these problems was the use of a stability augmentation system (SAS), which is simply a feedback control system designed to increase the relative damping of a particular mode of the motion of the aircraft. Such an increase in damping is achieved by augmenting one or more of the coefficients of the equations of motion by imposing on the aircraft appropriate forces or moments as a result of actuating the control surfaces in response to feedback signals derived from appropriate motion variables. After SAS, the following AFCS modes were developed: sideslip suppression SAS, pitch attitude hold, autothrottle (speed control system), much hold, height hold, and turn coordination systems. An integrated flight control system is a collection of such AFCS modes in a single comprehensive system, with particular modes being selected by the pilot to suit the task required for any particular phase of flight. In the past such functions were loosely referred to as an autopilot, but that name was a trademark registered by the German company Siemens in 1928. Today, AFCS not only augment the stability of an aircraft, but they can follow path and manoeuvre commands, thereby providing the means of automatic tracking and navigation; they can perform automatic take-off and landing; they can provide structural mode control, gust load alleviation, and active ride control. 1.5 BRIEF HISTORY OF FLIGHT CONTROL SYSTEMS The heavier-than-air machine designed and built by Hiram Maxim in 1891 was colossal for its time: it was 34 m long and weighed 3 600 kg. Even now, the largest propeller to be seen in the aviation collection of the Science Museum in London is one of the pair used by Maxim. It was obvious to Maxim, if to no-one else at the time, that when his aircraft flew, its longitudinal stability would be inadequate, for he installed in the machine a flight control system which used an actuator to deflect the elevator and employed a gyroscope to provide a feedback signal. It was identical, except in inconsequential detail, to a present-day pitch attitude control system. Two of the minor details were the system's weight, over 130 kg, and its power source, steam. The concept remains unique. Between 1910 and 1912 the American father-and-son team, the Sperrys, developed a two-axis stabilizer system in which the actuators were powered by compressed air and the gyroscopes were also air-driven. The system could maintain both pitch and bank angles simultaneously and, from a photographic Brief History 17 record of a celebrated demonstration flight, in which Sperry Snr is seen in the open cockpit, with his arms stretched up above his head, and a mechanic is standing on the upper surface of the upper wing at the starboard wing tip, maintaining level flight automatically was easily within its capacity. During World War I, aircraft design improved sufficiently to provide, by the sound choice of size, shape and location of the aerodynamic control surfaces, adequate stability for pilots' needs. Many aircraft were still unstable, but not dangerously so, or, to express that properly, the degree of damage was acceptable in terms of the loss rates of pilots and machines. In the 1920s, however, it was found that, although the early commercial airliners were quite easy to fly, it was difficult to hold heading in poor visibility. Frequently, in such conditions, a pilot and his co-pilot had to divide the flying task between them. The pilot held the course by monitoring both the compass and the turn indicator and by using the rudder; the co-pilot held the speed and the attitude constant by monitoring both the airspeed and the pitch attitude indicator and by controlling the airspeed via the engine throttles and the pitch attitude by using the elevator. From the need to alleviate this workload grew the need to control aircraft automatically. The most extensive period of development of early flight control systems took place between 1922 and 1937: in Great Britain, at the Royal Aircraft Establishment (RAE) at Farnborough; in Germany, in the industrial firms of Askania and Siemens; and in the USA, in Sperrys and at NACA (National Advisory Committee for Aeronautics - now NASA). Like all other flight control systems up to 1922, the RAE'S Mk I system was two-axis, controlling pitch attitude and heading. It was a pneumatic system, but its superior performance over its predecessors and competitors was due to the fact that it had been designed scientifically by applying the methods of dynamic stability analysis which had been developed in Great Britain by some very distinguished applied mathematicians and aerodynamicists (see McRuer et al., 1973; Draper, 1981; Hopkin and Dunn, 1947; McRuer and Graham, 1981; Oppelt, 1976). Such comprehensive theoretical analysis, in association with extensive experimental flight tests and trials carried out by the RAF, led to a clear understanding of which particular motion variables were most effective for use as feedback signals in flight control systems. In 1927, in Germany, the firm of Askania developed a pneumatic system which controlled heading by means of the aircraft's rudder. It used an air-driven gyroscope, designed and manufactured by Sperrys of the USA. The first unit was flight tested on the Graf Zeppelin-LZ127; the system merits mention only because of its registered trade name, Autopilot. However, the Germans soon decided that as a drive medium, air, which is very compressible, gave inferior performance compared to oil, which was considered to be very nearly incompressible. Thus, in its two-axis 'autopilot' of 1935, the Siemens company successfully used hydraulic actuators and thereby established the trend, still followed today, of using hydraulic oil in preference to air, which in turn was used in preference to Maxim's steam. In 1950, the Bristol Aeroplane Company built a four-engined, turbo-prop 12 Aircraft Flight Control transport aircraft which used electric actuators, but it was not copied by other manufacturers. At present, NASA and the USAF are actively pursuing a programme of reasearch designed to lead to 'an all-electric airplane' by 1990. The reader should not infer from earlier statements that the RAE solved every flight control problem on the basis of having adequate theories. In 1934, the Mk IV system, which was a three-axis pneumatic system, was designed for installation in the Hawker Hart, a biplane in service with the RAF. In flight, a considerable number of stability problems were experienced and these were never solved. However, when the same system was subsequently fitted to the heavy bombers then entering RAF service (the Hampdens, Whitleys and Wellingtons) all the stability problems vanished and no satisfactory reasons for this improvement were ever adduced. (McRuer and Graham (1981) suggest that the increased inertia and the consequently slower response of the heavier aircraft were the major improving factors.) In 1940, the RAE had developed a new AFCS, the Mk VII, which was again two-axis and pneumatic, but, in the longitudinal axis, used both airspeed and its rate of change as feedback signals, and, in the lateral axis, moved the ailerons in response to a combination of roll and important only in extra-atmospheric flight if all other external forces are essentially non-existent, it can be properly assumed that gravity acts at the centre of gravity (c.g.) of the aircraft. Hence, since the centres of mass and gravity coincide in an aircraft, there is no external moment produced by gravity about the c.g. Hence, for the body axis system, gravity contributes only to the external force vector, F. The gravitational force acting upon an aircraft is most obviously expressed in terms of the Earth axes. With respect to these axes the gravity vector, mg, is directed along the ZE axis. Figure 2.3 shows the alignment of the gravity vector with respect to the body-fixed axes. In Figure 2.3 represents the angle between the gravity vector and the YBZB plane; the angle is positive when the nose of the aircraft goes up. Φ represents the bank angle between the axis ZB and the projection of the gravity vector on the YBZB plane; the angle is positive when the right wing is down. Direct resolution of the vector mg, into X, Y and Z components produces: SY = mg cos [- O] sin @ = mg cos O sin @ SZ = mg cos [- O] cos @ = mg cos O cos cD Figure 2.3 Orientation of gravity vector with body axis systems. (2.45) Equations of Motion of a Rigid Body Aircraft 25 In general, the angles O and Q, are not simply the integrals of the angular velocity P and Q ; in effect, two new motion variables have been introduced and it is necessary to relate them and their derivatives to the angular velocities, P, Q and R. How this is done depends upon whether the gravitational vector seen from the aircraft is fixed or whether it rotates relative to inertial space. Aircraft speeds being very low compared to orbital velocities, the vertical may be regarded as fixed. In very high speed flight the vertical will be seen as rotating and the treatment which is being presented here will then require some minor amendments. The moment which relates the angular velocity and velocity of the body axis system with respect to the gravity vector is expressed depends upon the angular velocity of the body axes with respect to the vertical. ... Axis Transformations The physical relationships established so far depend upon two frames of reference: the Earth axis system and the body axis system. To orient these systems one to another requires the use of direction cosines. Any set of axes can be obtained from any other set by a sequence of three rotations. For each rotation a transformation matrix is applied to the variables. The total transformation array is obtained simply by taking the product of the three matrices, multiplied in the order of the rotations. In aircraft dynamics, the most common set of transformations is that between the Earth axis system and the body-fixed axes, XB, YB and ZB. The rotations follow the usual order: azimuth Ψ, pitch Θ, and roll Φ. ... 2.3.7 Linearization of the Inertial and Gravitational Terms Equations (2.14)-(2.16) and (2.39)-(2.41) represent the inertial forces acting on the aircraft. Equation (2.45) represents the contribution of the forces due to gravity to those equations. All these forces are proportional to the mass of the aircraft. Consequently, these terms may be conveniently combined with components to represent the accelerations which would be measured by sensors located on the aircraft in such a manner that the input axes of the sensors would be coincident with the body axes XB, YB and ZB. The external forces acting on the aircraft can be re-expressed as: where δX, δY and δZ are the aerodynamic forces and thrust forces. For notational convenience, AL, AM and AN are now denoted by L, M and N. Thus the equations of motion of the rigid body, for its six degrees of freedom, may be expressed as: ... The auxiliary equations of eq. (2.46) must also be used since they relate Φ to R, Q and P, T, O and Equations of Motion of an Aircraft (2.46) The set of six non-linear differential equations which constitute eq. (2.46) are nine first-order differential equations. Non-linear equations cannot be solved analytically and would require the use of a computer. Some simplification is possible, however, by considering the aircraft to comprise two components: a mean motion which represents the equilibrium, or trim, conditions, and a dynamic motion which accounts for the perturbations about the mean motion. In this form of analysis it is customary to assume that the perturbations are small. Thus, every motion variable is considered to have two components. For example: ... The auxiliary equations of eq. (2.46) can be expressed as: ... The equations which represent the trim conditions can be expressed as: ... Steady rolling, yawing and pitching motion can occur in the trim condition: the equations which define P0, Q0 and R0 are given by eq. (2.46) but being subscripted by 0 with Φ0, Θ0 and Ψ0. The perturbed motion can be found either by substituting eq. (2.57) into eq. (2.46) and subtracting eq. (2.58) from the result, or by differentiating both sides of eq. (2.46). When the perturbations are small, the sines and cosines can be approximated by the angles themselves and the unit value, respectively. Moreover, the products and squares of the perturbed quantities are negligible. Thus, the perturbed equations of motion for an aircraft can be written as: ... Equations of Motion of a Rigid Body Aircraft where g cos Θ0 and sin Θ0 have been used to represent steady orientations, and Y, O and the perturbations in those quantities. Equations (2.59) are now linear. Obviously, perturbation equations are required for the auxiliary set of equations given as eq. (2.46). Because the gravitational forces must be only the aircraft to the Earth axis system. However, the full set of perturbed, auxiliary equations is rarely used since it is complicated. But the components of angular velocity which represent the rotation of the body-fixed axes XB, YB and ZB relative to the Earth axes XE, YE and ZE are sometimes required. ... Steady flight means a flight in which the aircraft has ... The general case of the components of linear and angular velocity being zero. Possible steady flight conditions include level turns, steady sideslip and helical turns. Steady pitching flight must be regarded as merely a 'quasi-steady' condition because u and w cannot both be zero for any appreciable time if no rotations occur. Straight flight is motion with the components of angular velocity being zero. Steady sideslips and dives and climbs without longitudinal acceleration are straight flight types. Symmetric flight is motion in which the plane of symmetry of the aircraft remains fixed in space throughout the manoeuvre taking place. ... 2.4 COMPLETE LINEARIZED EQUATIONS OF MOTION 2.4.1 Expansion of Aerodynamic Force and Moment Terms ... 2.5 EQUATIONS OF MOTION IN STABILITY AXIS SYSTEM ... 2.6 EQUATIONS OF MOTION FOR STEADY MANOEUVRING FLIGHT CONDITIONS ... 2.6.1 Steady Pitching Flight ... 2.6.2 Steady Turning Flight ... 2.6.3 Steady Rolling (Spinning) Flight ... 2.7 ADDITIONAL MOTION VARIABLES ... 2.8 STATE AND OUTPUT EQUATIONS 2.8.1 State Equation ... 2.8.2 The Output Equation ... 2.9 OBTAINING A TRANSFER FUNCTION FROM STATE AND OUTPUT EQUATIONS ... 2.10 IMPORTANT STABILITY DERIVATIVES ... 2.10.1 Longitudinal Motion ... 2.10.2 Lateral Motion ... the axis OX, positive rudder deflection produces positive rolling motion, i.e. c, >O. The change in rolling moment coefficient with a deflection of the ailerons, Cl,A, is referred to as the aileron effectiveness. In lateral dynamics it is the most important control-related stability derivative. It is particularly important for low speed flight where adequate lateral control is needed to counter asymmetric gusts which tend to roll the aircraft. '' The change in yawing moment coefficient which results from a rudder deflection, is referred to as the rudder effectiveness. When the rudder is deflected to Cn8R7 the left (i.e. SR > 0) a negative yawing moment is created on the aircraft, i.e. cn < 0. ,R The change in yawing moment coefficient which results from an aileron results in adverse yaw if Cn < 0, for when a pilot deflects the deflection, C, &A' 8A ailerons to produce a turn, the aircraft will yaw initially in a direction opposite to that expected. When Cn > 0 the yaw which results is favourable to that turning 8A manoeuvre, and this is referred to as proverse yaw. Whatever sign Cn takes, its SA value ought to be small for good lateral control. 2.1 1 THE INCLUSION OF THE EQUATIONS OF MOTION OF THRUST EFFECTS 1. Many of the stability derivatives which are used in the equations of motion are the result not only of aerodynamic forces but of forces arising from flows induced by the propulsion system. Such flows profoundly modify the derivatives but the effects are usually difficult to predict, Equations of Motion of an Aircraft wind 4 Figure 2.7 Thrust alignment geometry. requiring special wind tunnel tests for their resolution. But where slipstream interference is minimal, such being the case when a subsonic jet has a central exhaust aft of the tail, the forces and moments associated with direct thrust make considerable contributions to various derivatives. The number of forces associated with the propulsion system include: (a) The forces acting on the inlet which result when the air mass entering the engine changes direction. (b) The moments caused by the angular velocity of a tube containing a mass of moving air. (c) The forces and moments resulting from the thrust itself. The angle which the thrust line makes with the relative wind is ET (see Figure 2.7) and is fixed by both the geometry of the aircraft and its trim condition. The angle of the thrust line with respect to the X-axis is fixed at - ao). Hence: ZT = - T sin (ET - a,) MT = eTT where the thrust offset eT is positive downwards. 3. Of course, thrust is a function of density, throttle setting, and the relative speed of the aircraft (on rare occasions it is a function of ao). Hence: dZT = sin (ET - ao) However: a T (COSET %= - au av cos2 a. + sin E, sin uo cos ao) (2.189) Inclusion of Motion of Thrust Effects axTaw av ax, - a T -- 38th (cos ET sin a. cos a. (cos ET cos a. + sin E, + sin ET sin2ao) sin ao) asth Z --a - aT au av azT - - -aT aw av azT - - -aT 88th 57 (sin E, cos2 a 0 - cos ET sin a. cos ao) (sin ET sin a. cos a 0 - cos E, sin2ao) (sin ET cos a. - cos ET sin ao) asth At the trim condition, however, the total moment must be zero, i.e. the thrust moment must be balanced by an equal and opposite aerodynamic moment. Thus: From eq. (2.195), however: pu0sec, = . - 2ToeT uo aT dM = eT [(% - (2.198) 2) (Ucos a. + w sin ao) + It is evident that the perturbations in moment due to thrust are influenced by the trim condition term, To/Uo. 4. Thrust can be written as: However, Cth is not an aerodynamic coefficient so that eq. (2.200) is misleading. The thrust contribution manifests itself chiefly in Xu and is expressed in the form: Equations of Motion of an Aircraft Figure 2.8 Resolution of thrust into forces and moments. where T, is the component of thrust along the axis OX. The partial derivative BT,IaU is found from data on the power plant. The direct contribution of thrust to other stability derivatives is usually negligible. 5. When the throttle setting, Sth, is increased there is a corresponding increase in thrust. Figure 2.8 shows how thrust is resolved into forces and moments. From Figure 2.8: 2.1 2 CONCLUSIONS The form of the equations of motion of an aircraft depends upon the axis system which has been chosen. Once a particular axis system is adopted, it is helpful to expand the aerodynamic force and moment terms, and to linearize the inertial and gravitational terms so that when small perturbations are considered the resulting equations will be linear and can be separated into longitudinal and lateral motion. Using the stability axis system is the most convenient for AFCS work. Sometimes, small motion is not of concern, however, and it is essential instead to consider steady manoeuvring flight such as pitching or turning. Not every motion variable of interest appears in the resulting equations of motion; such important variables as flight path angle, height, heading, and normal and lateral accelerations, are related, however, to these equations and this chapter shows how these variables can be obtained from a knowledge of the equations of Exercises 59 motion. The form of the equation lends itself to representing the longitudinal and lateral dynamics of the aircraft directly as state equations, with the other variables being obtained from associated output equations. Once the state and output equations are known it is possible to determine any transfer function relating a particular output variable to a particular control input. Not every stability derivative is significant in terms of its influence on the dynamics of the aircraft and only the most important need to be studied for their likely effects on the subsequent performance of an AFCS. Thrust changes do affect the motion of an aircraft, of course, but the thrust line does not always act through the c.g. of the aircraft, the origin of the stability axis system upon which the equations of motion are based. Consequently, special techniques are needed to introduced threse thrust effects into the equations of motion. 2.1 3 EXERCISES 2.1 Write down the state equation representing the small perturbation longitudinal motion of the aircraft CHARLIE-3. 2.2 Derive the transfer function relating the vertical velocity, w, in m s-', to the elevator deflection, SE, in radians, for the aircraft CHARLIE-4. 2.3 Using the stability derivatives of aircraft BRAVO-4 calculate the state and output equations, if the output variable is defined as the normal acceleration of the aircraft at its c.g. 2.4 The stability derivatives for VTOL aircraft in hovering flight are given below. Any stability derivative not listed should be taken as zero. (a) Calculate the transfer function relating normal acceleration, aZcg,to elevator deflection, SE. (b) Sketch the response of aZcgto a step deflection of the elevator of 0.03846 radian. (c) If the aircraft is hovering at a height of 100 m, calculate the sinking speed at, and the time of, ground contact after the application of a step deflection of the elevator. State any assumptions made. (d) In your opinion is the sinking speed obtained in part (c) excessive? Give a reason for your answer. 2.5 The lateral motion of the aircraft FOXTROT-:! is to be considered. Its rudder is not used at high Mach numbers. Derive the corresponding state and output equations, if the output variables of interest are heading angle, A, and change in roll angle, +. 2.6 For exercise 2 . 5 derive the corresponding transfer function relating, aycg,to aileron deflection, SA. 60 Equations of Motion of an Aircraft 2.7 An experimental VTOL aircraft in hovering motion has the following stability derivatives: Y , = - 0.14 Nb = 0.001 Y8, = 0.0 Ni = 0.002 Y8, = 1.02 Nh = - 0.66 L6 = - 0.012 Lf,= - 0.273 Li = 0.083 LA, = 0.7 N&, = - 0.05 NA, = - 0.53 Uo = 0.3 m sf1 LA, = - 0.12 p, r, 6, and 4 have their usual meanings of roll rate, yaw rate, sideslip angle and roll attitude, respectively. SA denotes the aileron deflection and SR denotes the rudder deflection. (a) Calculate the transfer function relating the yaw rate to the rudder deflection. (b) If the rudder deflection is an impulse function of 0.022 s, calculate by how much the heading of the aircraft will have changed some 10 s after the control deflection is applied. 2.8 A fighter aircraft, flying at 200m s-' and at a height of 104m has the following short period equations of motion: iu=-6a+q q = - 5.0~ -~ 0.69 - 12.OSE Derive the transfer function relating the pitch rate to the elevator deflection. If the aircraft's static stability is reduced to zero determine the pitch rate response of the modified aircraft to a step deflection of the elevator of - 1.0". Calculate the resulting steady state normal acceleration which the aircraft (c) would sense at its c.g. as a result of the manoeuvre of part (b). (d) Evaluate the response ratio (the acceleration sensitivity) of the aircraft. If the angle of attack is changed by 5.73", calculate by how much the load factor would change. (a) (b) 2.9 The linearized equations of perturbed longitudinal motion are given (in SI units) by: q = -0.659-0.2iuu = 225.0 Sth a - 1.2SE + 0.035 a - 9.810 - 0.18 u iu = q - 0 . 2 ~ -0 . 6 ~ ~0.035SE o=q (a) Determine the equilibrium flight speed of the aircraft. (b) Calculate the transfer function relating changes in forward speed to changes in thrust. Notes 2.10 For the aircraft CHARLIE-1: (a) Derive the state and output equations so that a change in flight path angle, y, as a result of a deflection, SE, of the elevator can be evaluated. (b) Find a transfer function relating change in y to a change in pitch attitude using the equations found in part (a). (c) Comment on the validity of the transfer function found in part (b). (d) Find the value of elevator deflection needed to produce a steady state value (if any) of - 1.0"of pitch attitude. Find the corresponding value (if any) of the flight path angle. 2.11 A large jet cargo aircraft, DELTA, is powered by four engines, each having a thrust of 182 kN. The mass of the aircraft is 264 000 kg. The flight condition is F/C#2. (a) Determine an appropriate state equation for the aircraft's motion. (b) Thence find the transfer function relating changes in forward speed, u , to a change in thrust, Sth. The pilot is located 25.0 m forward of the aircraft's c.g. and 2.5 m above it. (c) Calculate the steady normal acceleration experienced by the pilot if the angle of attack of the aircraft is changed suddenly by 2.85". 2.12 A high performance fighter is on approach at 165 knots. The linearized equations of perturbed lateral motion are given by: where Sdhtdenotes differential deflection of the horizontal tail. (a) Is it possible to find a combination of control surface deflections which will result in there being no lateral acceleration in the steady state, even though some of the state variables have finite values? (b) If your answer to part (a) was in the affirmative, determine the corresponding values of the steady surface deflections required. (c) Determine the transfer functions relating the lateral acceleration at the c.g. to each control surface independently, i.e. find: ay,fs) - 6.4(~)' aYcg(s) aYcg(s -and - s ~ (~) &dht(~) Can you decide from these transfer functions which control surface is the most important for manoeuvring the aircraft on approach? 2.14 NOTES 1. 2. For example, see chapter 4 of McRuer et al. (1973). This depends upon the assumption of constant aircraft mass. Equations of Motion of an Aircraft ml has been used to denote the perturbation in the pitching moment, M, to avoid confusion with the aircraft's mass, m. This form applies to linear, time-invariant systems only; when the system is nonlinear, the appropriate form is k = f(x, u, t) . For linear, time invariant systems only; when the output relationship is non-linear the appropriate form is y = g(x, u, t) . If the output equation is non-linear, the presence of measurement noise modifies y to become: y = g(x, u, 6 , t) . This assumes that the matrix (sI - A) is non-singular, which can be proved by recalling that 3-'{[sZ - A]-') = eA'. Although Uo is used in these equations, the correct value to be used is the true airspeed. For small the errors are insignificant if Uo is used instead of VT. If the elevator is located forward of the c.g. it is renamed canard. This description is increasingly common, although canard referred originally to an aircraft configuration which flew 'tail first', the forward tail surface being called a foreplane. It is this foreplane which is now considered to be a canard. 2.1 5 REFERENCES BABISTER, A.W. 1961. Aircraft FOGARTY, L.E. and R.M. HOWE. Stability and Control. Oxford: Pergamon Press. 1969. Computer mechanization of six-degree-of-freedom flight equations. NASA CR-1344, May. GAINES, T.G. and S. HOFFMAN. 1972. Summary of transformation equations and equations of motion. NASA Sp-3070. McRUER, D.T., I.L. ASHKENAS and D.C. GRAHAM. 1973. Aircraft Dynamics and Automatic Control. Princeton University Press. McRUER, D.T., C.L. BATES and I.L. ASHKENAS. 1953. Dynamics of the airframe. Bur. Aero. Rpt. AE-61-4 (Vol. 11) USA. THELANDER, J.A. 1965. Aircraft motion analysis. FDL-TDR-67-70, WPAFB, Ohio, USA. March. Aircraft Stability and Dynamics 3.1 INTRODUCTION The equations of motion have been derived in some detail in Chapter 2. Only under a large number of assumptions about how an aircraft is being flown is it possible to arrive at a set of linear differential equations which can adequately represent the motion that results from the deflection of a control surface or from the aircraft's encountering atmospheric turbulence during its flight. This resulting motion is composed of small perturbations about the equilibrium (trim) values. To achieve such equilibrium values requires the use of certain steady deflections of the appropriate control surfaces. Consequently, the entire range of the angle of deflection of any particular control surface will not necessarily be available for the purposes of automatic control, since much of that range is required to trim the aircraft. What is meant, then, by small perturbation is that any angle be sufficiently small to guarantee that the assumptions concerning any trigonometrical functions involved remain valid. For practical purposes, a change of angle of 15" or more should be regarded as large, and the designer should then consider the likely effects of continuing to use the small perturbation theory whenever such angular values can occur. Similarly, translational velocity should always be small in relation to the steady speeds; when the steady speed, such as Vo or Wo, is zero then changes of velocity of 5 m s-' should be regarded as being the limit of validity. However, it must be strongly emphasized that these are not firm rules but depend upon the type of aircraft being considered, its flight condition, and the manoeuvres in which it is involved. For the remainder of this chapter it is considered that all the assumptions of Chapter 2 hold, that any aircraft being considered is fixed wing and flying straight and level in a trimmed condition, and that its motion is properly characterized by eqs (2.109) and (2.110). For example, for longitudinal motion, eq. (2.112) is taken as the definition of the state vector x, i.e.: and the control vector u is defined as: 64 Aircraft Stability and Dynamics The state coefficient matrix A is then given by: xu xw 0 -g and the driving matrix B by: For lateral motion, the appropriate equations are (2.143) and (2.154), respectively where the coefficient matrix is: Y" 0 -lglUo Lb Lf, L:. 0 Nb N f , N:. 0 and the driving matrix is: 3.2 LONGITUDINAL STABILITY 3.2.1 Short Period and Phugoid Modes The dynamic stability of perturbed longitudinal motion is most effectively established from a knowledge of the eigenvalues of the coefficient matrix A. They can be found by solving the linear equation: I X Z - Al = 0 (3.5) I is a 4 x 4 identity matrix. By expanding the determinant, the longitudinal 65 Longitudinal Stability stability quartic, a fourth degree polynomial in h, can be expressed as: An aircraft may be said to be dynamically stable if all its eigenvalues, hi, being real, have negative values, or, if they be complex, have negative real parts. Zero, or positive, values of the real part of any complex eigenvalue means that the aircraft will be dynamically unstable.' Rather than solving the polynomial by numerical methods it is more effective to use a numerical routine to compute the four eigenvalues of A. It has been observed that for the majority of aircraft types, the quartic of eq. (3.6) invariably factorizes into two quadratic factors in the following manner: (1' + 2cphwphh + uEh)(h2 + 2cspwsph + wfp) (3.7) The first factor corresponds to a mode of motion which is characterized by an oscillation of long period. The damping of this mode is usually very low, and is sometimes negative, so that the mode is unstable and the oscillation grows with time. The low frequency associated with the long period motion is defined as the natural frequency, wph;the damping ratio has been denoted as tph The mode is referred to as the phugoid mode, a name improperly given to it by the English aerodynamicist, Lanchester, who coined it from the Greek word which he believed meant 'flight-like'. Unfortunately, +vyq implies flight as demonstrated by a fugitive, not a bird (Sutton, 1949). The second factor corresponds to a rapid, relatively well-damped motion associated with the short period mode whose frequency is w,, and damping ratio is c,,. As an example, consider the passenger transport aircraft, referred to as aircraft DELTA in Appendix B. If flight condition 4 is considered, the aircraft is flying straight and level in its cruise phase, at Mach 0.8 and at a height of 13 000 m. From the values of the stability derivatives quoted in the appendix, A is found to be: [- 0.033 0.0001 0 - 9.811 The eigenvalues corresponding to this matrix are found to be: hl, h2 = + 0.0033 f j0.0672 h3, A4 = - 0.373 + j0.889 (3.9)' (3. 10)' The eigenvalues of eq. (3.9) are seen to be those associated with the phugoid mode since the damping ratio, although positive, is very small (0.0489) and the frequency is very low (0.067 rad s-l), hence the period is long. Such an inference can be drawn because the solution of any quadratic equation of the form: 66 x2 Aircraft Stability and Dynamics + 2Cox + o2 = 0 (3.11) is given by: whenever 5 < 1.0. Complex roots occur only when the damping ratio has a positive value less than unity. From eq. (3.10) the eigenvalues can be deduced to be those associated with the short period mode, for which the frequency is 0.964 rad s-I and the damping ratio is 0.387. 3.2.2 Tuck Mode Supersonic aircraft, or aircraft which fly at speeds close to Mach 1.0, occasionally have a value of the stability derivative, Mu, such that Mu takes a large value which is sufficiently negative to result in the term wEh in the phugoid quadratic becoming negative too (see Section 3.6). When this happens, the roots of the quadratic equation are both real, with one being negative and the other positive. Hence the phugoid mode is no longer oscillatory but has become composed of two real modes; one being convergent, which corresponds to the negative real root, and the other being divergent, which corresponds to the positive real root. The unstable mode is referred to as the 'tuck mode' because the corresponding motion results in the nose of the aircraft dropping (tucking under) as airspeed increases. Aircraft DELTA in Appendix B will exhibit a divergent tuck mode in flight condition 3. 3.2.3 A Third Oscillatory Mode The c.g. of a modern combat aircraft is often designed to lie aft of the neutral point (n.p.) (see Section 3.3). When this is the case the stability derivative, M,, can take a value which will result in every root of the longitudinal stability quartic being real. As the c.g. is then moved further aft of the n.p., the value of M, changes so that one of the real roots of the short period mode, and one of the real roots of the phugoid mode, migrate in the complex plane to a point where they form a new complex pair, corresponding to the third oscillatory mode. When this has occurred, that mode is the main influence upon the dynamic response of any AFCS which is used. The phugoid mode has now become a very slow aperiodic mode, and there also exists another extremely rapid real mode. Too positive a value of M, can result in dynamic instability, for one of these real eigenvalues can become positive (see Section 3.5.2). Longitudinal Stability 3.2.4 s-plane Diagram The location of eigenvalues in the complex frequency domain is often represented by means of an s-plane diagram (which is simply a special Argand diagram). In Figure 3.1 are shown the locations (denoted by x) of eigenvalues for a typical conventional aircraft. For an aircraft which exhibits a tuck mode the locations are denoted by 0 and for an aircraft with a third oscillatory mode they are denoted by A . X Conventional aircraft o Aircraft with tuck mode A Aircraft with 3rd oscillating pair I - cr -5.0 I -4.0 n I -3.0 Figure 3.1 I -2.0 I -1.0 A - x -X - to I + IS s-plane diagram. A popular method of investigating how sensitive is an aircraft's stability to values of some particular stability derivative (and, consequently, some aerodynamic, inertial, or geometric parameter) is to illustrate how the eigenvalues travel around the s-plane as the values of the stability derivative are changed. This is a form of root locus diagram. Another effective way of determining to which stability derivative the aircraft's dynamic response is most sensitive is to carry out a sensitivity analysis on coefficient matrix, A (Barnett and Storey, 1966). It is important to remember that when the aircraft dynamics can be assumed to be linear those stability derivatives associated with the control surfaces play no part in governing the stability properties of the aircraft. Their importance for achieving effective automatic flight control, including stability augmentation, is paramount nevertheless. Aircraft Stability and Dynamics 3.3 STATIC STABILITY 3.3.1 Trim Condition In Chapter 2 where the derivation of the equations of motion for an aircraft was shown, a point was reached (in Section 2.6) where the set of equations governing the small perturbation motion about some equilibrium flight condition was considered. To achieve that equilibrium required a number of forces and moments to be balanced. The balance equations, for straight and level flight, were shown to be: Xo - mg sin O = 0 Zo + mg cos O = 0 Yo = Lo = Mo = No = 0 For the linearized equations, the control required to achieve trim is: where [BITis the generalized inverse of matrix B. When reference is made to the stability of these static components what is meant is the inherent tendency of an aircraft to develop forces or moments (or both) which directly oppose any deviation of this motion from equilibrium flight. The only forces which can change significantly as a result of disturbances are sideforce, lift and drag, the values of which depend upon the orientation of the aircraft relative to the oncoming airstream. Obviously, each motion variable can be considered from a stability viewpoint. Only the most significant criteria of aircraft stability are considered here. 3.3.2 Forward Speed Stability An aircraft is considered to be statically stable for any disturbance, u, in its forward speed if the value of the stability derivative, Xu, is negative. It is particularly important in the approach phase of flight that Xu should not be positive. This requirement may be understood from considering Figure 3.2. Suppose the aircraft in the figure is in a steady, trimmed flight condition corresponding to point A, i.e. the throttle setting is fixed and constant thrust is being produced which results in the aircraft's flying at some speed, VA. If, for any reason, the aircraft speed increases to, say, VA U, a drag force is generated which opposes the increase in speed, i.e.: + Another way of interpreting Figure 3.2 is to see that, at point A, if it is wished to increase the speed of the aircraft, the thrust has to be increased; to reduce its speed requires a reduction in thrust. Point B represents a lower speed (at which Static Stability Direction of (Thrust) Fx Xu> 0: unstable Figure 3.2 Speed stability diagram. many aircraft would typically fly on approach). There it is seen that any decrease in speed leads to an increase in the drag force which will result in a further reduction in speed. At point B, Xu > 0 and the situation is regarded as unstable. If the difference between the thrust available from the engines and what is required to sustain flight in a particular manoeuvre is small (sometimes this is expressed by saying that the thrust margin is small), or if the change in thrust from the engines, as a result of a change in throttle setting, is slow, then it is possible for an aircraft operating at point B to be in a position where recovery of the required airspeed is possible only by diving the aircraft. During the approach phase of flight, if this was not regarded as undesirable, it would certainly be regarded as unseemly. At speeds lower than VB the aircraft will tend to stall. If the unstable portion of the curve corresponding to maximum thrust intersects the line for which F, is zero at a value of speed higher than VStal1,the aircraft's speed will diverge, which will result in a stall, unless the pilot is able and willing to dive the aircraft. It is principally delta wing aircraft, such as the F-106, B-58 and Concorde, which tend to have positive values of Xu on approach. 3.3.3 Vertical Speed Stability An aircraft will be statically stable for any disturbance in the vertical speed, w, if the value of the stability derivative, Z,, is negative. This means that if, somehow, there is generated a positive velocity increment along the axis OZ, a force is generated which tends to oppose the initial disturbance in w. For this to be true, the lift curve slope of the wing must be positive for all values of angle of attack, an aerodynamic condition which is always satisfied. However, for wings of high aspect ratio (when span2/surface area is large) and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and hence Z,, is reduced. On delta wings, aeroelastic effects often increase Z,. 70 Aircraft Stability and Dynamics 3.3.4 Sideslip Stability Generally, the static stability requirement that the value of Yp be negative is unimportant. However, although Yp < 0 is the usual condition, it is of advantage to the pilot. Sideslip angle P is not easily detected by pilots when Yp < 0 because the condition causes symmetrical aircraft (which most are) to fly banked in steady sideslip manoeuvres. Also, if a pilot is turning at very low height, and there is a restriction on the bank angle which can be commanded, because of the proximity of the terrain, a negative value of Yp will allow a skidding turn to be performed. It can assist the side-step manoeuvre, which is sometimes performed when an aircraft on its final approach is not correctly aligned with the runway centre line. 3.3.5 Static Directional Stability An aircraft is said to have static directional stability if the value of the stability derivative, Nb, is positive. This means that the yawing moment N will increase as a result of a positive (sideslip) velocity v and the aircraft aligns itself with the relative airflow. The non-dimensional stability derivative Cn is referred to as the 'weathercock stability7. A large part of Cn is contribute8 by the volume of P the vertical tail.4 For supersonic transport aircraft, such as Concorde, the high Mach numbers and high values of the angle of attack, which commonly occur in operational flight, can cause considerable deterioration in the value of Nb. 3.3.6 Lateral Static Stability If there is a positive change in the sideslip angle then the aircraft's right wing drops and the aircraft slides to the right. Lb must be negative for stability; the 'dihedral effect' results in the right wing being pitched up to negate the sideslip. When Lb is negative, the spiral mode (see Section 3.4) will be convergent. 3.3.7 Longitudinal Static Stability It is explained in Chapter 1 that in the XZ plane, an intentional change of the aircraft's orientation can be achieved by deflecting the elevator, or flaps. Deflection of either surface produces a small, unbalanced force which, because of the distance of the point through which it acts from the aircraft's c.g., can result in a large pitching moment. Such a moment causes an aircraft to rotate about its c.g. until the steady moments adjust themselves to come into balance. How fast an aircraft will rotate, i.e. its angular acceleration, depends upon the size of the moment and the value of the moment of inertia about the axis OY, namely Iyy. Since the centre of pressure moves with changes in the angle of attack, any change in lift causes a change in the moment produced by the lift force Static Stability * IT Zero lift line + ZT #- f. IT Ma, Figure 3.3 Geometry of wingltail. about the c.g. Thus, an aircraft rotates to a new orientation when disturbed and, as a result, the moments due to drag, to the lift from the wing and from the tail, etc. must all change. If they change in a way that increases the extent of the rotation, this is an unstable condition. As a result, it is customary to take as a criterion of longitudinal static stability the sign of the stability derivative M,, for when M, < 0 any increase in the angle of attack, a,will cause an increase in the nosedown pitching moment, thereby tending to reduce the angle of attack. Once the configuration of an aircraft has been fixed, then, for any particular flight condition, the stability derivative M, depends principally upon the normalized distance xaclE from the mean aerodynamic centre of the wing to the aircraft's c.g. where xac is the distance from the a.c. of the aircraft and its c.g. and E is the chord length (the chord measured along the zero lift line of the wing see Figure 3.3). If the mean a.c. and the c.g. are coincident, that is, if the lift force acts through the c.g., then the stability derivative M, takes the value zero. This condition is known as neutral static stability. The c.g. location, which corresponds to this condition, is called the stick-fixed neutral point (n.p.). If the c.g. is located aft of the n.p., then M, has a positive value and the aircraft is statically unstable. If the c.g. is then located even further aft of the n.p., a condition is reached where an 'infinite' normal acceleration, a, is produced with no force being applied to the control stick. This particular locatyon of the c.g. is called the stick fixed manoeuvre point (m.p.). By varying the location of the c.g. the manoeuvre stability can be made zero, positive, or negative. The m.p. corresponds to neutral manoeuvring stability. The distance between the c.g. and the m.p. is called the manoeuvre margin. The distance of the c.g. from the n.p. is referred to as the static margin. To be statically stable the c.g. of an aircraft must be located forward of its n.p. Its m.p. for this condition must be aft of the c.g., but for conventional aircraft to be as manoeuvrable as possible, i.e. to produce as much acceleration as possible in response to a given control surface deflection, the c.g. should be only just forward of the n.p. In modern fighter aircraft this static stability is relaxed and the c.g. is often deliberately located aft of the n.p., thereby reducing the manoeuvre margin but, as a result, increasing the manoeuvrability of the aircraft, which is the desired result. In this case, however, it turns out that, for pilots to fly the aircraft 72 Aircraft Stability and Dynamics successfully, dynamic stability is required, which has to be provided by a stability augmentation system specially fitted for the purpose. It is a relatively simple matter to show that the static margin can be expressed by Cm = CL Cm - a (CLC - h) Xac = E where Cmis the coefficient of the pitching moment and CL is the lift coefficient. h, h,, xac and E are defined in Figure 3.3. Cm and CL denote dCm/da and dCL/da, respectively. The stability derivative M, represents the change in pitching moment which occurs as a result of a change in the vertical velocity, w. If moments are taken about the c.g. of the aircraft (nose-up moments being defined as positive), L. Hence, the static margin is related to the stability derivative M, a stable aircraft having a negative M. If the aircraft is unstable, the static margin is negative and M, becomes positive. The magnitude of the static margin is indicative of the aircraft's 'natural' pitch stiffness. The stability derivative Mq, being a measure of the damping in pitch, is also affected by the static margin... 3.4.2 Use of Stability Derivatives ... In these expressions the coefficient of the pitching moment, C,, is the pitching moment divided by ... the other quantities being those defined in Chapter 2. The stability derivatives are defined in the usual way, for example ... The equations of motion of the aircraft, expressed in terms of the stability derivatives, are given by ... (3.15) Transfer Functions Related to Longitudinal Motion 3.4 TRANSFER FUNCTIONS RELATED TO LONGITUDINAL MOTION 3.4.1 Relationship Between Transfer Function and State Equation The theory relating to the linearized equations of motion is given in Section 2.9 of Chapter 2. In this present section, some of the transfer functions corresponding to longitudinal motion will be derived, but the reader should be aware that a number of computer programs are available (see for example, Systems Control Technology, Inc., 1986; Larimer, 1978) for the automatic determination of appropriate transfer functions from a knowledge of the stability derivatives. These programs are usually based on the Leverrier algorithm (Faddeeva, 1959). The purpose of deriving analytically a number of transfer functions in this present section is to arrive at final forms, to see which parameters and terms are significant, and to note possible simplifications which can lead to useful approximations. It has been shown in Chapter 2 that if only a single control, δE, is considered, the linearized, small perturbation equations of longitudinal motion are given by: where The coefficient matrix, A, and the driving matrix, B, are given by: From eq. (2.164), the transfer function relating output variable, yi, to control input, ui, is given by: 74 Aircraft Stability and Dynamics Thus, every transfer function depends upon the variable chosen as the output and the control surface deflection used to change the motion variable. But it must always be remembered that when the control deflection is used to change some particular motion variable that same control deflection changes other motion variables simultaneously. It is this simple fact which sometimes causes great difficulty for the designers of AFCSs, and it is this fact which leads to so many systems, designed by means of the conventional theory of control for single input, single output, linear systems, producing aircraft performance which is unacceptable to pilots. Although transfer functions are useful, their use is limited, particularly for AFCS design for modern aircraft where many control surfaces may be used simultaneously. However, from eq. (2.164), it is evident that every transfer function relating to the motion of the aircraft must depend on the inherent characteristics of the aircraft through the resolvent matrix, [sI - A]-'. 3.4.2 Use of Control Matrix, C, to Select a Particular Motion Variable For the present, normal acceleration, and those motion variables such as h which are directly related to it, are not being considered. Thus, and, for further simplicity, since transfer functions are being considered, only a single output variable will be dealt with at a time. Consequently, eq. (3.28) now becomes: where C is a 1 x4 rectangular matrix. C contains only one non-zero element and that element has the value unity. The column in which this value is to be found depends upon which state variable is being taken as the output variable of concern. For example, if the output variable is chosen to be u, then: y = [1 0 0 0]x The other these relationships are: Thus, the unit element can be looked upon as a kind of pointer indicating which state variable has been chosen as the output variable. Quite often, the output matrix C is used to achieve conversion of physical units. For example, if the state variable q is defined in rad s-' but is required to work with pitch rate in degrees s-', then: Since defining q in degree s-' as an output variable results in y = [0 0 57.3 0]x. Transfer Functions Related to Longitudinal Motion 3.4.3 Transfer Function Notation It will be plain to the reader now that for every transfer function the denominator is identical. G (s) = N(s)/Δ (s) (3.34) The denominator polynomial is the characteristic polynomial of the aircraft, namely det[sI - A] which was dealt with in Section 3.2. When the roots of the polynomial are known, i.e. those values of s are known which result in: it will be seen that they are identical to the eigenvalues of A. The polynomial det[sI - A] is often called the stability quartic. Every transfer function for longitudinal motion has the same denominator, therefore, it is the numerator of each transfer function which distinguishes one transfer function from another. In the AFCS literature, the convention ... numerator polynomials are direct functions of the output variable and the control input used. The superscript yi denotes the output variable and uj denotes the control input. Thus, for the four state variables, the numerator polynomials are denoted ... it must be remembered that the output variable has... since the relationship between output and input is a ratio of polynomials ... In American reports especially, as N$(s). The superscript yi denotes the particular output variable and uj the particular input control... Control System Design Methods I 206 compressibility effects are significant, or when structural compliance exists. For these AFCS problems, particular care must be exercised if the design is based upon transfer functions obtained from a mathematical representation of the aircraft's dynamics which approximated the degrees of freedom involved. 7.4 PARAMETER OPTIMIZATION 7.4.1 Introduction It was shown in Chapter 6 how the flying qualities specified in terms of parameters such as short period damping, natural frequency of yawing motion, roll subsidence time constant, and so on. These parameters refer to idealized, low order models of the aircraft dynamics and once the controller structure and parameters have been specified because the settling time, or peak overshoot, or time-to-first crossover of the time response, produced by the corresponding low order model, is close to what is required from the aircraft motion when the aircraft has been subjected to some similar forcing function. The conventional design methods for s.i.s.o. control systems, dealt with in the earlier sections of this chapter, provide adequate means of achieving these figures of merit but require considerable experience, skill and judgement to produce acceptable designs. It would be helpful to have a design method which provides as a solution the structure of the control system and the best values for the corresponding parameters. A method which does this depends upon a performance measure which is a member of a class of performance indices. 7.4.2 Performance Indices For any control system, its output, y, is required to follow its input signal, r, as closely as possible. Any difference between the input and output is an error, e, where e = r - y. If the performance index is the integral of some function of the error, J, then: ... where e denotes the error. Time, t, is measured from zero, the instant at which an input is applied. If it is assumed that e(t) is of the form: the performance index, J, can be denoted as Parameter Optimization 207 where e is a constant. When n = 1, J is the integral of absolute error (i.a.e.); when n = 2, J2 is the integral of squared error (i.s.e.). The performance index, when n = 0, is defined as a special case, meaning that Jo= ∫ (1 − e = 0) dt ... Other forms of the performance index are available, for example, from eqs (7.95) and (7.96) that: Jo is, therefore, the settling time of the system. 1.s.e. (J2) is a much favoured performance index because it is easy to work with analytically, but the time response which results from an AFCS, designed on the basis of minimizing J2, is often unsatisfactory and, consequently, alternative performance indices, such as i.a.e. (J1) and i.t.a.e., (integral of the product of time and absolute error) are used, because they penalize large and persistent errors. Neither i.a.e. nor i.t.a.e. is easy to handle analytically, however, but design tables are readily available (see, for example, Newton et al., 1957). For further discussion of performance indices, the interested reader should consult Fuller (1967). 7.4.3 Parseval's Theorem and Definite Integral Table Integrals of the form: often need to be solved in AFCS work. By means of Parseval's theorem, a solution to eq. (7.98) can be found by evaluating it in the domain of the complex frequency, s, a procedure which is easier than solving for J directly in the time domain. Parseval's theorem states that the integral defined in eq. (7.98) can be expressed as: 208 Control System Design Methods / Of particular interest to AFCS designers is the case where: f(t) = f2G) = f (t) for then: Of course, eq. (7.101) is still the Parseval equivalent to: provided that, for t < 0, f (t) = 0. Suppose there is a variable, x (t), and it is necessary to evaluate its integral-squared value, I, where: Table 7.1 Phillips' integrals Let: where the subscript n refers to the degree of the denominator polynomial, d (s) where n ... Table 7.1. A more extensive table can be found in Newton et al. (1957). Parameter Optimization Let x(s) be a rational function of s of the form: If all the poles of the function lie in eq. (7.104) have negative values or, if any poles are complex with negative real parts, then I exists, and can be expressed as an algebraic function of the coefficients cj, where j = 1, 2, ..., n - 1 and i = 1, 2, ..., n. The results are summarized in Table 7.1. 7.4.4 Design of Optimal s.i.s.o. Linear Systems The method of achieving a design is procedural: a structure is assumed for example, some compensation element is inserted in the forward path of the closed loop system and the i.s.e. for a response to some specific input, is minimized by an appropriate choice of the unknown parameters of the compensation element. It may possibly be as simple a problem as finding some gain to that value which results in the lowest value of i.s.e. The procedure and method are illustrated by means of two simple examples. Example 7.10 A simple system is represented by the block diagram of Figure 7.17. The problem to be solved are: Figure 7.17 Block diagram for Example 7.10. 1. In response to a unit step input, r(t) = 1, find the value of K, which minimizes the i.s.e., if a is a constant. 2. In response to the same input find a which minimizes the i.s.e. If the forward transfer function is expressed as: K(s) = ... From Figure 7.17 it can be deduced that: Control System Design Methods 1 210 and so for a unit step input R(s) = 1/s: Thus, for E(s), referring to eq. (7.104): co = a, c1 = 1, do = K, d1 = a, d2 = 1, and from Table 7.1: Now, firstly: Therefore K = w which minimizes i.s.e. Secondly: For minimum i.s.e., a = hence 5 = W: VK and the optimum transfer function is: K, giving: = 0.5. Thirdly: If the value is not 0.5 ... In response to a unit step input, r(t) = 1, find the optimum value of a. Parameter Optimization 211 Let ... Example 7.11 A unity feedback system has the closed loop transfer function: Parameter Optimization Find the values of a, and a2 which minimize i.s.e. for a unit step input. Thus, from eq. (7.104): Therefore: co = al, c1 = a2, c2 = 0, do = 1, d1 = al, d2 = a2 and d3 = 1. From Table 7.1: and I3 is a minimum with respect to a1 when ... and I3 is a minimum with respect to a2 when: 7.4.5 Lagrange Multipliers Suppose that the variables, x(t) and u(t), are related by some differential equation, and it is required that u(t) is to be chosen to minimize: 6 = a J x2dt (7.105) 212 Control System Design Methods I subject to a constraint on u (t), namely: where C is some constant value to which the integral of u2 is restricted. For this problem, it can be shown that... Suppose that u (t) has been chosen, but does not satisfy eq. (7.106). The problem is then to solve for ... the difference between the right and left sides of (7.106) is then zero, i.e. ... At the start step input, the values of K and T which minimize the i.s.e. subject to the constraint that ... This is done by choosing the Lagrange multiplier, λ, as ... Lagrange multiplier ... The control, u (t), which minimizes eq. (7.108) is obtained subject to the constraint of eq. (7.106). Equation (7.106) is sometimes regarded as an 'energy' constraint, owing to its 'squared' nature. Therefore, rather than finding some control, u (t), from a class of control functions which minimize eq. (7.105), the equivalent problem which is solved is to choose u (t) from the class of control functions which minimize eq. (7.108). The procedure which is used is illustrated in the three examples which follow. Parameter Optimization Example 7.12 For the system represented by the block diagram in Figure 7.18 determine, for a unit step input, the values of K and T which minimize the i.s.e., subject to the constraint that ... Figure 7.18 Block diagram for Example 7.12. For unit step input: R(s) = 1/s: Therefore, for the error ... Control System Design Methods 214 ... Example 7.13 For the system represented in Figure 7.19 choose Kl and K2 such that, in response to a unit step input, yD, the performance index, J, is minimized. If J is to be finite, there has to be zero steady state error to a unit step input. Hence Kl must equal unity. Parameter Optimization Figure 7.19 Block diagram for Example 7.13. Now, for: Therefore ... Two solutions of K2 in a[λ]K2 = 0. From a solution of the quartic, a positive real solution of K2 can be found by iteration: K2 = 1.93. Note that the response of this system to a step input is very sluggish; it can be changed by increasing the weighting factor of 25 on yD2 term. Control System Design Methods 216 bandwidth, BW, is defined as: Figure 7.20 Block diagram for Example 7.14. Determine the gain which will minimize the i.s.e., subject to the constraint that BW ≤ 1. Note that: For i.s.e., ... For BW: ... Hence ... Example 7.15 CONCLUSIONS The chapter introduces the methods used for the design of AFCSs as a control problem and shows that, with linear feedback control, it is never possible to satisfy simultaneously the requirements for good stability, good tracking, and noise rejection. The use of feedforward was discussed and the equations for a generalized AFCS, which allowed the linear feedback control to be treated in the conventional manner, were developed. The chapter concentrates on the use of conventional control methods, such as root locus, Bode diagrams and Nichols charts ... Alternatively, the flying qualities ... extended by considering parameter optimization techniques to achieve feedback controllers which optimize a performance index, usually the integral of error squared, subject to the control system, some rate of change of a motion variable, or the bandwidth of the closed loop system. These methods are illustrated by means of examples. 7.6 EXERCISES 7.1 Design a closed loop system using linear state variable feedback for the open loop system shown in Figure 7.21. The desired closed loop poles of the system must have a damping ratio of not less than 0.45. And in response to a step input the peak overshoot of the response of the closed loop system that was observed 20 per cent and must not occur later than 0.5 s after the test signal has been applied. The complete response must have settled in 0.4 s. Aircraft dynamics Amplifier Rudder servo Figure 7.21 (a) Open loop system for Exercise 7.1 (a) Draw a root locus diagram for the system of Figure 7.21. (b) If ... poles of the system with positive unity feedback. (c) Use a pole placement method to obtain a feedback control law which will result in the required closed loop response. 7.2 The short period dynamics of an aircraft can be represented by the equations ... the aircraft BRAVO-2 as ... Control System Design Methods I (b) (c) (d) (e) 7.3 Repeat part (a) for the transfer function ... 7.4 The aircraft ALPHA has a yaw damper fitted ... response of the aircraft to ... Nichols diagram ... The pitch attitude control system of the aircraft DELTA-2 ... (a) Plot the Nyquist diagram for the transfer function ... (b) Determine ... The transfer function ... Dutch roll ... For the GOLF-2, the yaw damper ... Control System Design Methods 1 ... (b) Find the value of the gain ... The required transfer function ... The dynamics of a large transport aircraft, DELTA, for flight condition 2, are augmented by the addition of the state representation of the system ... Sketch the root locus ... 7.8 The block diagram of a roll attitude control system for CHARLIE-2is shown in Figure 7.23. The gain, K, of the controller is to be found such that the integral of error squared is minimized subject to the constraint ... Figure 7.22 Yaw damper system for Exercise 7.7. ... Figure 7.23 Roll attitude control system for Exercise 7.8. 7.9 The block diagram of a flight control system which is used to control the height of FOXTROT is shown in Figure 9.10(b) in Chapter 9 ... (a) Show that the closed loop pitch rate control system can be represented by ... the integral of error squared ... 7.10 The rate of change of height in response to a change in the collective of a hovering helicopter can easily be shown (see Chapter 13) to be governed by a transfer function ... Control System Design Methods I For a small helicopter with a single main rotor the values of the stability derivatives have been found to be: Z g00 = -0.4 and Zw = 1.4 When a simple height control system with a proportional controller, with a value = 6.0, and with unity feedback is used it is found that the dynamic response of the closed loop system is unacceptable. (a) Determine a feedback control law which will ensure that the height control system will have a closed loop transfer function: K (b) Compare the natural frequency and damping ratio of this system with the values which would obtain with the single height control system. 7.7 NOTE 1. y = mania, b) means that y will take either the value a or b, whichever is the larger. 7.8 REFERENCES BOWER, J.L. and P.M. SCHULTHEISS. 1958. Introduction to the Design of Servomechanisms. New York: Wiley. 1978. Linear Control System Analysis and Design. New York: McGraw-Hill. ERZBERGER, H. 1968. On the use of algebraic methods in the analysis and design of model-following control systems. NASA TN-D4663. July. FULLER, A.T. 1967. The replacement of saturation constraints with energy constraints in control optimization theory. Int. J. Control. 6(3): 201-27. KUO, B.C. 1982. Automatic Control Systems. Englewood Cliffs, NJ: Prentice Hall. MARKLAND, C.A. 1970. 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On pole assignment in multi-input, controllable linear systems. Trans ZEEE. AC-12: 660-5. TAKAHASHI, Y., M.J. RABINS and D.M. AUSLANDER. Control System Design Methods II 8.1 INTRODUCTION Whenever a well set of specifications are imposed upon the dynamic behaviour of an aircraft, and when those specifications cannot be met, an AFCS problem exists. If the required dynamic performance has to be achieved then an additional equipment must be used in conjunction with the basic aircraft, in all but the most trivial cases. Those conventional control techniques, outlined in Chapter 7, all depend upon an interpretation of the system's dynamic response, in terms of such parameters as settling time, frequency of oscillation of the transient, value of the peak overshoot, time-to-half amplitude, gain margin, phase margin, and so on. Inevitably, the design which results from using such methods is obtained as a consequence of some compromise, and it may not be optimal. To use the theory of optimal control, a specified performance criterion is set exactly and the corresponding control design is unique. How this unique solution may be found, and how the performance criterion can be chosen to reflect the handling and flying qualities criteria, are the subjects of this chapter. THE MEANING OF OPTIMAL CONTROL 8.2 An optimal control system is one which provides the best possible performance from its class when it responds to some particular input. To judge whether the system's performance is optimal requires some means by which the quality of the performance can be measured, and for this it is customary to adopt an integral of the form: J = ∫ f(x, u, t)dt ... If a system is normally considered to have been optimized if some control input, u(t), has been found such that the value of J is least over the period from to, when the response is considered to have started, to T, when the response ceases. In some cases J may be chosen so that it can be maximized over the chosen period: for example, this could be achieved by choosing the cost functional, Y( ), in eq. (8.1) to be negative. Equation (8.1) is not the only type of performance index of use in aeronautical studies, of course, but it is the most common. Y(x, u, t) is 223 Meaning of Optimal Control known as the cost, or pay-off, functional: it represents the cost of a system's having been at a particular point in the state space, corresponding to the particular control inputs, for the entire period of time (T - to). Posing an optimal control problem in this way has considerable merit, because it includes in its statement most of the important problems relating to flight control, namely: stability, the dynamic response of the closed loop system, and the determination of the required control law.' One of the chief problems in setting up an optimal control problem is the formulation of the performance index. The significance of performance indices has been lucidly explained by Fuller (1959). His approach is used in Section 7.4 of Chapter 7. When an optimal problem involves control limits, such as limits on the deflection of a control surface, or a limit on the rate of change of the position of a control surface actuator, it is often convenient to replace, say, the hard constraint, |SE| ≤ for example, by an integral constraint on the energy expended in the control activity. The equivalent constraint can be expressed as ... 8.3 THE STATE REGULATOR PROBLEM ... If the error vector, e, is defined as the difference between the actual state vector and the commanded state vector ... If the aim is to minimize the error ... it is necessary to place different weighting on each of the elements of the error vector ... A different value of weighting is required on each of the elements of the error vector, a square matrix, denoted here as Q, is used, i.e. ... If the error vector, e, is defined with respect to some operating point which can be chosen as the origin, i.e. ... the quadratic form ... e'Qe ... but it must be noted that Q must be positive semi-definite. ... Since the state, x, is associated with the control vector u, it is necessary to add a further term ... u'Gu ... G is positive definite ... the performance index is ... 8.4 THE LINEAR QUADRATIC PROBLEM ... If the control law is taken to be ... the algebraic Riccati equation ... The optimal closed loop system is ... Linear Quadratic Problem ... Example 8.1 ... Example 8.2 ... The Riccati equation ... 8.5 ... Controllability and Observability ... 8.6 The Time-Varying Riccati Equation ... 8.7 Output Regulator ... 8.8 Prescribed Degree of Stability ... 8.9 Model-Following ... Explicit model following ... Implicit model following ... 8.10 State Reconstruction ... 8.10.1 Introduction It is often the case that the state vector of the aircraft cannot be measured entirely: some state variables are difficult, if not impossible, to measure; others are not worth the expense of measurement. What can be done, however, is to reconstruct the state vector from measurements of the available output variables ... 8.10.2 Reconstruction Using the Output Equation ... If rank of C is n ... 8.10.3 Observers ... Luenberger (1966) ... a dynamic system, called an observer, whose input is the output of the aircraft and whose output is an estimate of the state vector ... 8.10.4 Full Order Observer ... 8.10.5 The State Reconstruction problem ... The block diagram representing the observed aircraft is given in Figure 8.8. i Figure 8.8 Observer system. 8.10.5 Optimal Observer Suppose that the dynamics of some aircraft are defined by the state and output eqs (8.238) and (8.239): k =Ax y = Cx + Bu (8.238) (8.239) It is intended to design an observer to provide an estimated state vector xE which will be close to the optimal state vector x, but requires as its inputs only the control vector u and another vector w which is related to the output vector y of the aircraft, i.e.: State Reconstruction 259 The forcing vector, w, is chosen to be: AK (-~ YE) where: YE . A CXE fiE= (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: rïE = (F - KC)xE + S - (A - KC)X By choosing the coefficient matrix F of the observer to be identical to that of the aircraft, namely: FAA (8.248) and by defining any difference between the estimated and actual state vector as an error vector, e, it can easily be shown that: Provided that X(A - KC) < 0, then as t -+ a,the error vector e will tend to zero and the observer's vector XE will correspond to the state vector x of the aircraft. To secure this desirable condition requires only that K be determined. As a first step, let K be chosen to be a stabilizing matrix. Imagine that the observer dynamics are defined by: Letting G 4 B (as before) results in: If F is chosen to be (A - KC), and: eAx-XE then: C = (A - KC)e = Fe 260 Control System Design Methods I1 Suppose that we have a system defined by: C = Me +Nv then if we choose as a performance index: then minimizing eq. (8.255) subject to eq. (8.254) will result in a control law: v = He (8.256) Hence: C = (M + NH)e If it can be arranged that: X(A - K C) = X(M + NH) (8.258) then the optimal closed loop observer will be the required observer provided that: M ' = A , N' = C and H' = - K (8.259) A block diagram representing the optimal closed-loop observer is shown in Figure 8.9. Figure 8.9 Optimal observer. Example 8.7 For the aircraft DELTA, at flight condition 2, the equation of motion representing the aircraft's dynamics, including the flexibility effects, is given by % = A x + Bu where X' and A [W Q iL iZ i3 i4 is hl 44 4 As h6 6A ijE 6E) State Reconstruction SE, and ?iE denote the respective deflections of the inboard and outboard sections of'the eleqator. X i represents the displacement of the ith bending modes. The corresponding matrix A is shown in Figure 8.10; matrices B and C are shown in Figure 8.11. Note that C results because the output is assumed to be solely the vertical velocity w. It is from this solitary measurement that the state vector is to be reconstructed. The resulting optimal gain matrix for the observer is : Figure 8.10 Coefficient matrix, A, for DELTA-2 (with flexibility effects). C = ~ 0 0 0 Figure 8.1 1 0 0 0 0 0 0 0 = = 0 0 0 01 0 i) and G of: diag[5 5 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 0.01 10 10 101 G 0 Driving matrix, B, for flexible DELTA-2. This corresponds to the choice of weighting matrices Q 0 2.0 Control System Design Methods 11 in the performance index: where e A x - XE and v = Ke. Some results of applying an optimal control law to the aircraft, assuming that every state variable is available for measurement, are shown in Figure 8.12. Also shown there are the results of applying the same control law, but with the state vector having been entirely reconstructed in the observer from the solitary, continuous measurement of w. 8.10.6 The Kalman-Bucy Filter In situations where noise contaminated measurements must be used (where for example, radio or radar receivers are used as sensors) then a Kalman-Bucy filter Optimal control with complete state vector Optimal control with reconstructed state vector Figure 8.12 Response of optimally controlled DELTA-2to initial angle of attack. 263 State Reconstruction may be used to reconstruct the state vector from the noisy output signals. Although its use results in optimal rejection of the noise signals which corrupt the measurement, it requires a dynamic system of the same order (usually) as the aircraft dynamics. The aircraft's linearized motion is assumed to be affected by the control surface deflections, u, and atmospheric turbulence, w,. The output signals, y, are affected by sensor noise, n. Hence: where: where 6( ) is a unit impulse function. where .&{ } is the expectation operator. The solution is obtained when a control u(t) has been found to minimize the performance index: + J = E {xl(T)Sx(T) + i: [xl(t)Q(t)x(t) + ul(t)G(t)u(t)]dt) (8.266) The separation theorem proposed by Lee (1964) allows the optimal control law to be found first. It can be shown to be: where P(t) is the time-varying solution of the matrix Riccati equation: ~ (t =) P (~) A+ A ' P (~)+ Q - P(~)BG-'B'P(~) (8.268) where: Equation (8.267) depends upon the best estimate of x which is obtained from the Kalman-Bucy filter defined by: The notation B(tlt) means the estimate of x(t) based upon measurements up to and including y(t). ~ (tis)the gain matrix of the Kalman-Bucy filter and is given by: where the error covariance, W(tlt), is obtained from: 264 w(&) = A W(t!t)(t)+ W(t/t)A' Control System Design Methods I/ + ES(t)E1- W(t/t)C1T-'(t)CW(t/t) (8.278) with: Example 8.8 An integrated flight control system has a height hold mode which can be represented by the transfer function: i.e. wph = 0.055 rad s-I and = 0.055. is composed of hcomm, which is The commanded height signal, h, 0 taken to be a random variable, with normal probability distribution and a mean value of 10 OOOft, and a variance of 2.5 X lo5ft2, plus Shcommof white noise. Thus ham, = hcommo+ ahcorn,. The statistical characteristics of the noise in the command channel are defined by: NCommis defined as 400 ft2 s. Height is measured continuously, but these measurements contain white noise, i.e.: where S{Sm> = 0 S{sm(t)sm(v>)= Nma(t - v) Nm is taken as 900ft2 S. The requirement is to design a system which will provide an estimate of height with the least possible variance. The height hold system can be represented in state variable form as: Let hcOmmo x3. Then, since h, representation becomes: = hcommo+ ShComm,the complete state State Reconstruction and: hm A y = [0.003 0.3 O]x + 6, then: + W C ' T - ' ~ -~ C X] ir = A x where W is the solution from: 0 = AW + W A ' + ESE' - WC'T-'CW From the problem statement: S = Ncomm= 400 T = Nm = 900 Now: I wll wl2 w13 w= ' wl2 w22 w23 wl3 w23 w33 WC' = Hence: I + o.3Wzl o.oo3w12 + O.3w2, 0.003 W13+ 0.3 Wz3 I o.oo3wll I 266 h Control System Design Methods /I = 0.003i1 + 0.3i2 The coefficients Wii are obtained from a solution of the ARE eq. (A). Steady state starting conditions are usually assumed, such as: 3.3 x lo6 8.11 CONCLUSIONS This chapter introduces the important topics of linear optimal control, controllability and observability, which are very important properties of the mathematical models of the aircraft upon which the designs of effective $FCSs are based. In such advanced systems as AFCSs with analytical redundancy (a topic which is not covered in this book) these subjects are of considerable importance and need to be thoroughly understood by the control system designer. The solution of the linear quadratic optimal problem by means of the algebraic Riccati equation (ARE) is presented, with particular reference to effective methods of obtaining the required feedback control law. Based upon this work, methods of designing an optimal output regulator, or a system with a prescribed degree of stability, or one which explicitly follows a model response are also presented. In all the methods, the result depends upon solving an ARE. Furthermore, an optimal command control system was presented, which is also based on the work of the LQP. The use of a dynamic feedback controller is also dealt with, before concluding the chapter with a study of a few techniques for satisfactorily reconstructing a complete state vector from measurements of a few, or even a single, output variable. 8.12 EXERCISES 8.1 For a system defined by the state and output equations S y = Cx D u the following matrices apply: + 0 1 0 l = [1 0 11 -20-2 B' = [- 1 1 - = Ax 11 C = [- 1 2 01 D = [O] Is the system controllable and observable? 8.2 A system has two components represented by the transfer functions: + Bu and The components can be connected in any one of three possible ways: 1. Cascade: Gl(s) G ~ (s) 2. Parallel: Gl(s) 3. In a closed loop configuration: + G~(s) Discuss the controllability and observability for each connection. 8.3 Write down an appropriate state equation and the corresponding output equation for the lateral acceleration at the pilot's station for the aircraft ALPHA-4.Is the aircraft completely controllable? Is it completely observable? What effect would losing rudder action have on the controllability? 8.4 Write down the state equation corresponding to the lateral motion of DELTA-1.If a performance index, J , is chosen to be: Establish: (a) Whether the feedback control law obtained as a solution to this linear quadratic problem can stabilize the aircraft. (b) The gains of the optimal feedback control law. (c) The eigenvalues of the closed loop flight control system. 8.5 The executive jet, ALPHA,is cruising at an altitude of 6100 m and a forward speed of 237 m s-l. The aircraft is controlled by means of its elevator and by changing its thrust. For the longitudinal motion the output vector has as its elements the pitch rate and pitch attitude, i.e. y' [q 01 (a) Compute the steady state response to a unit step change in thrust. (b) For Q = diag[l 1 11 and G = diag[l 1001 find the optimal feedback gain matrix. 8.6 For the aircraft detailed in Exercise 2.5 determine a feedback control law which will result in the acceleration response of the controlled aircraft being identical to that obtained from an aircraft which has been idealized and modelled by the equation: a - - - 10ay,o, + 10sa + s* Show that the feedback control law found does indeed provide model matching. 8.7 The aircraft BRAVO-3 is represented by the following state equation H = Ax where: x' 4 [U w q 01 u' 4 [&,I (a) Using weighting matrices: + Bu Control System Design Methods I1 Q = diag[l 1 1 11 G = [1] determine the optimal feedback control law. (b) Find the eigenvalues of the optimal closed loop system and compare them with those of the uncontrolled aircraft. (c) Calculate the response of the optimal flight control system for a period of 15.0 s to an initial angle of attack of 1". 8.8 A pitch rate damper is represented by the block diagram of Figure 8.13. It is of 10" s-I there is a steady observed that in response to a step command of q,,,, state error of 8.88". Controller 0.3 Actuator 6 ~ (s) c 10 st10 Aircraft dynamics 6~(s) Figure 8.13 Pitch rate damping system for Exercise 8.8. Use the method outlined in Section 8.9 to obtain an optimal control law which will both minimize the performance index: and will result in there being zero steady state error in the optimal closed loop flight control system. 8.9 The aircraft FOXTROT-3,using only its elevator for control, has an optimal pitch control system for which the feedback gain matrix, K, is given by: K = [0.0184 - 0.0855 - 2.905 - 14.0351 The actuator dynamics have been ignored. It is found, however, that only the pitch rate and pitch attitude can be measured on the aircraft. (a) Show how the motion variables u and w may be reconstructed if the elevator deflection can be measured also. (b) Draw a block diagram of this complete flight control system. Include all the gains involved in your scheme. 8.13 NOTES 1. The pay-off functional is introduced as a postulate. The reader may like to be reminded of the view of the postulating method, expressed by the philosopher Bertrand Russell: it has a great many advantages, all of which coincide exactly with those of theft in comparison with honest labour. References 2. 3. 4. 8.14 269 A necessary condition that a matrix Z be positive definite is det Z > 0; hence Z is non-singular. This restriction is required only when the upper limit of the integral is m . If it is finite, then the pair { A , B) does not need to be completely controllable. If rank of C is p then C f = [C1(CC')-'1. If rank of C is m then c?= [(c l c) - l C 1] . REFERENCES ANDERSON, B.D.O. and J.B. MOORE. 1971. Linear Optimal Control. 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What Northrop called the kind of AFCS it proposed to use to remedy the poor, inherent flying qualities of its YB-49, a flying wing bomber aircraft, was a 'stability derivative augmentor'. However, on the standard form for installation drawings, the title block was insufficiently wide, so that the name was reduced by a draughtsman to 'stability augmentor' to fit the available space (McRuer and Graham, 1981). All similar systems have been called stability augmentation systems (SASs) ever since, although their purpose remains as it was originally: the values of a number of specific stability derivatives of an aircraft are to be increased by means of negative feedback control. Although all stability derivatives could be so altered, only a few are usable candidates, since it is only by means of their alteration that any required change in the flying qualities of an aircraft can be effected. In general, SASs are concerned with the control of a single mode of an aircraft's motion. The general structure of such an SAS is shown in the block diagram of Figure 9.1, in which it can be seen there are four principal elements: aircraft dynamics, actuator dynamics, sensor dynamics and flight controller. These elements are essential and are always present in any SAS. When the SAS is switched off, the aircraft can be controlled directly by the pilot moving the appropriate control surface(s) through his cockpit controls. The flight controller, of course, is not then active. When the SAS is switched on, the control surface is driven by its actuator which is controlled by the flight controller. In SAS studies, command inputs are usually considered only secondarily; the flying qualities of the aircraft are enhanced by the control action of the feedback control system in such a manner that the effects of atmospheric, or other disturbances upon the aircraft's motion are suppressed. Sensor noise also affects the quality of control. However, many types of SAS cannot be switched off by a pilot, but always remain active, from the moment the master electrical switch is on. In the event of any failure of the SAS, the aircraft has then to be controlled solely by means of inputs from the pilot's cockpit controls. If an aircraft has a Actuator Dynamics -..-.. Pilots stick motion motion SAS oOn command Y c o ~ ~ Actuator dynamics Flight controller Atmospheric disturbances , Aircraft dynamics Aircraft motion variable, y Control surface deflection from guidance system Motion feedback voltage Figure 9.1 Stability augmentation system. direct link between some particular cockpit control and its corresponding control surface, then the aircraft is said to possess, in the event of an AFCS failure, the property of manual reversion. In some cases, the forces which need to be supplied by the pilot are beyond the limits of human performance. In some modern aircraft, particularly CCVs, which depend upon active control technology for their successful operation, no such manual reversion is provided, reliance being placed upon some form of redundancy in the AFCS to ensure continuous operation of the SAS. Sometimes when SASS admit pilots' or guidance systems' command signals, the system may be referred to as a command and stability augmentation system (CSAS). The principal SAS functions which are found on modern aircraft are: pitch rate SAS, yaw damper, roll rate damper, and relaxed static stability SAS. Essentially, certain desirable values of the non-dimensional stability Cn , Cn , and Cl , can be obtained more effectively derivatives, C, , C, , C n~' P using automatic"cont$ol than b i phgsical sizing of aerodynamic surfaces. 9.2 ACTUATOR DYNAMICS Actuators used in combat and transport aircraft are generally ele~troh~draulic.~ General aviation aircraft sometimes use electric actuators. Such actuator systems have their own dynamic characteristics which affect the performance of the closed loop SAS. In Appendix A , some information on the nature of the dynamic characteristics of such actuators is presented. When the design of an SAS (or any AFCS mode) is initially being considered it may be assumed, as a first 272 Stability Augmentation Systems approximation, that the dynamic response of the actuator, in comparison with that of the mode of flight of,the aircraft which is being controlled, is so rapid that it can be regarded as instantaneous. Making that assumption means that the actuator dynamics can be considered to be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually degrees per volt. If the aircraft is large, then it is known that the actuator must provide large hinge moments for control. It is improbable in such a case that the response can then be instantaneous and, consequently, the transfer function usually assumed in this situation is: S(s)/S,(s) = KXI(s + X) (9.2) Typically, X lies in the range 5-10.0s-l; 1 is the inverse time constant of the actuator. Even when it is known that the SAS design is likely to be much affected by the nature of the actuator's performance, it is customary to proceed with the design on the basis of either representation eqs (9.1) or (9.2) and then to simulate the final design, including the complete known description of the dynamic characteristics of the actuator, noting any loss of SAS performance as a result of including the more representative actuator dynamics. If a significant change is noted then some adjustment in the control law is normally tried in order to minimize the loss of dynamic performance. It is more important to realize that this final test must always be made, particularly to confirm that the existence of the higher order terms in the actuator dynamics do not cause instability in the SAS. Another feature of the actuator dynamics, which can sometimes have very grave effects on the SAS performance, is the existence of non-linearities. Such effects should be accounted for in the simulation study. There is a semantic difference which occurs between British and American usage; in British work, the actuator is usually taken to mean the device which converts electrical signals to mechanical signals, of low power, but of sufficient power to drive the hydraulic valve which controls the flow of hydraulic fluid to the powered flying control, which deflects the control surface. In American work, 'actuator' covers the whole system from the command voltage, S,, to the control surface deflection, 6. This is the usage followed in this book. When an electric actuator is used, the representation of eq. (9.2) is normally employed; the values of K and X will depend upon the actuator's characterization. It should also be remembered that the hinge moment is not a linear function; large deflections cause considerable loading of the actuator which affects the dynamic performance. For flight critical conditions, such effects should be studied by means of the simulation of the system, before committing to a final design. Finally, it must be remembered that no AFCS is generally allowed to Longitudinal Control 273 make use of the full range deflection of the control surfaces: it is usually limited to deflections +- 10 per cent of the trimmed value (although even this limited range of deflection is not allowed at the limits of the control surface deflection). It is then said that the SAS has 10 per cent control authority. Such limits on control authority were set by aviation authorities to ensure the safety of the aircraft in the event of AFCS failures which cause the actuator to be driven 'hardover'. The unwanted acceleration about the c.g. of the aircraft which could result was, therefore, strictly limited. This safety measure is inimical of good manoeuvring performance, however, and on military combat aircraft and CCVs considerably greater authority - sometimes 100 per cent - is now allowed, and is, in a few cases, essential. 9.3 SENSOR DYNAMICS Every sensor used in an AFCS is a transducer. In modern aircraft, its purpose is to measure motion variables and to produce output voltages or currents which correspond to these motion variables. Some of the electronic sensors, such as radar altimeters, or radar, process the information so quickly in comparison with the aircraft's response that it is customary to regard their transducing action as instantaneous. Often, however, such sensors have in-built filters to improve their noise characteristics, and the time constant of such filters is often considered as representing that of the sensor. The inertial instruments, such as gyroscopes and accelerometers, do have well defined dynamics characteristics (see Appendix A), but the sensors employed in AFCS are chosen to have bandwidths and damping such that they can be considered to be instantaneous in their action. A sensor is frequently represented, then, by its sensitivity, i.e. : K, can have units, such as V rad-' for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an accelerometer (more commonly: Vg-l). If the problem is concerned with an elastic aircraft, in which structural bending is significant, then sensor dynamics can be significant and the full representation should be used. For SASs, the most common sensors are gyroscopes and accelerometers. What usually affects the performance of the SAS more strongly than a sensor's dynamics is its location on the fuselage. 9.4 LONGITUDINAL CONTROL (Use of Elevator Only) 9.4.1 Introduction In Section 2.8 of Chapter 2 the state and output equations used with aircraft dynamics are presented. In SASs, the controller output is the command voltage to 274 Stability Augmentation Systems the control surface actuator which provides the appropriate deflection. If the actuator dynamics are to be represented by, say, eq. (9.1) no change to the form of the state equation is required but, if eq. (9.2) is used to represent the actuator dynamics, there is an additional differential equation to be accounted for, namely: sj = - ASj + KAS,,I (9.4) where S j is, of course, uj, one of the control surface deflections. This additional equation is usually made to augment the state equation by choosing x, + to be uj. For example, consider the state equation for longitudinal motion with a single control input, SE - i.e. eq. (2.108). Let: then: kg = - AX^ + KXU hence: 0 0 B= 0 0 - KX, When the control u depends solely upon time, we speak of it as being the control function; when it depends upon the motion (or other) variables of the aircraft, we refer to it as the control law, namely: U = f(~) (9.13) The control law of eq. (9.13) means that the control is based on output feedback; whether the control law is linear or non-linear depends upon the nature of the functional, f( ). The customary forms of feedback control for AFCSs, and hence an SAS, are linear, i.e. the control takes the form: 275 Longitudinal Control When: then full state variable feedback is involved. If: applying a control law such as eq. (9.14) to the aircraft dynamics represented by eq. (2.108) results in the controlled aircraft, the closed loop dynamics taking the form: Obviously, use of the control law has resulted in a change in the dynamic response of the controlled aircraft. From experiment and practice, it has emerged that only a limited number of forms of linear control law are effective for stability augmentation. This number includes the following: Other control laws, involving such motion variables as pitch attitude, 0, change in forward speed u, height h , and flight path angle y, are dealt with in Chapter 10. It should be noted here that control designs involving full state variable feedback are a mixture of SAS and attitude control; they are dealt with, therefore, in the next chapter. There are two methods excepted: pole placement and modelfollowing, for, even though the resulting control law in each case is one involving full state variable feedback (FSVF), the design intention is to improve the basic stability of the aircraft dynamics. It should be noted also that the control law of eq. (9.19) involves the feedback of more than a single motion variable: = a, cg w- Uoq + Z,W + ZSESE SE = K, Z,U + K, Z,W + K, ZGESE S, = (1 - K, Z ~ ~) - ' KZ,U , + K, Z,w(l (9.21) = Z,U .'. :. (9.22) (9.23) z -K,~Z~~)-~ (9.24) Of course, if the normal acceleration is not measured at the aircraft's c.g. but at some other station, XA, then: 276 Stability Augmentation Systems = (z,- xAMU)u+ (Z, - XAM,)W - xAMqq + (Zs, - *AMaE)sE (9.25) Obviously, from eq. (9.25) the choice of location from the sensor measuring normal acceleration can have a profound influence on the control law, eq. (9.19), and hence upon the stability of the controlled aircraft. 9.4.2 Pitch Rate SAS The stability derivative which such systems try to augment is M,, and, thereby, the damping ratio of the short period motion is increased. The block diagram of a typical, conventional pitch rate SAS is shown in Figure 9.2. The feedback signal is obtained from the rate gyro used to measure pitch rate, q. Since there is a sign change present inherently in the aircraft dynamics associated with the relationship of pitch rate to elevator deflection, the feedback signal is added to the A number of books and papers show a sign change command signal, q,,,. between the signal representing commanded elevator deflection, SEc, and the actual angular displacement of the elevator, SE. In such cases, the feedback voltage, vf, is shown to be subtracted from the command signal. Controller Actuator Aircraft dynamics Rate gyro - - K vf(4 4 * K,,,,, = 0.01745 rad V-' (Io V-') K, = 5.73 V rad-' s-' Figure 9.2 Pitch rate stability augmentation system. From Appendix A, it can be seen that a typical value for the sensitivity, K,,, of a rate gyro is 100 mV degree-' s-' (5.73 V rad-' s-l). The problem is solved when some suitable choice of Kc,,, is made to cause the damping of the short period motion to be increased. The usual assumption involved (but it remains no more than an assumption) is that a control system, typified by Figure 9.2, affects only the short period motion of the aircraft. Its phugoid motion is assumed to be unaffected by the control and its is also assumed that the phugoid motion does not affect the operation of the SAS. As a consequence, only the short period approximation needs to be used to represent the aircraft dynamics. The transfer function shown in the block representing aircraft dynamics in Figure 9.2 corresponds to aircraft FOXTROT at flight condition 3. The actuator dynamics a value of are assumed to be represented by a fixed gain (servo gearing), K,,,: 1" V-' has been assumed. It can be deduced from the short period dynamics that Longitudinal Control - 15 - 10 -5 -- 1 0 w, ?? m 0- - -5 - -10 -5.0 0.0 (a) I I I I I I I 0.5 1.0 1.5 2.0 2.5 3.0 3.5 I 4.0 I 4.5 5.0 Time (s) Time (s) Figure 9.3 (a) Response of FOXTROT-3 to a (0)= 1'. (b) Pitch rate response of FOXTROT-3 (vertical scale expanded). the damping ratio of the uncontrolled aircraft's motion is 0.16 and its frequency is 5.34 rad s-l. The response of the uncontrolled aircraft to an initial disturbance in its angle of attack of lo is shown in Figure 9.3. Note how the phugoid mode is evident chiefly in the speed response. The aircraft's rating is shown in the handling qualities diagram of Figure 9.4; on the same diagram the desired handling qualities point is shown as point Z. This point corresponds to a need to Stability Augmentation Systems Figure 9.4 Handling qualities diagram. increase the damping ratio to 0.6 and to ensure that the short period frequency is not less than 6.0 rad s-l. These increases must be achieved by an appropriate choice of Kcon, (which effectively results in Mq being augmented). The control law of the SAS is, therefore: A root locus diagram corresponding to the system represented by Figure 9.2 is shown in Figure 9.5. It can be calculated from that diagram that the required system gain K, is + 0.36. Hence: Figure 9.5 Root locus diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram corresponding to the system is shown in Figure 9.6(a); the gain margin is infinite. It should be appreciated that for the aircraft dynamics there is a 180" shift of phase introduced at all frequencies, corresponding to the negative sign in the numerator of the transfer function. Consequently, the change in phase angle due to the dynamic terms is about + 130" to - 90" over the range of frequencies. Figure 9.6(b) shows the Nichols diagram corresponding to Figure 9.6(a). The gain margin, being infinite, means that any value of Kcon,will obtain, w is in rad s-' 0.01 1 -300 (b) I -250 I I I -200 -150 -100 Phase angle (degrees) I -50 Figure 9.6 (a) Bode diagram for FOXTROT-3. (b) Nichols diagram for FOXTROT-3. 280 Stability Augmentation Systems from the standpoint of the stability of the closed loop system. However, to satisfy the handling qualities specification (point Z) only a single value will suit. Any of the other methods of Chapter 8 can be used, but these methods require FSVF for their synthesis. Hence, the basic uncontrolled system is better represented as in Figure 9.7, and the problem is merely to determine the feedback gain matrix, K, in the control law: Actuator Figure 9.7 Aircraft dynamics Open loop control system. For example, using the pole placement technique of Section 7.3 of Chapter 7, if the desired closed loop poles are chosen to be: phugoid: XI, h2 = - 0.004 + j0.04 short period: h3, h4 = - 5.0+ j8.0 it is easy to determine the required matrix of feedback gains, namely: From an examination of the relative magnitude of the elements of K i t is tempting to consider that the feedback control law can be represented by: as before, but now K, has the value of 0.3824. That this approximation is inappropriate can be seen from considering the closed loop poles, defined in eq. (9.32), which are achieved when the control law of eq. (9.31) is used, instead of eq. (9.30), which produces the desired closed loop poles of eq. (9.29). phugoid: XI, h2 = - 0.0075 f j0.04 short period: h3, h4 = - 4.9 f j3.3 If, however, eq. (9.30) is approximated to: the closed loop poles corresponding to the short period mode are not greatly affected being: phugoid: hl, h2 = - 0.007 + j0.025 short period: h3, X4 = - 5.0 f j8.1537 Equation (9.33) should be compared with the desired closed loop poles of eq. (9.29). 28 1 Longitudinal Control The response to an initial angle of attack of the controlled aircraft, using the control law defined in eqs (9.28) and (9.30), is shown by the dashed line in Figure 9.8; the solid line represents the same response for the control law defined by eqs (9.28) and (9.31). I1 -0.08 Y 0 I 1 I I I 2 I I I 3 I I 4 I I I 5 Time (s) Figure 9.8 Pitch rate response for pole placement system. Using the LQP solution is relatively straightforward but depends, of course, upon the choice of weighting matrices, Q and G. For the choice of eqs (9.34) and (9.33, namely: Q = diag[O.Ol 0.01 0.5 0.21 (9.34) the resulting feedback matrix, obtained from solving the ARE, is: K = [- 0.056' 0.046 2.4 18.1441 (9.36) Using the optimal control law results in the controlled aircraft having roots of: phugoid: XI, h2 = - 0.0934 + j0.0915 short period: X3, X4 = - 28.134 + j27.556 The closed loop response to an initial change of angle of attack of lois shown in Figure 9.9. Note how large changes of pitch rate have been penalized: the peak value at 0.2 s is only - 0.013" compared to a peak value of - 0.07 at 0.3 s for the uncontrolled aircraft. However, the long, drooping response, which has not settled by 5.0 s and which has arisen because of the dominant effect of the pitch attitude feedback, can only be reduced by penalizing the use of the elevator less heavily and allowing greater peak values of q . Other choices of Q and G matrices are needed. The dotted curve in Figure 9.9 shows the closed loop response which obtained for an arbitrary choice of weighting matrices: Stability Augmentation Systems Time (s) Figure 9.9 LOP response for FOXTROT-3. The corresponding feedback gain matrix, K, was: 9.4.3 Phase Advance Compensation One common form of an SAS for pitch rate is to use a dynamic control law defined by: Either form (a) or (b) of Figure 9.10 may be used: their characteristic equations will be identical. Consequently, they have identical responses to initial conditions and to atmospheric turbulence. The SAS function is identical whichever structure is adopted. However, the command function will be affected. Figure 9.10(b) is preferred whenever the aircraft has manual reversion in the event of any SAS failure: it permits a direct input, p,, from the primary flight control. For the aircraft FOXTROT at flight conditions 3 (see Appendix B) the corresponding Bode diagram is shown as Figure 9.11, from which it can be seen that the gain and phase margins are both infinite. A phase advance network can safely be introduced; its transfer function is chosen to be: G,(s) = (1 + s)/(l + s0.1) (9.42) Longitudinal Control Compensation network Elevator actuator Aircraft dynamics - Rate gyro - Kq AFCS operational Actuator Compensation network Figure 9.10 Rate gyro Pitch rate SAS. (a) Series Compensation. (b) Compensation in feedback. Figure 9.11 Bode plot of FOXTROT-3. 284 Stability Augmentation Systems When Kq is chosen to be 0.5 the modified Bode diagram that is also shown in Figure 9.11 applies. The dynamic response of the closed-loop system to an initial change of angle of attack of + lo is shown in Figure 9.12. The use of this phase advance network has added damping to the short-period response, but has reduced the frequency. Generally, phase advance networks tend to make the closed loop system perform less well in the presence of sensor noise. It has been shown in Section 7.2 of Chapter 7 how dynamic feedback controllers may be represented in state variable form. The procedure can be used with the phase advance compensation scheme: Then: Let: From eq. (9.45): Time (s) Figure 9.12 Pitch rate response with phase advance compensation. Longitudinal Control 285 Let: x A x5 (Note q = x3) then: . SB = Kfi where: K = Afi + Bs, 9.4.4 Additional Feedback Terms Sometimes, to achieve required handling qualities, an additional feedback term based upon the normal acceleration measured at the c.g. is included in the feedback control law used in pitch SAS thus: (9.57) 8~ = Kqq + Kazazcg From the point of view of modern control theory, eq. (9.57) is no more than another expression of a full state variable feedback control law. From the point of Stability Augmentation Systems - Controller and actuator 4 6~(s) Y (4 Aircraft dynamics + s J~F- I [a (4 zcg Rate gyro Accelerometer Figure 9.13 Pitch rate and acceleration feedback SAS. view of a flight control engineer, eq. (9.57) is the practical alternative, requiring only pitch rate and normal acceleration to be measured. Both variables are relatively straightforward to measure using rate gyros and accelerometers located at the aircraft's c.g. The block diagram of a SAS, using the control law of eq. (9.57), is represented in Figure 9.13, with its corresponding dynamic response shown in Figure 9.14. This response should be compared with that of the uncontrolled aircraft which is shown in Figure 9.3(b). The closed loop responses shown in Figures 9.8, 9.9 and 9.12 should also be inspected for comparison purposes. Acceleration feedback is generally considered to 'stiffen' the system, i.e. the short period frequency is invariably increased. This can be appreciated easily by considering FOXTROT-3 controlled by the law of eq. (9.57), where: -0.01 10 I I I I I 2 I I 3 ' I 4 I I 5 Time (s) Figure 9.14 Response of blended control to a(O)= lo. Longitudinal Control r 7 e - L' f - "p, j i ~ l i ~condition ht 2 I F ~ l i g hcondition t 3 Of -0.035 light condition 4 -!! -0.040[I 0 I I I I 2 I I 3 I I I 4 I 5 Time (s) Figure 9.15 Response of pitch rate SAS for four flight conditions. The corresponding eigenvalues of the controlled system are: phugoid: Xl, X2 = -0.0066 f j0.005 short period: h3, X4 = - 26.4 f j43.55 In American papers, a control law such as eq. (9.57) is often referred to as 'blended feedback control'; such a control law is usually arranged to give a possible, invariant flying qualities throughout the flight envelope of the aircraft. The closeness with which this ideal is approached depends upon the ratio of the feedback gains, K, and K,. Usually, at low dynamic pressures (i.e. 1 1 2 = ~) being not very large) the controlled aircraft is arranged to behave as if it were a pure pitch rate SAS; at high dynamic pressures, the system behaves more noticeably as a normal acceleration control system. Such blended feedback systems can 'mask' the natural ability of an aircraft to provide a 'stall warning'. This occurs because the control system tries to maintain good flying qualities until close to the point of the aircraft's stalling. To provide an illustration of this point, Figure 9.15 shows the transient response of the aircraft FOXTROT for all four flight conditions using the same fixed pitch rate feedback control law devised for flight condition 3. The transient responses for the same four flight conditions of the same aircraft, for the fixed blended feedback control designed for flight condition 3, are shown in Figure 9.16. It can be seen how effectively the blended control law of eq. (9.57) has provided invariant response, and this would ensure that the aircraft's handling qualities would remain acceptable as the aircraft traversed the region confined in the flight envelope. The performance of such systems is greatly affected by the sensor locations; it has been supposed, for the present, that both sensors were located at the c.g. of the aircraft. The matter is considered anew in Section 9.12. Stability Augmentation Systems -Flight condition 1 --Flight condition 2 ----Flight condition 3 -.-.- Flight condition 4 Time (s) Figure 9.16 Blended control - pitch rate response for four flight conditions. The importance of pitch rate and normal acceleration stability augmentation systems cannot be overemphasized. ... 9.10 RELAXED STATIC STABILITY ... Stability Augmentation Systems ... Figure 9.17 ... Figure 9.18 Static stability ... BRAVO-2 ... BRAVO-3 ... Figure 9.19 ... Figure 9.20 ... Figure 9.21 ... 9.11 SENSOR EFFECTS ... Figure 9.22 ... Figure 9.23 ... 9.8 LATERAL CONTROL ... CHARLIE-4 ... Figure 9.24 ... Figure 9.25 ... Figure 9.26 ... Figure 9.27 ... Figure 9.28 ... Figure 9.29 ... Figure 9.30 ... Figure 9.31 ... Figure 9.32 ... Figure 9.33 ... Figure 9.34 ... 9.8.5 Spiral Mode Stabilization ... Figure 9.35 ... ar6itrarily large without causing the dutch roll motion to be unstable. If, however, a proverse yaw effect is evident, i.e. NkA is positive, the damping of the dutch roll motion is augmented by the spiral mode stabilization. Often spiral mode stabilization is obtained by means of a kind of 'piggy-back7operation involving the yaw damper: the feedback signal from the yaw rate gyro is also used to drive the ailerons. That technique is referred to as aileronlrudder interconnection (ARI) or control crossfeed (see Section 10.6.5). 9.9 CONCLUSIONS This chapter deals with stability augmentation systems which are closed loop control systems used on aircraft to remedy those deficiencies in flying quality which are due to basic aerodynamic or geometric inadequacies in the aircraft. Feedback control is used to augment some particular stability derivatives, thereby improving the parameters which directly govern the flying qualities. Both lateral and longitudinal motion systems have been considered and the most common types of SAS, such as pitch, roll and yaw dampers, are treated. A number of methods of designing such SASShave been discussed and the effects on the closed loop performance of actuator and sensor dynamics have also been dealt with. SAS are important since they invariably form the innermost loop of an integrated AFCS. Exercises 371 9.10 EXERCISES 9.1 The linearized equations of perturbed lateral motion for a Tristar (L-1011) passenger aircraft in a cruising flight condition are given by: fi = - 0.13P - r + 0.04+ + 0.02SR +=p Using appropriate approximations, design a yaw damper to increase the damping of the dutch roll mode from its uncontrolled value of 0.14 to a new value of 0.67. Calculate the natural frequency of the yaw damper. 9.2 The short period dynamics 'of a fighter aircraft are represented in the s-plane diagrams of Figure 9.36. Design an SAS (ignoring actuator dynamics) to obtain a closed loop damping ratio of 0.6. 9.3 A VTOL aircraft of the AV8B type has the following linearized equations of motion: + X8,SE + XsTST + XsNSN + (Uo + Z q) q + ZS,SE + ZsTST + ZsNSN + Mqq + Ms,SE + Ms,ST + MS,SN u = X,U - g 0 w = Z,W 4 = M,w where u , w , q and 0 represent the changes in forward velocity, vertical velocity, Figure 9.36 Pole zero map for Exercise 9.2. 3 72 Stability Augmentation Systems pitch rate and pitch attitude respectively. The control inputs are represented by the elevator deflection, SE, the change in thrust, ST, and the deflection of the reaction nozzles, SN. At hover, the corresponding stability derivatives are as follows: Xs,=-0.56 Zs,=-0.1 Ms,=O.O It is required that in hover the vertical velocity be controlled such that it has a characteristic equation of motion of the form w 5w = 0. (a) Design a feedback control system to achieve this requirement. (b) Sketch a block diagram of the resulting control scheme. (c) If the aircraft is equipped with sensors for pitch rate and angle of attack, discuss the consequences for the synthesis of the control law determined in part (a). + 9.4 A strike aircraft has the following linearized equations of lateral motion: p = - 0.lp - r + O . l + + 0.048, p is the sideslip angle, in radians p is the roll rate (rad s-l) r is the yaw rate (rad s-') + is the roll rate (rad) SA is the aileron deflection (rad) 8, is the rudder deflection (rad) Only the roll and yaw rates of the basic aircraft are to be directly controlled to improve the handling qualities. The desired handling qualities are assumed to be those obtained from the dynamics of some ideal aircraft which has the model equation: where Assume that the dynamics associated with the measurement of any motion 3 13 Exercises variable, and also those associated with the control surface actuators, are negligible. (a) Obtain a feedback control law which will provide the required handling qualities. (b) Show that this law results in perfect matching. (c) By means of a block diagram show how the control law of part (a) could be implemented. 9.5 The state vector of an oblique-winged research aircraft is defined as The aircraft has been provided with five controls such that ur = [6EL 6ER 8AL &AR 6 ~ 1 where aELdenotes left stabilizer deflection, SER right stabilizer deflection, 6AL left aileron deflection, 6 As right aileron deflection and 6R rudder deflection. For a particular flight condition the corresponding matrices A and B are: - 0.0075 0 A = B = 0.19 0 0 - 32.2 0 0 0 0 0 0-634.4 0 0 634.40 0 0 0 0 0-1.0 0 0 0 0 1.0 0 0 0 0 0 0 0 0 0 0 0.24 0.05 0 0 0.006 0 - 1.0 0 0 0 0 01.0 0 0 0 0 0 0 0 0 0 1.0 0 0 0 0 0 0 0 0 0 0 1.0 0 0 0 0 - 24.39 0 0 0 - 5.86 - 0.03 0.84 0 0 - 6.3 0 0 0 0 0.002 - 0.71 0 6.14 0 0 0 - 0.13 - 0.1 - 0.67 0 - 1.734 1.734 - 0.77 - 0.77 0 0 0 0 0 0 - 0.09 - 0.09 - 0.04 - 0.04 0 - 0.012 0.012 0 0 0.054 0 0 0 0 0 0 0 0 0 0 0 0 8.0 - 8.0 - 6.53 - 1.075 0.1 I 19.18 - 19.18 6.1 - 6.53 - 0.012 - 0.012 - 1.075 0.585 - 0.585 0 - 4.3 (a) Find the eigenvalues corresponding to the phugoid, the short period and the convergent modes of longitudinal motion, and also those corresponding to the heading, spiral convergence, roll subsidence and dutch roll modes of the 3 14 Stability Augmentation Systems aircraft's lateral motion. (b) Find an optimal feedback control law for the weighting matrices: Q = diag[l 10 0.1 0.1 10 10 1 1 1 11 (c) 9.6 Calculate the corresponding closed loop eigenvalues. For the oblique winged aircraft of Exercise 9.5 it is desired to have the closed loop dynamics match those characterized by the vector differential equation k = L x , where L is defined as: L = diag[-2.0 -0.5 -1.0 -4.0 -10.0 -10.0 -3.5 -3.5 -5.0 -5.01 (a) Find a feedback control law to achieve these model dynamics. (b) Calculate the eigenvalues of the resulting closed loop system. (c) How do these values compare with the eigenvalues of the model aircraft? 9.7 A hypothetical aircraft is considered to have the following matrices when it is flying at a height of 6 000 m and a Mach number of 0.8. The state vector is defined as: x' = [h Uoy w 0 q] and the control vector as: The output vector is defined as: Y =A X Suppose that the equations characterizing the required stability augmentation system are given by: Exercises The matrices A, and H' are given as: 0 1 0 0 0 - -9-6 A,= 0 0 0 0 0 -10 0 0 0 0 0 0 1 0 0 0 -9-6- Find a suitable feedback control law. 9.8 A prototype fighter aircraft has been built with a vertical fin of reduced size such that the directional stability derivative, Nb, becomes negative and the derivatives N i , N&, and Lb are reduced in amplitude. The change of sign of Nb makes the aircraft directionally unstable. (a) Show that for such an aircraft there must always be two unstable roots. (b) The prototype aircraft has been flight tested at sea level and at a Mach number of 0.4 and the following stability derivatives were determined: Y, = - 0.2 Lb = - 5.0 Li = - 2.5 L: = 3.0 Nh = - 2.03 N i = - 0.07 LL, = 20.0 LQ, = 1.4 N : = - 0.2 N&, = 0.3 N&, = - 3.0 Find the eigenvdues of the aircraft. Using the two degrees of freedom approximation evaluate the transfer s) , thence design a yaw damper to improve the dutch function, r (~) / 6 ~ (and roll mode. (Note that the stable roots of the corresponding stability quartic are both real). (d) Will the feedback control scheme devised in part (c) stabilize all the aircraft's modes? (e) Using any suitable method determine a feedback control law which will result in the controlled aircraft being stable. (f) Discuss in operational terms the benefits which relaxed static stability (see Chapter 12) is likely to bring. (c) 9.9 (a) A flight control system has been designed for the aircraft ALPHA-2to reduce the acceleration experienced as a result of encountering turbulence. It is necessary to provide as a feedback signal some measure of the angle of attack, but no suitable sensor is available. An accelerometer is used, but it has to be located 1.55 m ahead of the aircraft's c.g. Show that the measured acceleration is approximately equal to 3 . 2 ~ (b) If the accelerometer is placed as detailed in part (a) what is the measured value of the initial normal acceleration in response to a step deflection of the elevator? 316 Stability Augmentation Systems 9.11 NOTES 1. All-electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric actuators in their place. The aircraft will have no hydraulic supply. These distances should not be confused with the distances lxp and lzp,quoted in Appendix B, which represent the distances between the pilot and the c.g. of the aircraft. In a number of high speed aircraft, spoilers are used instead of the ailerons; the principles to be discussed are not materially affected, whichever is used. In some of the earlier, single-axis, autopilots which consisted solely of yaw rate feedback to the rudder, it was because they did oppose the almost steady turn associated with an unstable spiral mode that they were successful. 2. 3. 4. 9.12 REFERENCE 1987. Eighty years of flight control: triumphs and pitfalls of the systems approach. J. Guid. Cont. 4(4): 353-62. McRUER D.T. and D.C. GRAHAM. Attitude Control Systems 10.1 - INTRODUCTION Attitude control systems find extensive employment on modern aircraft. They form the essential functions of any AFCS, in that they allow an aircraft to be placed, and maintained, in any required, specified orientation in space, either in direct response to a pilot's command, or in response to command signals obtained from an aircraft's guidance, or weapons systems. It is through their agency that unattended operation of an aircraft is possible. In AFCS work, attitude hold, the commonest function, is often referred to, especially in the USA, as a control wheel steering (CWS) mode. Stability augmentation systems, which are dealt with in Chapter 9, often form the inner loops of attitude control systems; the attitude control systems then form the inner loops for the path control systems, which are discussed in Chapter 11. It is often the case that attitude control systems need to use simultaneously several of the aircraft's control surfaces, or they may require the use of feedback signals which depend upon motion variables other than those being controlled directly. Attitude control systems are, consequently, more complex in their operation than stability augmentation systems. 10.2 PITCH AITITUDE CONTROL SYSTEMS Pitch attitude control systems have traditionally involved the use of elevator only as the control in the system. A block diagram of a typical system is shown in Figure 10.1; the assumptions adopted in Chapter 9 about the representations of the dynamics of both the elevator's actuator and the sensor of pitch attitude are still maintained here. Therefore, the feedback control law being considered in this section can be generally expressed in the form: As the feedback gain, KcK,, is increased, it is found that the aircraft's short period frequency w,, also increases, although its damping ratio, c,,, decreases; however, the damping ratio of the phugoid, c,,, increases. The period of the phugoid motion also increases until the mode becomes over-damped and, consequently non-oscillatory. An early view (c. 1940) that the best results are Attitude Control Systems I Actuator dynamics Aircraft dynamics Controller Attitude gyro I Figure 10.1 Block diagram of pitch attitude control system. obtained when the value of the feedback gain is chosen such that the phugoid mode is critically damped, thereby making the phugoid motion aperiodic, will be seen to be incorrect. In general, feedback of pitch attitude causes the damping of the phugoid mode to increase at the expense of the damping of the short period mode. Whenever the feedback signals, being used in an AFCS for longitudinal motion, depend solely upon motion variables which do not result in the augmenting of the stability derivatives Xu, Z,, Ma,or M,, then the total damping of the system is unchanged by the application of feedback. Consequently, the total damping of an open loop system can then be redistributed only among the resulting closed loop modes as a result of linear feedback control. If the phugoid damping is increased, for example, it can only be at the expense of the short period damping. If Kc is so chosen that the phugoid mode is heavily damped then, in the pitch attitude response of the controlled aircraft, the phugoid motion will be almost completely absent. The response of a pitch attitude control system used for FOXTROT-2is shown in Figure 10.2. The controller gain, Kc, was chosen to be 1.0 VIV, with the sensitivity of the attitude gyro, KO,being taken as 1 Vldeg. It has sometimes been claimed that whenever the pitch attitude of an aircraft is tightly controlled the phugoid mode cannot exist; but such a claim is incorrectly expressed (Stengel, 1983). What is meant is that, with such control, the roots of the phugoid mode are usually real and negative. When the phugoid mode is so heavily damped, any changes, which occur in other motion variables (such as speed and height) as a result of the pitch command signal, are small and the responses associated with such variables are well damped, with long period. It is for such reasons that the use of pitch attitude feedback to the elevator has been, and will go on being, one of the most successful feedback control techniques used in AFCSs. However, such a system is said to be type 0 - see Figure 10.1 - and there must then exist, in response to any step command or disturbance, a steady state error - see Figure 10.2. Moreover, the loss of short period damping to augment the damping of the phugoid mode has resulted in a rather unsatisfactory dynamic response because the stability margins have been degraded. The steady state error can be removed by including an integral term in the control law; the inclusion of this additional term, however, may further reduce the damping of the short period motion. A third term is then added to the control law, one involving feedback of the pitch rate (thereby implementing an SAS function) such that: Pitch Attitude Control Systems -0.4 0 I 1 I 2 3 I 4 I I I I I 5 6 7 8 9 Time (s) 1 1 1 0 Figure 10.2 Response of system of Figure 10.1. A block diagram of a pitch attitude control system using such a control law is shown in Figure 10.3. The use of such a three-term controller is not universal, however, and in many systems the degree of steady state error which exists with Elevator actuator . "(') + + Aircraft dynamics qp) 1 KI - \ Rate - gyro Kd 4 K4 K2 -Atitude l+-, F".. Integrating A" Attitude Control Systems Time (s) (a) Time (s) (b) the chosen values of controller gains, Kc and Kd, is acceptable. As a result, many pitch attitude control systems have a control law which consists of only two terms: This feedback control is very effective in general use. Step responses for a pitch are shown in Figures 10.4(a) and (b) for attitude control system for FOXTROT-2 the control laws: 6, = 0.50 + 0.6q + 0.7 J Odt (10.4b) Pitch Attitude Control Systems -0.4l 0 (4 (4 I 2 I 4 I 6 I 8 I I 10 12 Time (s) I 14 I 16 I 18 I 20 Time (s) Figure 10.4 (a) Step response of system of Figure 10.1. (b) Step response of system of Figure 10.3. (c) Response of system of Figure 10.3 to initial pitch attitude. (d) Step response of pitch attitude control with C* pre-filter. 322 Attitude Control Systems Figure 10.4(c) shows the transient response of the same system for the same aircraft and flight condition using control law (10.4b). Note that the steady state error has been removed. These responses should be compared with that shown in Figure 10.2. The improvements, which the use of an integral term and pitch rate feedback have made in the dynamic response, are evident from comparing these figures. A pitch attitude control system using a control law such as (10.3) can produce a better command response by first generating the command signal, ,,,8, from a pre-filter which follows the stick input, p,. One form of pre-filter is the C* criterion filter which has as transfer function: where the time constants have been chosen to suit the aircraft being dealt with. The resulting step response of the pitch attitude control system which uses this pre-filter will correspond to the C * criterion discussed in Section 6.5 of Chapter 6. With the use of this pre-filter it is often possible to change the values of Kq and KO so that the transient response to disturbances can also be improved. Figure 10.4(d) shows the step response of the system whose response without pre-filter was given in Figure 10.4(a). For the system corresponding to Figure 10.4(d) the feedback gains were changed to Kq = 5.0 and KO = 5.0. Note the improved response of Figure 10.4(d). There is, however, a common flight situation in which too tight control of pitch attitude can be disadvantageous: when an aircraft is flying in the presence of atmospheric turbulence, the pitch attitude control system tends to hold the pitch angle at a constant value. This fixity of attitude opposes the natural tendency of an aircraft to nose into the wind, thereby reducing the acceleration being experienced by the aircraft. It also results in the angle of attack coinciding with the gust. The net result of these two effects is that the accelerations experienced in gusty conditions are higher than they would be otherwise, with a consequent increase of the load being imposed upon the structure of the aircraft. When a pitch attitude control system is operating a problem can arise if the aircraft is banked at some large angle. The problem depends upon whether a rate gyro or a vertical gyro has been used as the means of providing the feedback signal representing pitch attitude rate in the control law. The rate gyro produces a signal which is related to the body axis system, the vertical gyro signal is related to the Euler axis system, i.e.: 8 cos + + 4~cos o sin + 8 = q cos 4 - r sin + q = (10.6) (10.7) At large bank angles these signals q and 0 cannot both be zero simultaneously. Nor are they equivalent signals. For the wings-level flight situation, either gyro can be used with no discernible difference in performance; but in turning flight the system performance will be quite different, depending upon which gyro has Roll Angle Control Systems 323 been used. If the vertical gyro is used, the operation of the pitch attitude system must be restricted to a limited range of bank angles. 10.3 ROLL ANGLE CONTROL SYSTEMS 10.3.1 Introduction Roll angle is generally controlled simply and effectively by the ailerons at low-tomedium speeds on all types of aircraft; on military aircraft, at high speed, spoilers are used. Such spoilers, on the wing of an aircraft, are a very effective means of producing roll moments, but these moments are generally very non-linear, and are quite often accompanied by a proverse yaw moment as well as producing considerable drag. Roll control for swing-wing aircraft is usually produced by means of control surfaces, moving differentially, and located at the tail. Swing-wings generally contain spoilers to augment the roll control power of the tail surfaces. These spoilers are activated whenever the wings are forward of some value of sweep angle, typically 40-45". Except at high speed, a differential tail is not very effective at producing rolling moments, since the differential deflection which can be applied is necessarily restricted to allow the same surfaces to be used (symmetrically) for longitudinal control. Associated with the rolling moments produced by this method is a large, adverse yawing moment. Unless particular care is exercised in the design of the basic aircraft, it is possible for the spoilers and the differential tail to produce rolling moments which oppose, and yawing moments which aid. In this section the symbol 6A will be used to denote any means of producing rolling moments. The complete transfer function relating bank angle to aileron deflection is given by: Typically, Ts can be very large; see Table 10.1. Therefore, the spiral mode can correspond to either a slow convergent or a divergent motion. In an early (and excellent) textbook Langeweische (1944) on flying, stated that 'any aircraft which was spirally stable was unpleasant to fly in rough air, for it was wallowy and unsteady and wore you out'. However, for unattended operation, neutral and divergent stability are undesirable since any disturbance can cause an aperiodic, divergent motion of the aircraft, which pilots have referred to as the 'graveyard spiral'. One of the most important functions of any AFCS operating on lateral motion must be, therefore, to attain to a high degree of spiral stability, but it must also improve the other lateral flying qualities so that a pilot is not 'worn out' Attitude Control Systems Table 10.1 Spiral mode time constants TS Aircraft -85.28 23.42 35.34 68.03 ALPHA CHARLIE DELTA FOXTROT 2849.00 111.11 97.09 103.20 534.76 97.09 126.58 534.76 -5000.00 - 128.20 - 37.88 1792.10 whenever he is flying in atmospheric turbulence. An effective technique of achieving good spiral stability is to provide the aircraft with good lateral static stability. To achieve the degree of dynamic stability desired in roll requires the use of a roll attitude control system. Such a control system is a feedback control system which maintains the roll attitude in the presence of disturbances and responds rapidly and accurately to roll commands from the pilot or a guidance system. For most aircraft, the following assumptions hold: (1) TR T,, and (2) the quadratic term in the numerator of eq. (10.8) cancels the quadratic term in the denominator. When these assumptions are true, or are nearly so, the aircraft's roll dynamics may be represented by a single degree of freedom approximation: * where: K+ = L i A TR = - (LA)-' p = d+ldt 10.3.2 A Typical System A block diagram representation of a typical, roll attitude control system, in which the actuator response is assumed to be instantaneous, is shown in Figure 10.5. It can easily be shown that: Hence: Roll Angle Control Systems Controller Aircraft dvnamics Attitude gyro 1 Figure 10.5 Bank angle control system. For a specific damping ratio of this roll attitude control system, the. value of controller gain needed is given by: It is interesting to consider what must be done to this value of controller gain if it is hoped to maintain the damping ratio of the closed loop system at a constant value throughout the flight envelope of the aircraft. In Section 2.10 of Chapter 2 it is noted that the stability derivatives LA and LbA could be expressed as: Consequently: z A Therefore, the gain must be steadily increased with increase in forward speed. Example 10.1 For CHARLIE-2 it can be shown that: Suppose 5 = 0.6 is required. Therefore: Attitude Control Systems where: w2,4 0.21 Kc But w, = 0.74 rad s-l, hence: Kc = 2.6VlV Figure 10.6 shows the step response of this example system. Time (s) Figure 10.6 Step response of bank angle control system. 10.3.3 Phase Advance Compensation Sometimes the cancellation of the numerator and denominator quadratics is inexact. In that case, the rolling motion contains a significant component of dutch roll oscillation which may lead to serious difficulties for a pilot flying that aircraft. Whenever this situation is likely to arise, the control law is changed from ?iA = Kc& to: where: The additional rate term in eq. (10.18) introduces damping and corresponds to a phase advance term. The corresponding block diagram is represented in Figure 10.7. With a control law such as eq. (10.18), the closed loop system has a transfer function given by: Roll Angle Control Systems Controller Aircraft dynamics ll.--+-Fbl Attitude gyro Figure 10.7 Bank angle control system with phase advance. By proper selection of values of Kc and Tc it is possible to achieve the transfer function of eq. (10.21): where: Example 10.2 Consider CHARLIE-2once more. Let: Hence: 0.21Kc = 1/TcT6 Attitude Control Systems Since there are three unknowns, Tc, T+, and Kc, and only two equations it is necesssary to choose one and evaluate the others. Suppose Kc is chosen to be 10.0, then, by elementary algebra, it can be found that there are two possible values of T, which can be used, namely Tc = 3.107s or Tc = 0.2742s. If the former value is used, the resulting value of T+, is 0.1633, whereas, when Tc is chosen to be 0.2742, the corresponding value for T+, becomes 1.737, i.e when Tc = 3.107s then: +(s) = c o m m 4.41 (1 + ~0.153) When Tc = 0.2742 the result is: +(s) 4.41 c o m m (1 + s 1.737) The response of the system corresponding to eq. (A) is ten times faster than the response obtained from a system corresponding to eq. (B). Hence, system (A) would be the preferred system because the quality of rolling motion from the aircraft would be better than the flying qualities specified in Chapter 6. = 10.3.4 The Use of a Roll Damper as an Inner Loop Using phase advance compensation is often unsuccessful in practice, in situations where the feedback, or the command signals, are subject to noise interference. An alternative scheme, which permits a designer to use considerable freedom in arriving at the required dynamic performance of the roll angle system, is to employ as an inner loop the roll damper SAS discussed in Section 9.5. The roll damping of the aircraft can be considerably augmented by such an inner loop, to values even greater than that needed by the roll angle system, in order to sacrifice some in the outer loop, thereby achieving good steady state performance and the required transient response. A block diagram representing a typical system is shown in Figure 10.8; the actuator dynamics are represented as a simple first order lag. It should be noted that using this technique requires that there be available another motion sensor, namely, a rate gyroscope, for use in the roll damper, or, alternatively, the signal from the roll attitude gyroscope must be washed-out in an appropriate filter. From Figure 10.8 it can be established that the closed loop transfer function of the roll angle control system is: 329 Roll Angle Control Systems Aircraft dynamics Aileron actuator 10 s+ 10 Roll angle controller horn* - Controller Kc, - 7 - "(s)- Lki (s-L'p) - P(S) 1 ' S 4(~) - Rate gyro 0.1 t Roll damper Attitude gyro 1.0 Figure 10.8 Bank angle control system with roll rate inner loop damper. By using the roll damper as an inner loop, the frequency of the roll angle system can be controlled by Kc2 and the damping by Kc1 ' The chief difficulty experienced with such systems is associated with locating the sensors to avoid the unwanted effects of structural flexibility. The effect of the aircraft's angle of attack should also be considered. (See the discussion on the roll rate gyro in Section 9.5 of Chapter 9). Example 10.3 For CHARLIE-2the single degree of freedom approximation for rolling motion as a result of aileron deflection can be approximated by the transfer function: If the system used as a roll angle control system is that represented by Figure 10.8, then the corresponding closed loop transfer function is: Kcl and Kc2 can be obtaine~dfrom any of the methods outlined in Chapter 7. System A. If Kc2 is chosen to be, say, 10.0 and Kcl is selected to be 31.55, then the characteristic polynomial of the roll angle system becomes: which is identical ti:) the polynomial which obtained for Example 10.2. Attitude Control Systems 330 However, since phase-advance is not being used, there is no numerator term and the factor (1 s3.107) is not cancelled. As a result, the response of this system, although heavily damped, is sluggish. + System B. A better choice of Kcl is 95.156 (KC2remains fixed at 10.0), for this results in the system being critically damped, i.e.: The step responses for systems A and B are shown in Figure 10.6; the superiority of B is evident from inspection. 10.3.5 Use of a Yaw Term in the Roll Control Law If the control law being used in a roll angle control system is modified to become: 8 ~ =~ Kcl+~+ Kc2y ~ , (10.24) The mode associated with the 'yawing' motion of the aircraft can then become a subsidence mode, with its damping being increased substantially, as Kc2 is increased. The dutch roll damping is decreased, however. From experiment and flight tests, it has been found (McRuer and Johnston, 1975) that the best practical arrangement results when: The step responses of a roll angle control system, used with CHARLIE-2, and using the control law eq. (10.24) for three values of the ratio, Kc{KcZ, are shown in Figure 10.9. These results should be compared with those shown in Figure 10.6. It is evident from Figure 10.9 that the best choice is a ratio, Kc{Kc2, of unity. 10.3.6 Some Problems Arising with Roll Control In fighter aircraft, the pilot usually controls the roll angle indirectly through a CSAS, a commanded role damper, since such CSASs are necessary to assist the aircraft to provide the rapid roll performance which is essential for modern aerial combat, or for evasive manoeuvres during low level strike missions. To achieve the performance required inevitably means the use of high loop gains. Such high values of gain cause a number of problems, although it is worth noting that the gains of such CSASs are often fixed throughout the flight envelope. Among the problems are the following: 1. The command signal from the pilot must usually be 'damped'. If the input signal to the CSAS corresponding to a small deflection of the pilot's stick is too large then pilot-induced oscillations may result. This is particularly Roll Angle Control Systems Time (s) Figure 10.9 Step response of bank angle control system with yaw term added. likely when the aircraft is being used on a precision tracking task. This problem is general for any high gain CSAS. The obvious remedy of reducing the value of input signal corresponding to the stick deflection often results in the system's performance being inadequate. 2. When the speed of the aircraft is low, and the dynamic pressure is relatively small, such as during a landing approach, the response of the aircraft is sluggish. To achieve the roll response required in this condition means that a pilot has to apply large deflections to the control stick. These large values can result in limiting of the command signals. At high speeds, when the dynamic pressure is large, the much more rapid response of the aircraft, in association with the high loop gain, can result in limiting of the feedback signal. Both limiting conditions can result in degraded roll performance if the roll control system is not well designed. 3. A system with a too high value of loop gain precludes control of bank angle by use of the rudder, which is a technique often used by pilots in making S-turns during landing, or during manoeuvres in aerial combat. This problem can be overcome by carefully scheduling the control gains with the correct flight parameter. 4. On swept-wing aircraft, as the stall condition is approached, it is essential to reduce the value of the loop gain by a substantial amount to avoid very large deflections of the control surfaces. Such large deflections lead to the aircraft's rapidly departing from its trimmed state into a stall. 10.3.7 Roll Ratchet Caused by Excessive Roll Damping In-flight experiments with modern fighter aircraft have indicated that excessive values of rolling accelerations are experienced by pilots when trying to reach some Attitude Control Systems 332 desired value of roll rate. To avoid such accelerations the pilot inust apply more slowly, through the primary flying control, the input to the roll control system. But, frequently, a pilot's reaction is instinctive and sudden, with the result that the closed loop system, formed by the pilot and the aircraft dynamics, oscillates in roll. The oscillatory motion is typically of high frequency (1.8-3.0 Hz) and when it occurs is referred to as 'roll ratchet'. The phenomenon arises with aircraft in which the roll damping is excessive. Suppose the closed loop transfer function of a roll damper system is given by: +(s) = c o m m K s (l + ST) If the damping is large T -+0 and eq. (10.26) can be approximated to: = -K +(s) c o r n I s When a pilot closes the command loop around a roll damper SAS the system may be represented as shown by Figure 10.10. The form of mathematical model used to represent the pilot is explained in Appendix C ; the model used represents a proportional gain, K,, followed by a pure time delay, T (representing the pilot's reaction time) of about 0.13s. Therefore: Model of pilot + Kpe-n - Roll damper ~cornrn(~) K - ~ - 4(s) * S Figure 10.10. Pilot-in-the-loop roll ratchet. When the loop gain (K,K) has a value of, say, 12, and the time delay function is approximated by e-"' = (2 - ~ s) / (2 TS), then: + +(s) 92.512(2 - 0.13s) s2 3s + 185 +cornm(s) + Therefore, the system will oscillate with very little damping (5 -- 0.01) at a frequency of 13.6rad s-' in response to a unit step function. The result of applying a unit step function to a digital simulation of eq. (10.28) is shown in Figure 10.11(a). The roll ratchet oscillation is clearly evident, at a frequency of 13 rad s-l. Figure 10.11(b) shows two step responses for the Roll Angle Control Systems 1.0 445.1 0.8 - - - ---=- 4comm(s) 0.6 k l2 s+k s+12 - 4 0.4 - 0.2 - 0.0 0 I 2 I 4 I 6 I 8 I 1 10 Time (s) (a) Roll ratchet frequency o = 12.7 rad s-' -1.0 0 (b) 2 4 6 8 10 Time (s) Figure 10.11 (a) Bank angle control system: pilot reaction instantaneous. (b) Bank angle response with pilot reaction time of 0.3s. same simulation, but for the situations where T in eq. (10.26) is not entirely negligible, T being 0.01 in case A and 0.2 in case B. From the figure it can be seen that roll ratchet is only evident in case A; in case B, where T has increased, i.e. the roll damping has been reduced, the roll ratchet vanishes. Readers should refer to Chalk (1983) for further discussion of these topics. 10.3.8 Unwanted Pitching Motion Caused by Rolling Motion In Section 2.6 of Chapter 2 it is shown that, in a steady turn, there occurs a steady pitch rate, the value of which is: 334 Attitude Control Systems g tan qss = - uo + sin + = r sin + It is necessary to use as feedback a signal proportional to this steady state pitch rate, to oppose the pitch rate signal being used in the pitch attitude control system, otherwise the pitch attitude control system will not perform properly in banked turns. This matter is discussed in Section 10.2. To obtain this signal, qss, requires that the output signal from the yaw rate gyroscope be multiplied with that from a resolver driven by a bank angle servomechanism (or the product can be determined in an on-board digital computer). 10.4 WING LEVELLER In small, general aviation aircraft there is a need, sometimes, for a regulating system which will hold the wings level in the presence of atmospheric disturbances. Although any roll angle control system performs this function, in such a class of aircraft the use of a roll attitude gyro may be avoided by means of setting the command signal, to zero and using a tilted rate gyro in a wing leveller system such as that represented in Figure 10.12. This system has proved to be very effective. The principle of the tilted gyro is the same as that explained in Section 9.8. ,,,+, - Aircraft dynamics Controller ~A(s) Kc Tilted rate gyro P(4 US) , - 4s) 7 Us) cos (a+aR) + 0.1 sin (a+aR) -= Figure 10.12 Roll rate system with tilted gyro. P(S) Co-ordinated Turn Systems 10.5 CO-ORDINATED TURN SYSTEMS 10.5.1 Introduction A co-ordinated turn is one in which both the lateral acceleration, a,, , and the sideslip velocity, v , are zero. In such a turn the lift vector is perpendi&!lar to the aircraft axis OY. Co-ordinated turns reduce adverse sideslip and, therefore, roll hesitation. In such turns, there is minimum coupling of rolling and yawing motions. Provided that the side force due to aileron, Y: A , and the side force due to the yaw rate, Y,, are both negligible, then zero sideslip angle (P = 0), zero sideslip velocity (v = P/Uo = O), and zero lateral acceleration (a, = 0) are all equivalent conditions. Sometimes, particularly in early textbook? on flying techniques, a co-ordinated turn was assumed to be one in which the lateral acceleration experienced in the cockpit was zero - a condition displayed to pilots by the turn-and-bank indicator, with its black ball centred between the vertical lines. However, this condition is not one which finds much use in AFCS studies since the acceleration at the cockpit is a function of the distance from the aircraft's c.g. The acceleration at the pilot's station features in AFCS work only in relation to ride control systems, which are dealt with in Chapter 12. 10.5.2 Conditions Needed for a Co-ordinated Turn For a body axis system the side force equation is: Y= m (~ W - P + UR) Following the development detailed in Section 2.4 of Chapter 2, it can be seen that the rate of change of sideslip angle can be expressed as in eq. (2.75), i.e.: If R0 = 0, WdUo = a0 and, if a co-ordinated turn is achieved, i.e. if: p =0 then: If the aircraft has been trimmed so that olo is zero, then: 336 Attitude Control Systems Therefore, in a co-ordinated turn, the rate of turn develops in proportion to the bank angle, 4. Of course, neither Yv nor YzA is generally zero, nor may they be neglected. Consequently, if p is to be zero, so that eq. (10.35) obtains, a steady deflection of the ailerons is required to maintain the co-ordinated turn. The value of aileron deflection required is given by: There are a number of factors which may delay the establishment of a coordinated turn. They include the following: 1. An aileron deflection usually induces a yawing moment. 2. The build-up of yaw rate, as a result of any change in bank angle, is delayed by aerodynamic lag. 3. The action of the yaw damper, which is commonly fitted to aircraft, tends to reduce any transient yaw rate. As an illustration of how these factors affect the turn, consider an aircraft, such as B, which is: CHARLIE in Appendix NkA > 0 (10.37) Whenever a positive roll rate is required i.e. BA < 0 , a negative (adverse) yawing moment results. This can be seen from eq. (2.85): For all aircraft, N' and NAR are both negative. If the yawing moment is negative, the sideslip is positive. 10.5.3 Sideslipping as a Result of Sensor Signals in Lateral AFCSs If the rate gyro used to measure the yaw rate in a yaw damper is of the strapdown variety (i.e. it is fixed to the aircraft and is not mounted on gimbals) its signal is a measure of the body rate, rather than of a windbody rate (i.e. one which has been measured in relation to the stability axes). However, the equations used in the yaw damper design have been derived using stability axes, so that the signal produced by such a gyro is given by: rbOdy = r, cos (YO + ps sin a. (10.38) In a rolling manoeuvre, with a positive angle of attack, the component due to roll rate in the signal from the strap-down gyro will increase. But if this signal is used as the feedback signal in a yaw damper, that feedback signal will be increased, causing further rudder action, which results in an increased sideslip angle. For negative angle of attack in a rolling manoeuvre, the effect is to reduce the sideslip motion. Co-ordinated Turn Systems 10.5.4 Horizontal Acceleration During a Turn The situation is represented in Figure 10.13: fc denotes the centrifugal force, VT the tangential velocity, ω the angular velocity, m the mass of the aircraft, and R the radius of the turn. From Figure 10.13 it can be seen that the centrifugal force is given by: ... The total acceleration is the vector sum of a, and the acceleration due to gravity. The maximum value of acceleration is always experienced at the same bank angle, irrespective of aircraft velocity, VT. If the aircraft, however, is subject to some maximum value of ay, then the lateral acceleration is limited to some maximum value which corresponds to VT. For a given speed, Uo, and a constant rate of turn, ω, the bank angle required for a co-ordinated turn is given by: Although turns are invariably made at values of bank angle too large for the linearization of sin and cos to hold, the results obtained above are correct. The number of turns which are completed in a manoeuvre may be calculated from: ω + Attitude Control Systems 10.5.5 A Steady Sideslip Manoeuvre This flight condition of non-symmetric, rectilinear translation is often used in light aircraft to correct for the presence of a cross-wind on the landing approach. For small values of sideslip angle, the drag on the aircraft increases; as a result, the aircraft's lift/drag ratio decreases. In this flight condition, rates of change are zero, i.e. p = q = r = 0; and the resulting sideslip angle β and the control surface deflections SA and SR required for the manoeuvre can easily be found, provided that the matrix A is non-singular. The control deflections required tend to be very large, since powerful controls are needed to sideslip an aircraft at large angles. If A is singular, it implies that the bank angle required for the manoeuvre is zero. In this situation, the bank angle term on the r.h.s. should be transferred to the l.h.s. of eq. (10.51) and the P term should be transferred from the l.h.s. to r.h.s. The new matrix A which results is then nonsingular. The control deflections required to produce the specified sideslip angle can then be determined, along with the resulting bank angle. 10.6 SIDESLIP SUPPRESSION SYSTEMS 10.6.1 Introduction It can be deduced from the discussion on co-ordinated turns that sideslip angle is the motion variable whose control is central to the achievement of a co-ordinated turn. Sideslip Suppression Systems 339 turn. There is no particularly good method of measuring sideslip: the vane sensors which are used in some low speed aircraft are affected by problems concerning the local aerodynamic flow around the vane. They are also physically vulnerable. Some types of stagnation point sensor are useful for sensing flow direction, but have not yet found general application for AFCSs. Thus, the obvious means of controlling sideslip angle, by using a feedback control law based on sideslip sensing, is rarely used on high performance aircraft. However, its design and use will be covered first, to indicate the effectiveness of such systems, before presenting some other methods which are commonly used. These include: lateral acceleration feedback, computed yaw rate feedback, and control cross feeds. Further discussion of this topic can be found in McRuer and Johnston (1975). 10.6.2 Sideslip Feedback Figure 10.14 shows a typical system in which the sideslip angle, β, is sensed and used as a feedback signal to drive the rudder so that the sideslip motion is eliminated. Note that the system includes a yaw damper as its inner loop. For example, the yaw damper system for CHARLIE-4 used a yaw rate gyro with a sensitivity, K, of 0.1 V deg-1, a value of controller gain, Kc, of 10, and a washout time constant of 1.0 s. The state equation for the yaw damper was defined in eqs (9.74)-(9.76). The command input there was taken as rCom; from Figure 10.14 it can be seen that when the sideslip suppression system is added, the command input to the yaw damper is now: Wash-out network 1 + Pilot's Rudder actuator rudder + command β + - s + 1 ... Figure 10.14 Sideslip suppression system. The block diagram of the sideslip suppression system is represented in Figure 10.14. Sideslip suppression system, has written as: where: K [P r + ...] Because of the perturbed airflow surrounding the vane of a sideslip sensor, the output signal is prone to contamination by noise. Consequently, to avoid feedback of local flow disturbances, it is customary to use vane sensors of low sensitivity. To illustrate the effectiveness of the system, a value of sensitivity, Kβ, for the sideslip sensor in Figure 10.14 of 0.05 V deg-1 has been assumed. Figure 10.15 shows the system response to an initial sideslip disturbance of 1.0°, Kc2 = 100.0, Kc = 10.0, Kβ = 0.05 V deg-1. The responses correspond to Kc2 = 0, is the response of the yaw damper only; the other responses should be compared to this one to observe the relative effectiveness of the sideslip suppression system. The effect of the sideslip suppression system can also be appreciated from an examination of the system eigenvalues. Table 10.2 shows the eigenvalues of the basic uncontrolled aircraft (CHARLIE-4), the yaw damper only, and the combined system with sideslip suppression. The corresponding values of controller gains Kc2 and Kc2are indicated. From Table 10.2 it can be seen that the sideslip suppression has increased the stability of the spiral mode, thereby reducing the sideslip transient more effectively. It is worth appreciating, finally, that a considerable degree of sideslip suppression results from the action of the yaw damper on its own. 10.6.3 Lateral Acceleration Feedback When an accelerometer is located at the centre of percussion, an instantaneous centre of rotation at which occurs the centre of pressure of the aerodynamic force. Figure 10.15 Response to P(0) = 1o. (a) Uncontrolled aircraft. (b) Yaw damper system - wash-out time constant = 1 s. (c) Sideslip suppression system. Attitude Control Systems 342 Table 10.2 Eigenvalues of sideslip suppression systems Eigenvalue Uncontrolled aircraft Yaw damper and wash-out network Sideslip suppression - sideslip feedback as a result of the rudder deflection, the acceleration which is sensed (assuming linear relationships) is: Thus, the sideslip suppression system can now have a block diagram like that shown in Figure 10.16. The system requires, however, that the sensitivity of the accelerometer be high, since Vr is usually small, e.g. for CHARLIE-4, Yv = -0.056. In addition, the acceleration threshold of the accelerometer must be low if the lateral acceleration feedback system is to be effective for small values of sideslip angle. This low threshold value means that the system is subject to spurious inputs from structural effects, and, possibly, in very high performance aircraft, from the effects of Coriolis acceleration. The state equation of eq. (10.53) also applies for this system, and for the example of CHARLIE-4, the only change which occurs is to the element a4 of A. It now becomes: Rudder actuator Rβ * st4 Wash-out network Attitude Control Systems ... Controller and integrator KF, KR - Sideslip controller Accelerometer K, Ka' Figure 10.16 Block diagram of lateral acceleration control system. 343 Sideslip Suppression Systems This type of sideslip suppression system is sometimes referred to as a 'directional stiffening' (see McRuer and Johnston, 1975) since it augments N β. 10.6.4 Sideslip Suppression Using Computed Yaw Rate It was shown in Section 10.5 that, in a co-ordinated turn, the rate of turn develops in proportion to the bank angle, i.e. ψ sin φ. By using this signal as a feedback signal in the system shown in Figure 10.17, the yaw damper will not oppose the turning motion as a result of the yaw rate which is generated when the bank angle is not zero. In other words, if the sideslip angle has a value other than zero, the feedback operates. The error signal in Figure 10.17 is given by: ess = Kc2KEx (1 + ...). If the system is effective, if the resolver is accurate; with modern, airborne digital computers computing equations such as eq. (10.55) can be carried out to very good accuracy. With old-fashioned, electromechanical resolvers, the accuracy was often difficult to achieve and, as a result, the rudder could be held over at either extreme of its range of deflection during a turn. This tendency was usually diminished by adding a forward path control system... 10.6.5 Control Crossfeed... Attitude Control Systems ... Figure 10.17 Block diagram of computed yaw rate sideslip suppression system. Attitude Control Systems K. (a) (b) Time (s) Time (s) Figure 10.18 Response to P(0) = 1o. (a) Computed yaw rate system Kc = 10.0. Sideslip Suppression Systems 345 aircraft, the system never enjoyed much popularity. The responses of such a system, for CHARLIE-4,using the same parameters for the inner loop as were used in the yaw damper design (Section 9.7), are shown in Figure 10.18. The sensitivity of the resolver was chosen to be 1V deg-1. The value of the gain of the controller is indicated at the appropriate response curve in Figure 10.18. 10.6.5 Control Crossfeeds Introduction It must be remembered that roll rate is often required when either stopping or completing a lateral manoeuvre during the final approach. Such manoeuvres are usually controlled by use of the ailerons; however, the use of ailerons can result in a significant yawing moment if the stability derivative, Nδ, is relatively large. This yawing moment can make a substantial contribution to t i e sideslip which can arise in manoeuvres of this kind. As a result, it has long been a practice in lateral/directional control systems to incorporate a control crossfeed to remove that source of sideslip. There are two types of crossfeed (sometimes referred to as 'interconnects'): aileron-to-rudder interconnect (ARI) and bank angle-to-rudder crossfeed. ARI is the most common, being the most effective. A control system, which uses ARI to maintain at zero the sideslip induced by both aileron deflection and roll rate (RA) is represented in Figure 10.19. The rudder signal from the crossfeed gain, Kc-, has been found from flight studies (see McRuer and Johnston, 1975) to be: However, the student should understand that if the ARI is to be a permanent crossfeed, the crossfeed gain must vary with flight envelope, and be active just at the terminal phases of flight, being switched in, say, when the flaps are deployed, or when the landing gear is lowered, gain scheduling of Kcf will be needed (it should vary inversely with the forward speed of the aircraft). Additionally, if the structural modes of the aircraft are significant, then some form of frequency compensation must be used. In aircraft with a variable configuration, such as a swing wing, the sign of the crossfeed signal may also need to be changed as a function of the sweep angle to maintain dynamic stability of the system. The presence of the wash-out network in the crossfeed path is required to permit the aircraft to produce steady sideslipping manoeuvres, unopposed by the system. Such manoeuvres are most frequently required in cross-wind landings. One concern of flight control system designers using ARI, to suppress sideslip to improve turn coordination, must be the situation when the aircraft is subjected to asymmetric thrust when an engine (or engines) has failed. In such a situation, the pilot will 346 Attitude Control Systems Figure 10.19 ARI system. need to command a constant aileron deflection to counter the resulting yawing motion caused by the engine failure. The ARI system may aggravate this control problem. From Figure 10.19 the following relationships can be established: 1 - ecfT2 10FCaA + 10Kc2Kc&co,m + KclKcs - 10Kc2Kc& (10.65) ewo = ewo + KRr (10.66) For CHARLIE-4 (see Appendix B) ... Wash-out network 1 Rate gyro Wash-out network Cross-feed gain Rudder actuator s s+4 10 + ... Aileron dynamics ' CHARLIE-4) Aileron actuator Rate error Attitude error Figure 10.20 Bank angle-to-rudder crossfeed system. Sideslip Suppression Systems 347 Let then: For CHARLIE-4, Kcf = 9.5156VN? Kc = 10.0 V N , Kcf = 0.035 V deg-l, KR = 10 V deg-l s-l. Therefore, the 2 system can be represented as: ... The speed of response of the system depends critically upon the values chosen for T2 and Kcf. For this particular aircraft and flight condition, Kcf = 0.035 is the best choice. Large values of T2 (slow wash-out) result in poor sideslip response. A value of T2 = 0.05 s, very short, gives good response for this example. The responses of this system are shown in Figure 10.20. It is evident how effectively the sideslip has been suppressed, and it can be seen that the bank angle response has not been seriously affected by the crossfeed (cf. Figure 10.6). Roll Angle to Rudder Crossfeed This type of control crossfeed is used mostly with roll attitude control systems. Its use is not to be flexible. For such aircraft, the time constant needed for the wash-out network can be very long, and the value of damping ratio for the closed loop system. The situation arises because the Attitude Control Systems ... Figure 10.20 Step response of Figure 10.19. The frequency of the dutch roll mode is very low, as are the frequencies of the structural bending modes, so that feedback of lateral acceleration cannot be used, otherwise there would be considerable coupling of the rigid body and structural motion. Furthermore, the relatively slow response of the rudder fitted to such large aircraft precludes the use of high loop gain to suppress unwanted sideslip motion. Consequently the following technique is employed. (See Figure 10.21.) The crossfeed signal, ecf, is introduced into the summing junction of the yaw damper as if it were a command signal for some value of yaw rate which corresponds to zero sideslip. It has already been shown in this chapter that in a co-ordinated turn the yaw rate is given by: ψ sin r= ... However, for small bank angles (which is likely to be the case for large transport aircraft) the command signal for yaw rate can be taken as: ... Using such a crossfeed the rudder channel some phase advance compensation is frequently required. The response of the system of Figure 10.21 can be represented by means of eqs (9.74)-(9.76) save that the fifth row of the matrix A, now becomes ... 349 Sideslip Suppression Systems Pilot's rudder command Rudder actuator ... Figure 10.21 Block diagram of roll angle to rudder interconnect system. For CHARLIE-4, it can be shown that: Hence: The response of this system to an initial value of sideslip angle of 1o is shown in Figure 10.22 where the responses of the yaw damper and ARI system to the same disturbance are also shown for comparison. Bank angle to rudder crossfeed is evidently less effective at suppressing sideslip than ARI. Its use results in the stability of the spiral mode of the aircraft being reduced. If this type of crossfeed is used to suppress sideslip, it is usually necessary, therefore, to use a roll angle control system of the kind discussed in Section 10.3 to augment the stability of the spiral mode. When this is done, there is a marked improvement in the sideslip suppression capacity of this type of crossfeed. Reliability Although reliability is of the greatest importance in AFCS work, there are situations in which the loss of a feedback path will result in no more than a downwards change of level of an aircraft's flying qualities. In these situations, the dynamic stability of the aircraft can be recovered by the pilot. However, there are instances in which the loss of a feedback path can result in an aircraft response so sluggish that it becomes necessary to disconnect at once the other feedbacks so that the aircraft is no longer under automatic control. In such cases, if the airframe should fail, it is necessary to immediately disconnect the bank angle signal from the rudder. This is a difficult engineering problem and its partial solution is to be found in the technique of using redundancy in the feedback paths. Readers should refer to McRuer and Johnston (1975) for further discussion of control crossfeeds. 10.7 DIRECTIONAL STABILITY DURING GROUND ROLL The geometry of the situation is represented in Figure 10.23 in which ψ denotes the lateral displacement from the desired track (the runway centre-line), ψ is the azimuth angle, ? is the heading angle of the aircraft, P is the sideslip angle and u is the gear slip angle. Now: The lateral acceleration which the aircraft experiences during its ground roll is: ... (10.80) With this acceleration being used as a negative feedback signal, and with the feedback gain represented by K,, then: Y = NBP + NRr, Figure 10.24 Block diagram of a lateral guidance system... Directional Stability During Ground Roll Figure 10.23 Ground run geometry. where Np is the aerodynamic weathercock stability, Na is the contribution of the undercarriage to the track stability, Yp is the aerodynamic sideforce stability derivative and Yu is the combined sideforce stability derivative from the tyres of the undercarriage. where: Taking Laplace transforms, eq. (10.83) and (10.84) can be re-expressed as: where (i) represents a null matrix. From this equation, the characteristic equation can easily be shown to be: s3 + [Yr - Yp]s2 + [N b - N & - K- YV] ... 352 Attitude Control Systems Examining the quadratic term, and noting that Y s = 0, and that NA > 0, it is then evident that the sideforce contributions of the tyres of the undercarriage contribute to the damping of the motion during ground roll. However, suppose V represents the ground speed and Vw represents the component of headwind which arises when the aircraft is moving on the runway in the presence of a wind. Equation (10.79) then becomes: The presence of the headwind now results in the real root of the characteristic cubic being finite, rather than zero, with the possibility of some stability in track. When the headwind is positive, the real root is stable if: Therefore, it can be deduced that N b stabilizes the ground tracking mode whereas N ; destabilizes it. A discussion of the dynamics of aircraft rotation and lift-off can be found in Pinsker (1967). 10.8 CONCLUSIONS Automatic control systems to regulate the attitude angles of an aircraft, or for changing an aircraft's attitude to a new commanded value, are introduced. To emphasize the principles of negative feedback control which are common to the many varieties of attitude control system used on aircraft, the pitch attitude control system is dealt with extensively. The use of a pre-filter in conjunction with these types of AFCS to obtain the required handling qualities in the controlled aircraft is briefly dealt with, before a roll angle control system is considered. The use in such systems of phase advance compensation networks, or a roll rate damper as an inner loop to achieve the required dynamic response is dealt with and gain scheduling as a means of maintaining at the same closed loop performance over as much of the flight envelope as possible is also treated. The unwanted results of tight roll control, such as roll ratchet or pitching motion due to rolling, are treated briefly before the means of achieving automatically controlled coordinated turns by a variety of methods is explained. The chapter concluded with the important subject of controlling direction stability during ground roll. 10.9 EXERCISES 10.1 A transport aircraft, flying at a Mach number of 0.8 and a height of 10 000 m has to aileron deflection, SA, (SA) as its transfer function, relating bank angle, φ, Exercises 353 as defined below. When the aircraft flies at half the height and at a Mach number of 0.4 its transfer function becomes (2): The block diagram of the bank angle control system is shown in Figure 10.24. Controller 1 Aircraft dynamics ω Attitude gyro 1 Figure 10.24 Block diagram of a bank angle control system for Exercise 10.1. + , to (a) Determine the closed loop transfer function relating the bank angle, for flight condition 1. (Hint: make the commanded bank angle, φcomm.) (b) What is the effect upon the dynamic response of the bank angle control system if the aircraft flies at flight condition 2? Assume the controller gain, K, remains unchanged. (c) If the value of Kc is 2.5, and if the value of the commanded bank angle is 5.0°, sketch the closed loop response for both flight conditions. 10.2 If the experimental aircraft of Exercise 2.7 is flying at 15 m s-l, and has the same stability derivatives that were listed in that question, calculate the lateral acceleration at its c.g. for a flat, co-ordinated turn in which the yaw rate is 0.33 rad s-1. (The aircraft may be assumed to have zero sideslip velocity.) 10.3 In the sideslip suppression system, represented by the block diagram in Figure 10.25 a sideslip signal is used as feedback to drive the rudder so that sideslip is eliminated. The wash-out filter in the inner loop can be regarded as a blocking filter for constant manoeuvre commands, i.e. yaw rate feedback operates only during changes of flight state. The dynamics associated with the rudder servo are negligible. The equations of motion of the aircraft are: The sideslip suppression system is designed so that its closed loop response are obtained from an idealised model specified by the Attitude Control Systems Rudder servo + ... 354 Figure 10.25 Sideslip suppression system for Exercise 10.3. characteristic equation: s 3 + 9.0 s + 0.1 = 0 (a) By means of any suitable analytical method determine suitable values of the sensitivities of the rate gyro and the sideslip sensor. (b) Show that one of the values of K, and K p obtained in part (a) results in the sideslip motion being suppressed in any manoeuvre. 10.4 For the aircraft detailed in Exercise 3.5 design a simple bank angle control system, with full wing-tip fuel tanks, which will ensure that the aircraft can roll through a bank angle of 30° in 1.25 s in response to a step command. 10.5 (a) In a co-ordinated turn the aircraft and OY and the lateral acceleration at the c.g., ayc.g and the sideslip velocity, P, are both zero. The situation is represented in Figure 10.26. Derive, for small angles, a transfer function relating yaw rate, r, to bank angle, 4. (b) The aircraft CHARLIE is flying with a flight path angle of 2°, the appropriate lateral stability derivatives are: (Height = SL; Speed = 37.73 m s-1) Figure 10.26 Geometry of a co-ordinated turn for Exercise 10.5. Exercises 355 ... (c) Determine the transfer function for the aircraft, relating yaw rate to bank angle. (d) If, at t = 0, the aircraft is disturbed so that a bank angle of 10° develops, find how the sideslip will develop. Explain why the angle of sideslip rate is produced. 10.6 A helicopter has been found to produce a steady sideslip angle of 10° determine the steady deflections of the ailerons and rudder required to maintain the sideslip if the bank angle was to be zero during the sideslip manoeuvre? The high speed reconnaissance aircraft described in Exercise 3.9 is to be controlled such that its transient roll rate response is close to that shown in Figure 10.27. (Compare this response with that determined for the uncontrolled aircraft in Exercise 3.9.) Determine a suitable feedback control law. Time (s) Figure 10.27 Model roll rate response for Exercise 10.6. 10.7 For the L-1011 detailed in Exercise 8.6 the rudder is fixed to 'hold' the side force due to yaw rate when a co-ordinated turn is being executed under 'manual' control. If the steady sideslip angle is 5.73°, calculate the lateral acceleration and the centre of rotation of the aircraft. (Note: the equations of Exercise 9.1 were derived using SI units.) (b) Suppose the aircraft is flying over the North Pole at the cruising speed which corresponds to the equations of motion given in Exercise 9.1. Calculate the value of the Coriolis acceleration to which the aircraft will be subjected. Comment upon whether there would be any significant effect upon the performance of the aircraft's AFCS if the aircraft were to carry out over the North Pole a co-ordinated turn of the kind defined in part (a). 10.8 A transport aircraft with twin piston engines has the following equations of lateral motion: (a) Attitude Control Systems ... (b) A 'steady' state sideslip manoeuvre is required with a sideslip angle of 5.0°. What values of bank angle and aileron and rudder deflections are required to produce the manoeuvre? 10.9 The small trainer aircraft in Exercise 10.8 is to be flown at low speed with its starboard engine out. If the yawing moment due to the starboard engine failing is ... The propeller torque is such that after the failure the aircraft banks to starboard. Heading: (a) Sketch the response of the aircraft. (b) The pilot takes no corrective action for three seconds after the starboard engine fails. Determine the approximate maximum angle of sideslip which develops. (c) Calculate the aileron deflection needed to counteract the rolling moment produced by the sideslip. (d) With the starboard engine failed, determine the aileron and rudder deflections needed to fly the aircraft with a bank angle not exceeding 5°. Calculate the sideslip angle which results from this manoeuvre. (e) Why is the effect of a lower speed on the ability of the pilot to control the aircraft with the remaining engine? 10.10 A training aircraft, with a roll control system, is flown by a novice pilot. The transfer function of the aircraft with the system is given by: ... The pilot was found to have the following transfer function ... 10.11 ... REFERENCES ...

speed schedule, for CHARLIE-1,is given in Figure 11.41. At the start of the coupled glide path descent, the airspeed, Uol, is 85.0 m s-l; thirty seconds later, it is 65.0 m sf1. During that time the aircraft will have travelled a slant distance of: Time (s) Figure 11.41 Airspeed schedule for CHARLIE-1. The horizontal distance covered is actually 2 248 m (assuming aircraft descends along the glide path). The height at the start of this manoeuvre is 320 ft. A typical set of parameters, corresponding to CHARLIE-1,is given in Table 11.3 and the corresponding dynamic response to an initial displacement, d, of 100 ft above the glide path, is shown in Figure 11.42. It is evident how effective this system is in restoring the aircraft to the glide path and maintaining it there subsequently. Flight Path Control Systems Table 11.3 Parameters of glide-path-coupled system (Flight Path Control Systems Figure 11.42 Response of glide-slope-coupled system. For the aircraft BRAVO-3 the system was simulated and the results are presented in Figure 11.40, and using the values of the parameters listed in Table 11.3, it can be shown that the closed loop dynamics can be represented in that form, representing a generalized AFCS, which was explained in Section 7.2 of Chapter 7. Aircraft dynamics where: X & [U w q θ SE] ui' (SE) - - (0.021 0.122 - 0.2 - 0.512 0.65 11.00004 - 0.006 - 0.402 A = - - 9.81 0.292 - 1.96 - 0.4 0 0 1 0 0 0 0 0 0 - 10 For CHARLIE-1 (11.49) ILS Glide-path-coupled Control System Output equation y = Cx where: y - 4 [a q 0i to dol 0 015 0 0 0 C= 0 1 0 0 0 Controller dynamics = Acxc + B,y + E (11.54) Controller output equation is: Yc = Ccxc + DcY (11.55) with: s - Ac YC (11.56) From Figure 11.40 and Table 11.3 it can be deduced that for the glide path coupled control system the control law is: Furthermore, it can be seen that: If we let: I with 4 %, 396 Flight Path Control Systems Then if the third term on the r.h.s. of eq. (11.57) is omitted then: ay = 0.42 + 1.042 + 0.12 e = 0.042 Let : Hence: Hence, the third term on the r.h.s. of eq. (11.57) can be written as: ILS Glide-path-coupled Control System Hence: - A, = 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 Let: then the closed loop dynamics of the glide-path-coupled system can be expressed as: irr-Kv+L where: (11.78) 398 Flight Path Control Systems Table 11.4 Eigenvalues of glide-path system at two values of range Range = 4000 m r-1 = 0.0 A2 = - 0.005 A3 = - 0.013 A4 = - 0.1 A5 = - 0.45 A6 = - 1.0 A7 = - 3.2 A8 = - 4.0 Range = 2000 m Range = 200 m 0.001 0.0024 A5 = - 0.0026 A5 = - 0.004 A6 = - 8.91 Å7 = - 1.57 A6 = - 3.58 -1.9 A7 = - 2.58 - 4.1 K7 The eigenvalues of the closed loop system just described, which correspond to values of range, R, of 4000m and 200m, are shown in Table 11.4. It can be deduced from these values of the closed loop roots that at a range of 4 000 m the glide-path-coupled system is stable, but when the range has reduced to 200 m it is unstable. But even in the case of R = 200 m, the instability is mild. 11.8 ILS Glide-path-coupled Control System the glide path at the c.g. However, if the aircraft receiver is installed at, say, the nose of the aircraft, the dynamics of the system are affected, as follows. From Figure 11.43, the height measured at the receiver is: (assuming θ is small). Therefore the flight path angle at the receiver is: Glide path receiver(' - 1- Figure 11.43 Glide path receiver located at aircraft nose. 399 Automatic Landing System Thus, the effect of locating the glide path receiver in the aircraft's nose is to introduce a phase advance term into the closed loop dynamics. 11.10 AUTOMATIC LANDING SYSTEM Although the contribution to the development of airborne automatic landing systems has been international, the basis of most of the operational systems in service is the system developed in the UK by the Blind Landing Experimental Unit (now disbanded) of the Royal Aerospace Establishment. It makes use of the ILS, and the entire automatic landing segment is made up of a number of phases which are shown in Figure 11.44. At the start of the final approach phase (point 1 in Figure 11.44), the aircraft being considered is assumed to be guided on the glide path by a glide-path-coupled control system of the type described in Section 11.9, and to be steered onto the runway centre-line by means of the ILS localizercoupled control system described in Section 11.8. What has been described above is a category (1) automatic landing. What distinguishes landings into the various categories are the conditions of visibility. These categories are summarized in Figure 11.45; it can be seen that each category is defined as a combination of the decision height (DH), i.e. the minimum Point 1 Outer marker radio beacon 2 Middle marker radio beacon 3 Start of flare phase 4 Start of KOD manoeuvre 5 Point of touchdown G 10 0 8 m hard shoulder -, L 8 000 m Figure 11.44 / 3 000 m runway BLEU aircraft automatic landing. Flight Path Control Systems 60 - -E , • 8 • ,P 00 h , y. > 30I • t'E - I = I I I Category IIIa 200 I I I I 400 600 800 1 000 1 200 RVR (m) Figure 11.45 Definition of landing categories. I permitted ceiling for vertical visibility for the landing to proceed, and the runway visual range (RVR). Categories I and II allow only coupled glide path approaches down to the DWRVR combinations defined in Figure 11.45. At the DH, the pilot either continues the flight under manual control to land, or executes a go-around manoeuvre either to attempt once more to land at the airport, or to divert to an alternative. Category IIIa allows the aircraft to make an automatic landing by providing an automatically controlled flare phase, with the pilot taking over control of the aircraft at the point of touchdown. Category IIIb allows the aircraft to use automatic flare and roll-out, with the pilot assuming control only at some distance along the runway after touchdown. Category IIIc is totally automatic landing with automatic taxi-ing: no system has yet been certified as being able to provide category IIIc performance. The DHs which obtain for categories IIIa and IIIb vary with airline and aircraft type. A summary of some airlines and aircraft types, given merely for illustration, with no intention of being definitive, in Table 11.5. The automatic flare control system is arranged to provide a flare trajectory corresponding to that shown in Figure 11.46(a). Therefore the system represents the path of the aircraft's wheels as the landing is carried out. During this flare manoeuvre, the flight path angle of the aircraft has to be changed from - 2.5° to the positive value which is recommended for touchdown; in other words, during the flare manoeuvre the control system must control the height of the aircraft's c.g. and its rate of change such that the resulting trajectory corresponds as nearly as possible to the idealized exponential path shown in Figure 11.46(a), while at the same time causing the aircraft to rotate in a fashion similar to the representation of Figure 11.46(b). The equation which governs the idealized, exponential flare trajectory shown in Figure 11.46(a) is: Automatic Landing System Table 11.5 Landing categories for different airlines Airline Aircraft types Minimum values RVR (m) British Airways Lufthansa Air France Swiss ~ i KLM DELTA TWA r ~ DH (ft) Trident 3 Tristar Concorde B-757 A300 A300 Concordea DC-10 B-747 Tristar Tristar " A t Paris, Charles de Gaulle airport, only. A t Zurich only. The distance from ho to the point of touchdown depends on the value of ho, the flare entry height, and the approach speed of the aircraft, Uo. Usually the point of touchdown, which is aimed for, is 300 m from the runway threshold which is the nominal location of the glide path transmitter (see Figure 11.35). Assuming that the airspeed does not change significantly throughout the flare trajectory (a not unreasonable assumption), then: Glide outh centre-line Exponential flare trajectory y' ; CL,- ~ ntr.i CLI CIIIJJ height Point of touchdown x , I Ground Glide path transmitter (a) Glide path centre-line Point of touchdown Runway threshold Ground Figure 11.46 (a) Flare trajectory. (b) Rotation of aircraft during flare. Flight Path Control Systems so: Uo sin y = Uo sin(- 2.5°) (assuming heights are measured in metres). Hence: The constant of integration is found from the initial condition that h(0) = h0 . Hence: CONDITION (A) of CHARLIE-1 of 57.3 m s-l). From eq. (11.84) it can easily be shown that: If the time to complete the exponential flare is taken as 57 then: (x * 300) = u057 = (286.57 (11.87) From eq. (11.86): ho = - h.o / - (11.88) hence: - 2.5 = - h.d = - , h.o = 2 . 5 T From Figure 11.46(a), ho = x.tan2.5° = 0.0435x, therefore: 7 = (0.0435)12.5) = Hence, substituting eq. (11.90) in eq. (11.89) yields: x = 300 + (286.5 x 0.0435)/2.5) -,. + ~,. • r = - L - r = r=75.2m (11.91) ho = 3.25m = 10.65 ft (11.92) 1.3 s (11.93) Hence, the ideal flare manoeuvre is assumed to take 6.5 s to complete. The flare law, which is given in the automatic flare trajectory is given by: (11.94a) h = - (h,f,T), the Kr_brsl is taken, as a gain. Thus the ideal flare trajectory is given by: Automatic Landing System A Figure 11.47 Automatic flare controller em, - Pitch attitude control system kinematics 1 - + T = ~ Radio altimeter Figure 11.47 Block diagram of automatic flare control system. Automatic Landing System 403 Note that the pitch attitude control system is used: changing 0 results in a change in flight path angle, and consequently, a change in height. Because the heights involved are very low, an accurate measurement of height is necessary for this control system: a low range altimeter is used. The control law used can be simply: but, to ensure accuracy, it is usual to add an integral to the proportional term so that: The addition of the integral term, and the need to remove, by filtering, any noise from the height signal obtained from the radio altimeter, tends to destabilize the closed loop system. Consequently, it is customary to include a phase advance network with the feedback terms to improve the stability, i.e.: 4.0 - -12.0 -12.0 - 16.0 0.0 1 4.0 1 8.0 12.0 Time (s) I 16.0 Time (s) I 16.0 Time (s) Figure 11.48 Flare trajectory response. I 20.0 404 Flight Path Control Systems where p = 0.6lt and TI 'F2. The results of a digital simulation of such an automatic flare control system for a particular flare entry condition is shown in Figure 11.48. Because the model flare trajectory is exponential it takes infinite time to reach zero height, but the height, h, as Kfrt is taken as a - 1.5 ft, thereby ensuring that the wheels will touch the runway at a time of much nearer 57 s. Obviously the methods of modern control theory can as easily provide a feedback control law to achieve automatic flare control. Either model following, or solving a LQP, will provide effective feedback control laws. 11.11 A TERRAIN-FOLLOWING CONTROL SYSTEM 11.11.Introduction The terrain following situation is represented in the sketch shown as Figure 11.49. A change of range, the horizontal distance, is denoted by Ax; Ah denotes a change of the height of the aircraft. The reference height (of the obstacle to be cleared) is denoted by ho and the flight path angle is denoted by y, as usual. For the aircraft to be considered, which is a representative (or generic) strike aircraft, there are two controls, namely a commanded change in normal acceleration measured at the aircraft's centre of gravity, a, which is denoted ai, and a cg' change in the rate of opening or closing of the throttle which is denoted by u2. It is customary in terrain-referenced navigation to use, as a technique for estimating the aircraft's position, a comparison of the return signals of the radar Pushover -+ — Ax Figure 11.49 Terrain following geometry. A Terrain Following Control System 405 altimeter, (an accurate, short range, vertical radar system), with those obtained from a three-dimensional terrain model which has been digitized and stored on an on-board computer; this complex method is used to avoid, as much as possible, active sensing of the terrain, thereby reducing the chances of being detected. However, there may be unmapped obstacles, so that terrain-following by looking ahead in a map database is not really safe and recourse to a forward-looking infra-red (FLIR) system, is often taken. For the purposes of explaining the terrain-following control system it will be assumed that range can be measured by FLIR or laser rangefinder and that changes in height are measured using a radio altimeter. 11.11.2 Equations of Motion The aircraft's equations of motion, assuming a stability axis system and small perturbations, can be expressed as: Note that the elevator is not being used as a control. To relate the aircraft's motion at point A to the obstacle, and thereby to design an effective control system to control the aircraft automatically to avoid the obstacle, it is necessary to transform these equations into earth-fixed horizontal and vertical axes. For the linearized situation being studied, the flight path angle at point A is assumed to be zero. Hence: ; h = Uoy - w, and Therefore: from which: and Therefore: Flight Path Control Systems Figure 11.50 Block diagram of terrain following system. Any change in thrust depends upon u2, and, as a first approximation, can be represented by a simple, linear model, namely: The system is represented in the block diagram shown as Figure 11.50. KE represents the gain of the engine and TE its time constant. From Figure 11.50 it may also be deduced that: x4 = - - - - - - - - - - - - b (11.108) By defining the state vector x as X = [U w h x y T]' and the control vector as u = [u1 u2]', it is possible to obtain as a mathematical representation of the aircraft dynamics involved in the terrain-following situation a state equation, namely: Table 11.6 State variables definition for terrain-following system State variable Motion variable Notation XI Vertical acceleration Rate of change of height Height Thrust commanded Change in thrust Change in airspeed h x2 X3 X4 X5 X6 h S THR, - T H u 407 A Terrain Following Control System The coefficient and driving matrices for the generic strike aircraft are given as follows: A = - 1 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 0 0 - 1 0 - z w uo xo ooxu - 1 y 0 The output variables are defined in the manner indicated in Table 11.7, then the output matrix, C, is given by: 1 0 0 0 0 - 0 1 0 0 0 - 0 C = 0 1 - 0 0 0 1 0 0 0 (11.112) 0 0 0 0 0 1 - 0 0 0 0 0 1- The following parameters and coefficients obtain for the strike aircraft at Mach 0.85 at a height of 40000ft. Table 11.7 Output variables definition for terrain-following system Output variable Y l Y2 y3 Y4 Y5 , - Output equation Motion variable XI Vertical acceleration Height Thrust rate Change in thrust Change in airspeed X3 X4 X5 X6 - X5 Flight Path Control Systems Hence: The aircraft dynamics are represented in Figure 11.50. 11.11.3 The Control System By framing the terrain-following problem as a LQP, with an appropriate performance index to be minimized, the feedback control is found as a solution to the LQP. The performance index is chosen to be: where: z denotes the vector defining the desired flight path. This vector is defined in terms of vertical acceleration, height, required thrust rate, thrust and speed. These desired inputs have to be available as instantaneous values of the variables. Therefore: el denotes the error in the vertical acceleration, e2 the deviation in the aircraft's height from the desired path, e3 the error in the thrust rate developed, and e5 the deviation in the aircraft's speed from the reference value. A method of solving such an LQP was discussed in Section 8.4 of Chapter 8, from which it can be shown that: A Terrain Following Control System 409 where: i + A - B G - + B' 'L - C'(Qe + QC - 0 (11.1) ((11.119), (11.122), (11.14) and (11.15) the parameters shown in Table 11.8 were used. Table 11.8 Elements for weighting matrices in LQP The reference vector, z, is chosen to be: i.e. the aircraft path requires constant speed, no-change in thrust, no vertical acceleration and a constant height of 200 ft, i.e. the aircraft encounters a sudden step change in the terrain of 200 ft. Using matrices A, B, C, Q and G so far given, the resulting control law is given by: The appropriate variables, hi, h2, k3, k4, k5, and k6, which make up the vector, h, are shown in Figure 11.51. These represent the solutions to eq. (11.120) for the forward vector z given in eq. (11.123). The resulting terrain-following trajectory for an initial aircraft height of 300ft is shown in Figure 11.52; the variables, yi, corresponding to the deviations being controlled, are shown in Figure 11.53. Note how height (y2) changes from 300 to 95 ft and there is a peak height of 350 ft before settling to the required height of 200 ft, after 1 minute of flight time. In executing that manoeuvre the 4 10 Flight Path Control Systems Curve A = hi Curve B = h2 Curve C = h3 Time (X h2 x) Time (X h3 x) Stand hi: 6 , H /4- Time (X 1s2 s) (0 Figure 11.51 Solution of eq. (11.120) for control law Time (s) Time (S) Figure 11.52 Terrain-following trajectory. A Terrain Following Control System Curve A = h4 Curve B = h5 Curve C = h6 Time (X h5 x) (X h6 x) Stand h4 Time (X 1s2 s) Figure 11.53 System response. 4 1 1 0 4 12 Flight Path Control Systems Aircraft has to undergo a - 2g to + 2g (approximately) change in about 10 s. The airspeed is also required to increase from the equilibrium value of 933.3 to 1303.33 ft s-l, i.e. the aircraft is required to go supersonic at t = 30 s: this is most improbable. To avoid such a speed excursion it is necessary to solve the problem again with a much larger value of q5, to penalize such deviations in airspeed. 11.12 CONCLUSIONS Although the chapter is devoted to path control systems, it is opened with detailed studies of automatic control systems which control aircraft flight variables such as airspeed, Mach number and height, rather than path variables such as heading or bearing to a transmitter. Direction and heading control are treated next, so that they can be used as elements in the automatic tracking systems which depend upon the radio transmissions and appropriate airborne receivers for VOR, ILS localizer and glide path to obtain the appropriate guidance commands for their tracking systems. From a detailed consideration of ILS localizer and ILS glide-path-coupled systems it was the next step to consider automatic landing, including a flare phase, and then finally a terrain following system which employs LQP theory. 11.13 EXERCISES 11.1 A business jet aircraft uses a speed hold system to assist the pilot with ILS coupled approaches. The block diagram of the system is shown in Figure 11.54. The aircraft dynamics are represented by: Uo = 72m spl W = 980658 Maximum thrust = 54655N Xu = - 0.0166 LID ion approach 2; = - 0.175 = 8.0 Controller Throttle andactuator ~ engine Engines Change in thrust Aircraft dynamics Pitot system Figure 11.53 Block diagram of a speed control system Exercises 411 (12. Exercises 413 (a) If the system has 20 per cent authority, calculate the gain of the engines, K E . (b) If the time constants associated with the engine, the throttle actuator and pitot system are all negligible show that it takes 167s to achieve a new commanded speed when K1 = 1.0. (c) If the effective time constant of the engines is 1.0 s, K2 = 7.5 and K3 = 5.0 what is the maximum value that K1 can take before the closed loop system becomes unstable? 11.2 (a) For the landing system represented in Figure 11.54 an aircraft is coupled to the glide path via its receiver and its AFCS. For any given departure from the glide path measured by the perpendicular distance, d, show that the angular error of the aircraft from the nominal glide slope increases with the integral of the flight path angle. Assume the airspeed is constant. transmitter Figure 11.54 Glide path geometry for Exercise 11.2. (b) The flare manoeuvre, performed at touchdown, results in the rate of descent of an aircraft being decreased in an exponential manner. If an aircraft has a constant forward speed of 80 m s-' and the distance to the touchdown point from the runway threshold, at which the glide path transmitter is located, is 500 m, calculate the following for a glide path angle of 2.5°: (i) The approximate time to execute the flare manoeuvre. (ii) The height of the aircraft at the start of the flare phase. Exercises 413 (iii) The ground distance travelled during the manoeuvre. (c) Draw a block diagram of a complete automatic landing system which uses the glide path transmission to measure the departure of the aircraft from the approach trajectory. Show clearly all the significant variables of the system and indicate the function of each block. Detailed transfer functions need not be shown. 11.3 The bank angle control system developed in Section 10.3 of Chapter 10 for CHARLIE-2 is used as the inner loop of the direction control system whose block diagram is shown in Figure 11.18. With a value of controller gain, KT, of 2.0 the response was very slow, the settling time being about 28.0s, although the response was well damped with no oscillation in the aircraft's heading evident. It is known that increasing the value of KT to 16.0 leads to an oscillatory response, which is unacceptable. (a) Find a value for the gain, KT, of the controller of the direction control Flight Path Control Systems system 11.22 such that the direction response to a step command exhibits no overshoot and settles within 7.0 s. (b) What is the maximum value which KT can take before the direction control system becomes unstable? (c) If the controller gain, KT, is set to a value of 8.0 determine the response of the system to the sideslip cross-wind profile shown in Figure 11.41. 11.4 A cargo aircraft, with four turboprop engines, has been proposed for use as a gunship for counter-insurgency (COIN) operations. A control system is to be used so that the errors in positioning the aircraft in relation to its target are minimized, even in the presence of head and cross-winds. Three aerodynamic control surfaces - elevator, ailerons and rudder - are to be used. The state variables of the aircraft have been defined as follows: change in airspeed, u change in angle of attack, a change in pitch rate, q change in sideslip angle, f4 change in roll rate, p change in yaw rate, r change in bank angle, change in pitch attitude, and the control vector is defined as: u [SE SA SR1' where the angle of attack error, ET, = target azimuth error, E' T = target azimuth error, E' T = head-wind component, up cross-wind component, wg integral of target elevation error (= - e' t) integral of target azimuth error (= - r (- e' t) * The controls used on the aircraft are: SE elevator deflection SA aileron deflection SR rudder deflection Exercises 415 A = - 0.0013 2.1 15.7 - 0.5 9.7 0.0 - 9.8 - 0.0013 -1.5 - 1.95 1.0 0 0 0.003 2.7 - 0.2 - 0.52 1.00 0 0 0 0 0 1.0 0 0 - 0.1 0.0 0 - 4.0 0 0 - 0.06 - 0.9 0.00 0.0 0 0.4 0 0 0 0 0 0 0 0 0 0 0 0 0 1.0 0 0 0 0 0 0 0 0 0 0 0 0 -1.0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 ...

Elements of B and C matrices used to determine the corresponding feedback gain matrix. (d) Determine the eigenvalues of the closed loop system. Compare these with the values corresponding to the uncontrolled aircraft. Comment upon whether the closed loop system has the required response to a crosswind? 11.5 The following stability derivatives relate to a VTOL aircraft in hovering motion: ... and is used to control the If a feedback control law, ST = h , aircraft, show that its change of height is characterized by a critically damped transient mode. At what time after a step input of h, does the rate of change of height reach its maximum value? Exercises 4?7 The block diagram of a speed control system used with the aircraft described in Exercise 2.9 is shown in Figure 11.55. 11.6 Controller Throttle servo Engine dynamics Aircraft dynamics - Accelerometer Figure 11.55 Block diagram of a speed control system for Exercise 11.6. Making use of appropriate numerical approximations determine the maximum permissible value of accelerometer sensitivity for stability of the closed loop speed control system. 11.7 The glide slope receiver in the aircraft CHARLIE measures the angular deviation of the aircraft from the glide slope. The receiver is mounted in-line with the aircraft's c.g., but directly below the pilot. (a) Determine the transfer function y (s) / 0 (s) which is appropriate to this location of the receiver. (b) Show from Figure 11.40 must be modified to take account of this new location of the receiver. (c) Is the location of the receiver beneficial, or otherwise, to the performance of the coupled system? 11.8 For BRAVO-3,using the system represented in Figure 11.5 with K, = 1.5 and an accelerometer time constant of 0.5 s, find a suitable value for the gain of the controller such that the Mach hold system is effective. What happens to the system's performance if the accelerometer fails in service? 11.9 Design a flight control system, suitable for localizer coupling, for the aircraft approach. It can be assumed that over the approach phase the aircraft speed is constant. At what range from the runway threshold will your system become unstable? Show how your system performs in the presence of a constant cross-wind from the right (i.e. from starboard) of 20 knots. ALPHA's 11.10 The VOR-coupled control system, represented by the block diagram of Figure 11.32, uses the directional control system of Figure 11.18 as its inner loop. The value of gain chosen for the directional controller was 2.0. Use the value of K*, evaluated in answer to Exercise 11.3(a), with the system of Figure 11.32, when the value of the gain of the VOR coupling unit is chosen to be 25.0 and the value of the gain associated with the integral term is 0.025. Compare the response obtained with that shown in Figure 11.33. Which is better? Suppose Kc is increased to 0.25. What is the likely effect upon the performance of the VORcoupled control system? 11.14 NOTES 1. 'All-weather' is merely a euphemism used in aviation circles to describe the bad weather which the British habitually enjoy. ILS is being replaced by MLS, the microwave landing system, which involves the use of transmission frequencies which avoid those errors which limit the ILS. However, the principles involved in providing the guidance signals are sufficiently similar for the account being given here based on the ILS to suffice. Although it is not shown on the diagram, the speed control system (of the type represented in Figure 11.9) is assumed to operate and it has the following parameter values: KE = 30 000, KCu = 3.0, Kel = 0.4. 2. 3. 11.15 REFERENCES BLAKELOCK, J. 1965. Automatic Control of Aircraft and Missiles. New York: Wiley. KAYTON, M. and W.R. FRIED. 1969. Avionics Navigation Systems. New York: Wiley. McRUER, D.T., I.L. ASHKENAS and D.C. GRAHAM. 1973. Aircraft Dynamics and Automatic Control. Princeton University Press. 1979. Airplane Flight Dynamics. Kansas: Roskam Publishing. ROSKAM, J. Active Control Systems 12.1 INTRODUCTION Although there is now available a large amount of published material relating to active control technology (ACT), which is being added to continuously, there are few satisfactory definitions of what ACT is. It has been called an extension of conventional feedback control systems which provides a multi-input, multi-output capability to allow full exploitation of the complete six degrees of freedom of an aircraft (Ostgaard and Swortzel, 1977). A later definition proposed, however, that ACT should be considered to be 'the use of motion feedback control systems, in the absence of passive design features, to achieve specific design objectives' (Houghton, 1978). Neither definition is complete. In this textbook, it is proposed to use a definition which is based upon those earlier versions, namely: Active control technology is the use of a multivariable AFCS to improve the manoeuvrability, the dynamic flight characteristics and, often, the structural dynamic properties of an aircraft by simultaneously driving an appropriate number of control surfaces and auxiliary force or moment generators in such a fashion that either the loads which the aircraft is subjected to, or its trajectory, or both, are controlled in a way that would otherwise be impossible. Using the word that is used, ACT means that the aircraft will have been designed from the outset to use the feedback control system to provide the means by which the aircraft is to perform its mission. ACT provides the means to enhance the performance and operational flexibility of an aircraft. Modern aircraft are designed to attain maximum aerodynamic efficiency with considerably reduced structural weight, an achievement owing much to the use of new materials. The current design requirements for many aircraft, and the missions they are to perform, are such that the resulting configurations are considerably changed from the familiar designs of earlier times. In meeting the new requirements, the designs have typically exchanged: thin, lifting surfaces, a long, slender fuselage, a low mass fraction structure, a design admitting a high level of stress, and low load factors. These features have resulted in aircraft which are of the required structural lightness, but which, consequently, exhibit considerable flexibility. Such aircraft can develop structural displacement and accelerations of large amplitude as a result of the structural deflections and the rigid body motion of the aircraft. The structural deflections can arise either as a result of some manoeuvre Active Control Systems 420 command from the pilot, or from a guidance or weapons system, or from encountering atmospheric disturbances. The structural vibration which results can impair the life of the airframe because of the repeated high levels of stress and the peak loads to which the aircraft is subjected. With such new aircraft, a new class of flight control problems has emerged, including: the need to minimize the loads experienced by the aircraft, either completely or at just a few specific locations; the particular requirement for transport aircraft to precisely control the location of an aircraft's c.g. over its entire flight envelope; the suppression of flutter; the reduction of the amplitude of the disturbed motion of an aircraft which is caused when it encounters turbulence. To solve such problems by using modifications of the components of the airframe, such as increasing the damping or stiffness of the structure, or modifying the planform and the size of the conventional control surfaces, usually by increasing them, even in the few cases where there are feasible solutions, would impose severe economic penalties upon the aircraft's operation, mostly in terms of reduced payload, or reduced performance in terms of range or speed. Consequently, ACT was proposed to meet the evolving demands for more effective and efficient aircraft. 12.2 ACT CONTROL FUNCTIONS It is generally agreed that the most beneficial effects of using ACT will be secured by using any, or all, of these six ACT functions: relaxed static stability (RSS), manoeuvre load control (MLC), ride control (RC), flutter mode control (FMC), gust load alleviation (GLA) , and fatigue reduction (FR) . 12.2.1 RSS By relaxing the requirement for static stability it is possible to achieve better dynamic response to the aerodynamic controls and to reduce the trim drag and thereby enhance the aircraft's manoeuvrability. It is necessary when doing this to restore the aircraft's dynamic stability and its handling qualities by using an ACT system. When the need for static stability is relaxed, the empennage required on the aircraft is smaller: an empennage is sized to provide the aircraft's trim and manoeuvre requirements. With a small empennage, some savings in the weight of the aircraft are possible. An SAS for an aircraft with RSS - aircraft BRAVO of Appendix B - is discussed in detail in Section 9.4 of Chapter 9. By using RSS on an aircraft it becomes feasible to provide 'carefree manoeuvring'; sometimes the RSS function is an element of an aircraft's 'enhanced manoeuvre demand' systems. ACT Control Functions 12.2.2 MLC MLC is a technique of redistributing the lift generated by the wing of an aircraft during a manoeuvre. By the symmetrical deflection of control surfaces, mounted at proper stations in the trailing edge of the wing, in response to load factor commands, it is possible to reduce the increments in the stress by arranging for an inboard shift of the centre of lift of the wing. This shift also reduces the bending moment at the wing root, which is a major factor in the fatigue life of a wing. MLC is sometimes referred to as active lift distribution control (ALDC), or a structural mode control (SMC) system, which it is called on the USAF bomber, the B-1. Occasionally such systems are provided simply to ensure that any loads which arise from the execution of some particular manoeuvre do not exceed some specific limit. The purpose of the RC system is to improve ride comfort for the crew or passengers by the reduction of objectionable levels of acceleration which are caused by the rigid body motion of the aircraft, such as a bomber, an RC system is required to reduce the accelerations on the crew compartment so that the crew can perform their tasks effectively. In a transport aircraft carrying passengers the requirement may be for a reduction in accelerations to be achieved over the whole length of the passenger cabin. For an interdiction aircraft, carrying out high speed strike missions at low level, the principal need is to prevent the pilot's ability to track his target from being impaired by the accelerations at the cockpit which are a result of flying in turbulence. Obviously the mission requirements considerably influence the purpose and the design of the system used to improve an aircraft's ride characteristics. 12.2.4 GLA GLA is a technique which controls the contribution of the rigid body and the bending modes to the complete dynamic response of an aircraft to a gust encounter. Its purpose is to reduce the transient peak loads which arise from such encounters. Since their purpose is similar to that of RC, the two functions are often achieved by a single system. Moreover, a successful GLA system will contribute to the reduction in structural loading so that MLC and GLA are quite likely to be used in conjunction with each other. 12.2.5 FMC By properly controlled deflection of certain auxiliary control surfaces it is possible to damp the flutter modes of an aircraft without having to increase structural Active Control Systems 422 weight. There may also be an increase in the flutter speed. The principal benefit of FMC in flutter speed is the increase in permissible wing-mounted stores which can be carried within the same speed envelope. The benefit for bomber and transport aircraft cannot be so direct, and it may be limited on those aircraft to a possible reduction in the weight of the wing. Of all the ACT functions, FMC is the most sensitive to configuration, particularly to planform and thickness of the wing. To reduce the rate of fatigue damage, FR systems minimize the amplitude and the number of transient bending cycles to which the structure may be subjected during flight in turbulence. This ACT function has not yet been implemented physically on any aircraft, and at present, its objective is achieved indirectly as a result of the combined action of the other five ACT functions. It remains, however, as potentially one of the most economically advantageous ACT functions. 12.3 SOME BENEFITS EXPECTED FROM ACT The potential benefits from applying ACT functions depend upon several aircraft parameters. However, the only function which will provide its benefits independent of the speed range of the aircraft is ride control. GLA, MLC and FR will be particularly beneficial for STOL aircraft because of the low wing-loading which such aircraft have, presuming that the STOL aircraft being considered are provided with aerodynamic rather than propulsive lift. To obtain the performance needed for commercial operation, supersonic transport aircraft must operate over a very wide range of dynamic pressure which greatly affects the handling qualities. Some form of stability augmentation is required. If RSS is decided upon to obtain a reduction in drag, thereby conserving the fuel consumption, the SAS is required to provide the dynamic stability in addition to improving the handling qualities. SST, and modern bombers such as the B-1, have a long slender fuselage, the forward position of which acts as a cantilevered beam mounted forward of the stiff structure. This construction typically results in fuselage bending at low frequencies. Such low vibration frequency causing considerable discomfort to passengers and crew alike. Hence, on such aircraft, an RC system is needed. On the B-1, the SMC system also acts as a ride control system at low altitudes; the function is referred to in this aircraft as a low altitude ride control system. The cruise phase of flight usually takes place at high altitude, over long stage lengths. Consequently, new transport aircraft have their configurations designed particularly for energy conservation. High aspect ratio wings and RSS will be employed to effect some drag reduction. As a Gust Alleviation 423 result, the aircraft's handling qualities in high altitude turbulence will be poor and, therefore, a GLA system will be required. The principal area in which the many benefits of ACT will be seen to greatest advantage is that of aerial combat. Some present-day fighters require co-ordination of eight to eleven control surfaces to provide the aircraft with the controlled motion in six degrees of freedom, even though the two extra degrees of freedom in translation will inevitably be limited. Figure 12.1 represents the six degrees of freedom which ACT can provide. The extra degrees of freedom in translation provide that improvement in manoeuvrability which can lead to superior combat tactics. 4 Yawing moment Direct sideforce (Drag modulation) Rolling moment Direct lift force pitching moment Figure 12.1 Six degrees of freedom of an aircraft. 12.4 GUST ALLEVIATION 12.4.1 Introduction In Chapter 5 it is pointed out that the way through which an aircraft flies is constantly in turbulent motion. Consequently, the aircraft's aerodynamic forces and moments fluctuate about their equilibrium (trimmed) values. These changes cause the aircraft to heave up or down, to pitch its nose up or down, to roll about the axis OX, or to yaw from side to side about the aircraft's heading. These motions result in accelerations which are experienced by passengers and crew as unpleasant effects. To reduce these accelerations it is necessary to cancel the gust effects by other forces. The general principle of gust alleviation is that specially located sensors provide motion signals to a controller which causes appropriate deflections of suitable control surfaces to generate additional aerodynamic forces and moments to cancel the accelerations caused by the gust. Several methods of achieving such alleviation have been proposed - almost since the beginning of manned flight since early aircraft were particularly susceptible to being upset by gusts. One of the earliest pioneers of flight, Lilienthal, was killed in 1896 when his glider was so upset. A patent was granted in 1914 in the USA to a Mr A. Sperier for a 424 Active Control Systems 'stabilizing device for flying machines'. The device was claimed to counteract the disturbance and to prevent it from having an injurious effect on the stability of the machine'. In 1915 the very first NACA report by Hunsaker and Wilson (1915) contains a reference to the problem was made by early British workers in aircraft stability and control from 1914 up to the Second World War. The foundation papers which established the basis for suitable mathematical representations of turbulence were published by Von Karman (1937) and Taylor (1937). There was a proposal for a gust alleviation system in 1938 by a Frenchman which was eventually flight tested in the USA in 1954. In 1949, the Bristol Brabazon aircraft (then the largest in the world) was fitted with a GLA system. The Brabazon system used complementary deflections of the ailerons in response to signals from a gust vane mounted on a probe ahead of the aircraft's nose. Successful flight trials with this kind of system were carried out in the USA in the period 1950-1956 and experiments were carried out from 1955 to 1960 by the RAE in England using an AVRO Lancaster. All these attempts, except the Brabazon, were concerned solely with alleviating the effects of gusts on the rigid body motion. In every case, however, the results achieved were unsatisfactory. In the case of the RAE experiments with the Lancaster, a considerable loss of stability was observed which arose as a result of the larger pitching moment which was created by the symmetrical aileron deflection. This moment led to a decrease in the effectiveness of the alleviation system at large gust gradients. The American systems, like the Brabazon system, depended upon a gust vane to detect the aircraft's entry into the gust field by sensing either changes of pressure or a change of direction of the relative wind. They were unsatisfactory chiefly because it was not appreciated that any gust has components normal to the plane of symmetry of an aircraft and because secondary effects, such as changes in flight condition, downwash effects on the tailplane, the time delay between the wing's encountering the gust and then the tail, were not considered. Thus, gust vane systems tried, in effect, to provide control correction in advance of the actual gust and were really feed forward systems. They were unsuccessful because the control system could not be designed then to provide the necessary speed of response, nor made insensitive enough to the secondary effects mentioned earlier. In many of these tests, the operation of the GLA systems actually caused a deterioration in the gust behaviour of the aircraft to which they were fitted. The fundamental problem with GLA systems is that, when a gust has been sensed, the system cannot take action until it is too late to achieve much effect. These defects were noted and avoided by Attwood et al. (1961) who proposed in their patent application of 1955, which was granted in 1961, that the GLA should Gust Alleviation 425 sense linear and angular accelerations and should use auxiliary control surfaces to produce the countering forces and moments required to minimize the unwanted accelerations. Some further developments stemmed from that work including, the prototype XB-70 bomber, this system was introduced to reduce to sensitivity to side gusts in high speed, low altitude role; and also the prototype American bomber, the XB-70. It was an event in 1964, however, which accelerated the present interest in gust alleviation. A B-52E bomber of the Strategic Air Command of the USAF encountered severe turbulence, with an estimated peak velocity of 35 m s-l, on a low-level mission over territory in the western USA. Approximately 6 s after penetrating the gust field its yaw damper was saturated and the response of the then 'unaugmented' rigid body dynamics was such that about 80 per cent of the fin broke off. This event led in 1965 to an extensive flight development program, known as the load alleviation and mode suppression (LAMS) program, being carried out by the USAF and its contractors. The results of the programme was presented in the report of Burris and Bender (1969). The work was extended in 1973 and the GLA function was used in the RCS which was developed to provide improved ride quality (Stockdale and Poyner, 1973). 12.4.2 Gust Alleviation Control The objective of the gust alleviation control system is to reduce the structural vibration excited by turbulence may be reduced it either the amount of energy transmitted from the gust to the bending modes is reduced or any energy which is transmitted to the bending modes is rapidly dissipated. Both methods should be employed simultaneously for optimal effectiveness. To reduce the energy being transferred requires a countering moment (or force) from the deflection of some control surface. The method requires an accurate knowledge of the aircraft's stability derivatives. Of course, these derivatives change with flight condition, with mass and the mass distribution of the aircraft, with changes in dynamic pressure, etc. Consequently, the aircraft dynamics are known too imperfectly to admit of perfect cancellation of any gust forces or moments. Once the energy has been absorbed, its dissipation can be controlled by augmenting the damping of the elastic modes. It is difficult, however, to achieve a sufficient increase in structural damping by use of feedback control since the aerodynamic forces and moments available are limited. Only the disturbing forces and moments, which are not perfectly cancelled, are transmitted to the bending modes. Furthermore, if the structural modes are closely coupled, then it is often hard to move a particular mode of energy between the modes which have structural damping, thereby contributing to the behaviour of very lightly damped structures. To actively suppress the bending of a structure it is necessary to be able to sense either the structural displacements or the associated rates of change. It is possible to sense these quantities to provide motion signals for feedback in the GLA control system. However, as it had been discovered in the early GLA tests, the control surfaces suitable for controlling the rigid body motion are unsuitable 426 Active Control Systems for controlling the aircraft's bending modes and, consequently, auxiliary control surfaces are required. 12.4.3 Ride Quality Almost every modern aircraft has an SAS which is used to control its rigid body motion, and for which the locations of the sensors are carefully chosen to pick up the minimum of spurious signals from any structural motion. Such SASs do not control or deliberately alter the structural vibration of the aircraft. Yet it should be remembered that such SASs do provide a large amount of reduction of the unwanted motion produced by an aircraft in response to any gust disturbance. However, from operational records and simulations, it is known that those symmetrical structural modes with the lowest natural frequencies contribute substantially to the levels of acceleration which are present at various points of the fuselage, such as the cockpit. For example, it has been found that at the crew stations on the B-52E, without any SAS, 60 per cent of the total normal acceleration experienced by crew members is caused by the rigid body motion of the aircraft. The remaining 40 per cent, due to the rigid body motion and the other quarter was caused by the structural modes of higher frequency. If the accelerations are unacceptable, resulting in discomfort for passengers or crew or impairment of the pilot's ability to fly, then an RC system is needed to reduce the accelerations being experienced. 12.5 THE RIDE CONTROL SYSTEM FOR A BOMBER AIRCRAFT 12.5.1 The Aircraft Dynamics The differential equations which represent the B-52E heavy bomber can be expressed as: where x represents the state vector relating to longitudinal motion, and xl is that relating to lateral motion. In deriving the longitudinal equations it was assumed that the rigid body motion of the aircraft was adequately represented by the short period approximation. Included in both sets of equations are the dynamics associated with five structural bending modes: in the longitudinal set there are two modes 1, 5, 7, 8 and 12, and in the lateral set they were 1, 2, 3, 9 and 10. The 427 Load Alleviation System for a Bomber Aircraft control inputs which were employed for longitudinal motion were the deflections of the elevator and a horizontal canard; for lateral motion the three control inputs were the deflections of aileron, rudder and vertical canard. For longitudinal motion, the state vector x is defined as: and a hyphen the control vector u is defined as: u and y have their usual meanings of angle of attack and pitch rate, respectively. λi represents the vertical displacement of the ith bending mode. The corresponding state and control vectors are defined as: xl = [V Pr + 4 - Yl ?l Y2 ?2 ~3 3 5 7 ?9 9 Y10 (12.5) (12.6) The corresponding matrices A and Al and B1 are defined in eqs (12.7) to (12.10) (given in Figures 12.2 and 12.3). Since load alleviation is being considered, normal and lateral acceleration are motion variables of primary concern. If the measured normal acceleration at the pilot's station, location A, is taken as the output variable, y, it is easy to show, from the material presented in Section 2.7 of Chapter 2, that: (12.11) y = aA , = C x + DA u. Since the lateral acceleration at the pilot's station, location A, is also used as an output variable, yl, then yl = Cl xl (12.12) The Ride Control System for a Bomber Aircraft 428 -1.6 0.8 0.8 0 0 0 0 0 0 0 0 0 0 0 ... Figure 12.2 Coefficient and driving matrices for B-52E longitudinal motion (12.7) (12.8) Figure 12.3 Matrices A and B. Active Control Systems 429 (12.9) (12.10) (12.11). When bending effects are included in the aircraft dynamics, it is not correct to assume the output variables are located at the c.g., and in such cases the structural motion due to the structural modes can be seen to be the sum of the rigid body acceleration plus the component due to the deflection associated with each structural mode. Consequently, the matrices CA and DA in eq. (12.11) must be altered to account for the structural motion represented by the bending modes. For B-52E, location A is 4.4m from the tip of the nose of the aircraft (hence X = 17.48 m) and it can be shown (see Burris et al., 1969) that: It can also be shown from the work presented in Section 2.7 of Chapter that, at body station A, the lateral acceleration, with structural bending effects included, is given by: where vi(A) is the slope of the ith bending mode at body station A. Active Control Systems 430 Load Alleviation System for a Bomber Aircraft Table 12.1 Eigenvalues of uncontrolled longitudinal motion Lateral motion Eigenvalues have been denoted by λi ; the symbol hi does not denote here the displacement of the ith bending mode. For the B-52E, at the pilot station: CIA= [- 0.072 - 3.56 34.6 0.98 14.94 0.29 - 181.77 - 4.1 100.5 3.8 - 7531.0 - 18.54 - 3321.35 - 3.181 DIA= [1.401 - 0.94 - 1.0431 (12.16) (12.17) The eigenvalues corresponding to the matrices A and Al are given in Table 12.1. 12.5.2 Alleviation Control System Designed as Optimal Control System Since both the normal and lateral accelerations are linear functions of both state and control vectors, it is possible to use in a gust alleviation system the feedback control which results from minimizing the performance index, J, where: the performance index in eq. (12.18a) corresponds to longitudinal motion, that of lateral motion (in the lateral case Y = [q Q] and the performance index would be: J1 = (w1Q yI + uI1l Qu1I (12.18b) Active Control System Figure 12.4 Feedback control, u as an effective performance index. x and Q (0Q1) is a symmetric, non-negative definite matrix, w, and Q (G) is a symmetric, positive definite matrix weighting the elements of the control vector, u(ul). (See Section 8.4 of Chapter 8.) When Q (Q) was chosen to be a diagonal matrix such that the state variables corresponding to rigid body motion were weighted at unity, while those state variables associated with the flexible modes were weighted as 10.0, then when G (Gl) was taken as (12.15), the resulting feedback control matrix, obtained by solving the associated LQP, for longitudinal motion, is given by eq. (12.19) and for lateral motion by eq. (12.20). Both are shown in Figure 12.4. The eigenvalues corresponding to the closed loop alleviation control system are shown in Table 12.2. Note from a comparison of Tables 12.1 and 12.2 how, for 'longitudinal' motion, the damping ratio of the first bending mode (Ad) is doubled in the closed loop system whereas the damping of every other mode is affected: in every case increased, except for the seventh mode, where it is actually very much Table 12.2 Eigenvalues of closed loop gust alleviation control system Load Alleviation System for a Bomber Aircraft Time (s) -0.2 Input (a) -0.4 0 0.2 Time (s) 0 (a) Time (s) 0.2 Time (s) 0 Figure 12.5 Acceleration responses of uncontrolled aircraft. reduced. For lateral motion, the damping of the rigid body motion is very much increased, while the damping of the dutch roll mode, for example, has been changed from a very lightly damped, oscillatory mode to become two real modes (A3, A4). The same effect has occurred with the first bending mode (A5, A6) This substantial increase in the damping of the first bending mode has been achieved at the expense of the damping of the third and tenth lateral bending modes. Some acceleration responses as a result of an initial disturbance are shown in Figure 12.5 for the uncontrolled aircraft and in Figure 12.6 for the aircraft fitted with the optimal gust alleviation control system. It is evident that the accelerations occurring at the pilot's station have been reduced by the optimal control systems. With another choice of weighting matrix for the state vector, namely: for longitudinal motion, and: for lateral motion, it is found that the control law, which results from minimizing the performance index, depends almost entirely on the state variables which 2.0 - a(0)v 0 . - * -8 0 2 -2.0 - 1 2 3 Time (s) 4 5 + -3 0-5 Time (s) Figure 12.6 Optimal responses of gust alleviation system. (a) Normal acceleration. (b) Lateral acceleration. 1 Active Control Systems 432 contribute most to the output a, (or a,.), e.g. for longitudinal motion, using CA 4 defined in eq. (12.13), the resulting feedback matrix, K, was found to be: G was again taken as I2. The elements denoted by the symbol * were all less than 1 x 10-' and consequently can be neglected. The only gain associated with the bending mode is the 0.41 associated with the vertical displacement of the first longitudinal bending mode. The greatest contribution to normal acceleration comes from the rigid body motion and the first bending mode, as represented by u, the effect of increased values of the elements of the weighting matrix, Q, is to increase the rigid body motion. If we measure can be found the feedback gain matrix of eq. (12.23) cannot be synthesized directly and the optimal control law to achieve gust alleviation cannot be implemented. Some method of estimating x3 from other measurements would be required. A number of estimation methods were outlined in Chapter 8. 12.5.3 Sensor Blending Suppose an attitude gyro is located somewhere on an aircraft to measure pitch attitude. If the aircraft is flexible, the output signal from the attitude gyro will contain components proportional to the rigid body motion and to the bending modes. Assuming, for simplicity, that only the first two bending modes are significant, the output from an attitude gyro located at a point numbered 1 will be given by: If two more attitude gyros, are located on the same aircraft, but at points number 2 and 3, say, their output signals can be represented as: ~2 = q Let: + k22X1 + k23X5 = klga + kg2X1 + k3h5 = (12.24) (12.25) (12.26) Load Alleviation System for a Bomber Aircraft 433 then: where: I k I k12 k13 Km = k21 k22 k23 k31 k32 k33 Hence: I The elements of M are the bending gains. For example: By combining the signals obtained from these three, independently located, attitude gyros in the correct proportions it is possible to eliminate the contribution of the first bending mode which may then be used with the feedback control law which depends upon the vertical displacement of that mode, as in eq. (12.23). The location of each gyro has been governed by the requirements of the gust alleviation system. This is a very important assumption for sensor blending. 12.5.5 Model-following control for gust alleviation control system has been shown by using LQP theory. But if it was required that all the state variables, including the bending modes, of the aircraft system were to behave in a desired manner then the technique of implicit model-following dealt with in Section 8.7 of Chapter 8 can be easily applied to obtain the required control law. For example, if the performance index is to be minimized: where: it can be shown, from Chapter 7, it can be shown that: K = (C D - (TC - CA) For longitudinal motion of the B-52E, suppose that the model matrix, T, was chosen to be: The resulting feedback matrix K can be easily be shown to be: The resulting feedback control law which relies very heavily on the rates and displacements of the bending modes. If the simple feedback system were required to obtain the feedback signals required, the feedback control law for the first two bending modes can be controlled. Let: T 4 (mgl - 10.0 - 5.0 - 1.0 - 4.01 (12.39) The corresponding feedback matrix, K, was found to be: An acceleration response to an initial change in angle of attack for this model-following system is shown in Figure 12.7. The feedback control derived by this method is seen to be as effective for gust alleviation as that obtained from solving the LQP. Interested readers can find further discussion of load alleviation systems in Burris et al. (1969). " Time (s) -1 Figure 12.7 Model-following control system response to normal acceleration. C RCS for a Modern Fighter Aircraft 0.0074 0.031 Horizontal tail actuator p=20 7.5 ~ Symmetricalaileron actuator &~,Yrn~ ' - 20+ FAbm p=20 K2 Aircraft dynamics 0.046 K4 0.0067 Figure 12.8 Ride control system. 12.6 A RIDE CONTROL SYSTEM FOR A MODERN FIGHTER AIRCRAFT A modern fighter aircraft of the type represented by aircraft ECHO of Appendix B has an RCS fitted to provide a better aircraft path through low level turbulence so that the pilot's weapons tracking ability is not impaired by the accelerations which arise at the cockpit. A block diagram is shown in Figure 12.8 from which the following features will be noted: 1. There are two control surfaces employed: the horizontal all-moving tail and the symmetrical ailerons. 2. The motion variables sensed are angle of attack and pitch rate. 3. The control law being used in the pitch rate SAS is a proportional plus integral control. 4. The loop which acts through the symmetrical ailerons is washed out. 5. The control surface actuators are very rapid, both having a time constant of 0.05 s. The equations of motion corresponding to the pitch rate SAS and the symmetrical ailerons can be expressed for the echo at ECHO-2 is attitude control systems. 12.2 With the control gain K2 K4 may be found... Flight Path Control System... The feedback control gains K1 to K4 may be found by using any of the design methods outlined in Chapters 7 and 8, but the preferred method is to use the LQP of Chapter 8 to minimize the performance index which results from considering the ride discomfort (RD) index (of Section 6.6). It is shown there that: where the subscript PIT signifies horizontal tail. Thus, if α, q and the control deflections are minimized by the optimal control system, then fRDwill also be minimized. Thus, if the normal acceleration is taken as the system output, then it can be shown (see eq. (2.150)) that: y=Cx+Du For ECHO-2, the matrices C and D are given by: Conclusions 435 Thus, minimizing: where: results in the control law: where: Aircraft Positioning Control Systems 437 Figure 12.9 Acceleration step response for RCS. The response of the controlled aircraft to a step command in PC is shown in Figure 12.9. The response to low-level turbulence is shown in Figure 12.10. The r.m.s. value of the intensity of the vertical velocity gust was 0.3 m s-l. 12.7 AIRCRAFT POSITIONING CONTROL SYSTEMS 12.7.1 Direct Lift Control and Sideforce Generation There are four positioning tasks which require great precision in aircraft: air-toground weapons delivery; air-to-air combat; in-flight refuelling; and all-weather landing. By using direct lift control (DLC) and direct sideforce generation (DSFG) it is possible to furnish an aircraft with these capabilities. DLC is a technique of directly modulating the lift of an aircraft by using direct control surfaces, such as flaps. DSFG is a method whereby sideforce can be generated directly on an aircraft. Figure 12.11, By using the rigid body of the aircraft as the reference for the direct control of the aircraft's translational motion in the vertical and lateral planes, without the pilot and the rigid body of the aircraft could be readily controlled in the ... when DLC is combined with the SAS. When conventional controls are used, the translational motion is restricted to what can be achieved by either pitching or yawing the aircraft. Consequently, lateral motion and vertical motion are coupled. With DLC and DSFG the coupling to vertical motion from the lateral motion is avoided. The aircraft is effectively controlled in the normal and lateral directions, but by using DLC and DSFG, complete control of all six degrees of freedom can be attained. Active Control Systems -2 0 1 2 3 4 5 6 7 8 9 10 Time (s) Figure 12.10 Response to low-level turbulence. Note this capability can be provided by use of thrust vectoring, or can be generated by either aerodynamic or propulsive means. Aerodynamic methods are the more efficient. However, even with conventional aircraft, some degree of DLC and DSFG is possible by using auxiliary control surfaces as flaps, slats and drag petals. 12.7.2 Longitudinal Control The equation of short period longitudinal motion can be written as: Aircraft Positioning Control Systems 439 where: 12.7.3 Longitudinal Control In Figure 12.11 at constant surface incidence using Figure 12.11 DLC If constant angle of attack control is required, it is necessary to remove the influence of pitch rate from both the q and α equations. Thus, if a further control, say ul, can be found such that: and if another control, say, u2, can be found such that: then if a proportional feedback control law is used of the form: then there is a control law for manoeuvring the aircraft at constant angle of attack. 12.7.4 Constant Pitch Attitude In this case the aircraft is required to manoeuvre in the vertical plane as if it was pinned at the flight path, at constant pitch attitude. Therefore, DLC is used so that the flight path angle can be controlled. If the pilot is to be free to pitch the aircraft without changing the flight path angle, ... it is to control θ and the aircraft's normal velocity w, using an altitude hold system. 12.8 CONCLUSIONS Fixed attitude is required to: lift control or it is possible to use the entire aircraft to pitch at constant angle of attack, i.e. flight path at constant pitch attitude. 12.7.5 Vertical Translation Vertical translation requires that the aircraft's flight path should change in response to a pilot's command while its pitch attitude remains constant. 12.7.6 Sideforce and Lateral Control ... Active Control Systems Figure 12.11 DLC system. Aircraft Positioning Control Systems 439 For example, if the pitch rate is controlled to zero, using a pitch SAS by making use of gain, then the aircraft can be made to change its flight path at constant attitude. 12.9 EXERCISES 12.1 A pitch-pointing control system is to be fitted to an advanced fighter aircraft, such as ECHO... 12.2 ...

The flight control system of a modern aircraft, e.g. BRAVO-3, uses an SAS to ensure stable pitch response... 12.3 An aircraft ALPHA-1 has a control law: ... 12.4 Design a flight control system... 12.5 Cf. the aircraft FOXTROT...

12.10 REFERENCES ATTWOOD, J.L., R.H. CANNON, J.M. JOHNSON and G.M. ANDREW. 1961. Gust alleviation system. US Patent 2, 985, 409. 1969. Aircraft load alleviation and mode stabilization (LAMS) - B52 system analysis, synthesis and design. AFFDL-TR-68-161. WPAFB, Dayton, Ohio. HUNSAKER, J.C. and E.B. WILSON. 1915. Report on behaviour of aeroplanes in gust turbulence. NACA TM-1 (MIT). October. OSTGAARD, M.A. and F.R. SW~RTZEL. 1977. CCVs: active control technology creating new military aircraft design potential. Astronautics and Aeronautics. 15: 42-57. HOUGHTON, P.J. 1978. Active control technology. In: M. C. Colloquium, London. March. STOCKDALE, C.R. and R.D. POYNER. 1973. Control configured vehicle - ride control system. BURRIS, P.M. and M.A. BENDER. 1969. AFFDL-TR-73-83. WPAFB, Dayton, Ohio. turbulence. J. Aero. Sci. 4: 311-5. the statistical theory of turbulence. J. Aero. Sci. 4: 131-8. TAYLOR, G.I. 1937. Fundamentals of Helicopter Flight Control Systems 13.1 INTRODUCTION Helicopters are a type of aircraft known as rotorcraft; for they produce the lift needed to sustain flight by means of a rotating wing, the rotor. Because rotors are powered directly, helicopters can fly at zero forward speed: they can hover. They can also fly backwards, of course. At present, there are two main kinds of helicopter: those which use a single main rotor and a small tail rotor,' and those which have two main rotors in tandem. These helicopter types are illustrated in Figure 13.1. In the single main rotor type, the rotor produces vertical thrust. By inclining this lift vector a helicopter can be accelerated in both the fore and aft, and the lateral directions. This main rotor is usually shaft-driven and, as a result, its torque has to be countered, usually by a small tail rotor mounted at the end of the tail boom. Yaw control is achieved by varying the thrust developed by this tail rotor. - Main rotor & + - Rotation Rotation (a) Figure 13.1 Most common helicopter types. (a) Single main rotor. (b) Tandem rotors. 452 Helicopter Flight Control Systems In the USA and UK the main rotor rotates counterclockwise (viewed from above); in France, they use clockwise rotation. This has some significance in relation to the use of the tail rotor. To approach some point at which to hover, the pilot of a helicopter must make his aircraft flare to stop. Since it is customary for helicopter pilots to sit in the right-hand seat in the cockpit, the external view can be restricted in this flare manoeuvre, and, often, a sidewards flare is executed, which requires the pilot to apply more pressure to the left pedal in order to sideslip to the right, but this increased left pedal deflection demands a greater trimming moment from the tail rotor which has to be achieved by an increase in the thrust of that rotor. Pilots flying French helicopters do not have so great a problem in carrying out this manoeuvre. The two rotors of the tandem helicopters are normally arranged to be at the top and the front and rear of the fuselage. These rotors rotate in opposite directions, thereby ensuring that the torque is self-balancing. There is normally a significant overlap between the rotor discs; however, the hub of the rear rotor being raised above the hub of the rotor at the front. The resulting aerodynamic interference causes a loss of power, but the amount lost, being about 8-10 per cent, is almost the same as that lost in driving a single tail rotor. Every rotor has blades of high aspect ratio which are very flexible. These rotors are either articulated, in which case they use hinges at the root of the blades to allow the motion of the blades in directions normal to, and in the plane of, the rotor disc. A schematic representation of an articulated rotor hub is shown in Figure 13.2. At the blade hinge, the bending moment is zero; no moment is transmitted, therefore, through the root of the blade to the fuselage of the helicopter. Recent designs have eliminated hinges: these are referred to as hingeless, or rigid, rotors. Figure 13.2 Rotor hub of an articulated rotor. The out-of-plane motion of the blade, perpendicular to its radial direction, is referred to as its flapping motion. Motion about the vertical hinge causes the blade to deflect in the plane of the disc and such motion is referred to as lagging motion. In hingeless rotors, flapping and lagging motion are defined as the out-of-phase and the in-phase motion of the blade, and the pitch angles of its blades can be altered to cause a change in the blade's angle of attack, thereby controlling the corresponding aerodynamic forces. On a hinged blade, the pitch bearing is usually outboard of both the flapping and lagging hinges, but on a hingeless rotor the bearing may be found either in- or outboard of the major bending moment at the blade root. With any type of rotor there will be an azimuthal variation of lift as the rotor rotates. Such variation affects the degree of flapping motion and, consequently, the direction of the average thrust vector of the rotor. A cyclic variation of lift can be effected, therefore, by changing a rotor blade's pitch as the blade is being rotated. This altering of blade pitch is termed the cyclic pitch control: when it causes a pitching moment to be applied to the helicopter it is called the longitudinal cyclic, usually denoted by SB. If the applied moment is about the roll axis, the control is called the lateral cyclic, denoted by SA. Yaw is controlled by changing, by the same amount, the pitch angle of all the blades of the tail rotor; such a collective deflection of the blades of the tail rotor is denoted by ST. When the pitch angles of all the blades of the main rotor are changed by an identical amount at every point in azimuth, a change is caused in the total lift being provided by the rotor. This type of control is called collective pitch control, denoted by [small protected] Direct control of translational motion is by means of the collective control, since it is the means by which the direction of the thrust vector can be controlled. The importance of the collective to helicopter flight cannot be overemphasized: it is a direct lift control which allows the helicopter's vertical motion to be controlled quickly and precisely. Since there is considerable energy stored when the rotor rotates (as a result of its angular momentum) only small changes in the collective setting are needed to achieve vertical motion without any accompanying exchange of height for airspeed. Moreover, for small collective inputs, the ability of the helicopter's engine (or engines) to change speed is not of great concern. However, this simple means of controlling height makes difficult the control of a helicopter's horizontal speed: to slow down, it is necessary to pitch a helicopter nose-up. Thus, a pilot achieves deceleration by means of pitch attitude, while maintaining his helicopter's height with the collective control, which requires of the pilot greater control co-ordination. It is characteristic of helicopters during the approach to hover, and at hover, that any changes in the vehicle's speed require some adjustment of the collective which, in turn, causes a change in the helicopter's yawing motion, thereby resulting in the development of significant sideslip. These coupled motions subsequently result (in the absence of immediate and effective pilot action) in the helicopter rolling and pitching. This complex dynamic response is of particular concern when carrying 454 Helicopter Flight Control Systems the helicopter's approach on the glide slope, for it can lead to deviation from the desired flight path. With tandem rotors, matters are different. If both rotors are tilted, a change is caused in both the forward force and the pitching moment. With the rotors normally at different heights, a differential change of pitch provides only a pitching moment. Differential collective pitch between the rotors is used in a way which is analogous to the use of longitudinal cyclic; in this way a helicopter with tandem rotors is pitched. Lateral cyclic pitch on both rotors is used to produce pitching motion; yaw control is provided by tilting the rotors in opposite directions. If the c.g. of the helicopter is not located exactly midway between the rotors, then use of the lateral cyclic will inevitably produce a yawing moment. If such a tandem helicopter is rolled towards starboard (to the right), yawing motion towards port (to the left) will be induced. This characteristic is opposite, unfortunately, to that needed to produce a co-ordinated turn. The helicopter gives rise to a number of very distinctive AFCS problems, including the following: it is unstable; its control is effected through its response; it is capable of hovering motion; the pilot has to directly control its lift force, as well as controlling the motion about its three axes; and its speed range is narrow, the speeds involved not being very high (the upper limit is about 240 knots, i.e. 120 m s-1). Only the problems involving stability and control of the helicopter are dealt with in this book, and then only briefly. For more information, the reader should consult Johnson (1980), Mil et al. (1966, 1967) and Nikolsky (1951), which are outstanding books giving excellent and comprehensive coverage. Bramwell (1976), Gessow and Myers (1952), Lefort and Menthe (1963), McCormick (1967) and Payne (1959) provide further information as useful background material, although there are a number of errors present in Payne (1959) so it must be read carefully. 13.2 EQUATIONS OF MOTION 13.2.1 Introduction Any study of the dynamic response of a helicopter is complicated because each blade of the rotor has its own degrees of freedom, which are in addition to those of the fuselage. Yet, for small perturbations in the helicopter's motion, a knowledge of the motion of each blade is not required: only the rotor's motion as a physical entity needs to be considered. It is usual to assume that the rotor speed, Ω, is a constant. Because such analyses are invariably carried out in a bodyfixed axes system (see Figure 13.3) and it is assumed that all perturbations are small, the inertia terms can be linearized and the lateral and longitudinal motions may be considered essentially uncoupled. It should be Equations of Motion Figure 13.3 Helicopter axis system. remembered, however, that because of the rotation of the rotors, a helicopter does not have lateral symmetry (except for coaxial or side-by-side rotor configurations). There is, consequently, considerable coupling of lateral and longitudinal motions. For example, consider the roll coupling which can result from yawing motion. However the pedals in the cockpit are moved, a rolling acceleration is experienced because the tail rotor is generally above the roll axis. This can be easily seen from an examination of the equations governing rolling and yawing motion: L = ... (13.1) N = ... (13.2) For a tail rotor, if TT, represents the thrust produced by the tail rotor, and hT represents the height of the hub of the tail rotor above the helicopter's c.g. and l is the distance aft of the c.g. at which the tail rotor is located, then: It is simple to show that the ratio of rolling to yawing acceleration can be expressed as: Since I, < I, in general, then: 456 Helicopter Flight Control Systems Since hT < l, then usually the value of the ratio is in the range of 0.1-0.25. For more on equations of motion, readers should refer to Johnson (1980), Nikolsky (1951), Mil et al. (1966, 1967), Gessow and Myers (1952), Lefort and Menthe (1963) and Bramwell (1976). 13.2.2 Longitudinal Motion In wind axes, the linearized equations of motion are: ... where XH and ΔZ are increments in the aerodynamic forces arising from disturbed flight. AM the corresponding increment in pitching moment, γ the angle of climb, and OF the pitch attitude of the fuselage. Because it is assumed that the perturbations in u, w and OF are small, the increments in the forces and the moment can be written as the first terms of a Taylor series expansion, i.e.: where SBis the cyclic pitch control term, and 6c the collective pitch control term. The coefficients aX/du, aX/aw etc. (or in theshorthand Xu, Xw, etc.) are the stability derivatives. Thus: ... (13.12) If it is assumed that the axis system is chosen such that γ = 0, then: ... (13.13) The term M,w is usually included to account for the effect of downwash upon any tailplane which may be fitted. Because lift is generated by the rotating blades whose tilt angles are considered as the control inputs, it proves to be helpful to employ a nondimensional form of those equations. Let the radius of the rotor blades be denoted by R. The tip speed of any blade is therefore given by LR R. The blade area is STR' where the solidity factor, s, of the rotor is given by: where b represents the number of blades used in the rotor and c represents the chord of those blades (assuming, of course, that they are all identical). 457 Introduction Let: Ω = alflR ... non-dimensional time, T = ... The nondimensional form of inertia is defined as: ... (13.26) and the non-dimensional derivatives are defined as: ... Chapter 3. Using these definitions, the equations of motion for a helicopter can be written in a non-dimensional form ... (13.35) The significance of the prime is explained after eq. (13.45) ... Hence, if eqs (13.11) and (13.12) are divided by ... then ... (13.40) Following equations are obtained ... (13.44) This non-dimensional form of the equations of motion (eqs (13.43)-(13.45)) is due to Bryant and Gates (1930); it is, however, a cumbersome notation. The prime has been used here to denote that the form being developed is not the final one. For rotational convenience it is proposed to write ... (13.47) Similarly, the correction to the derivatives can be made. ... (13.48) ... (13.49) Equations of Motion 13.2.3 Lateral Motion To control lateral motion the following inputs are used: the deflection angle of the lateral cyclic, SA and the collective pitch angle of the tail rotor, ST. The corresponding equations of motion are: ... (13.50) ... (13.51) ... (13.52) 13.3 STATIC STABILITY The derivatives Yp and Yr are usually negligible in helicopter studies. Using the same procedure to non-dimensionalize these equations as that employed with the longitudinal motion produces ... (13.54) ... 13.2.4 Canonical Form If the following are chosen as state vectors: x' [U w q OF] or x = [VP r ψF φ]', and the following as control vectors: u = [δB δC]' ... and the following matrices result ... 460 Helicopter Flight Control Systems Note that in this formulation the equations of motion are in a canonical form, namely: (13.65) ẋ = Ax + Bu ... 13.3 STATIC STABILITY 13.3.1 Introduction Static stability is of cardinal importance in the study of helicopter motion since the several important problems which are peculiar to helicopters are brought about by the derivatives of those vehicles. For example, any disturbance of the speed of a helicopter causes a change in the force and moment acting on the main rotor which cause the rotor to tilt backwards, so as to oppose the change, thereby resulting in the transverse equilibrium of the helicopter. But then the static stability depends on direction of disturbed equilibrium occur in the first place? Suppose the helicopter motion about the transverse axis, OY, is its longitudinal equilibrium is disrupted. The angle of attack of the main rotor will then change; such a change causes a change in thrust and, consequently, a change in the reactive moment of the main rotor. That change disrupts the directional equilibrium. The practical interplay between the balancing forces means that a helicopter pilot must constantly try to restore the disrupted equilibrium so that controlling the flying a fixed wing aircraft. That is why the simple question: 'Do helicopters possess static stability?' requires the examination of a number of factors before an answer can be attempted. Three factors are involved: (1) the static stability properties, if any, of the main rotor; (2) the static stability properties, if any, of the fuselage, and (3) the effect of the tail rotor and any tailplane on any static stability properties. Further discussion of static stability can be found in Johnson (1980) and Mil et al. (1966). 13.3.2 Static Stability of the Main Rotor Speed In Figure 13.4 it is assumed that the helicopter is flying straight and level at a speed V. Subsequently, the speed is increased by a small amount, AV. The flapping motion of the blades therefore increases (see Section 4.9 of Chapter 4). Helicopter Flight Control Systems Figure 13.4 Static stability of main rotor with speed. As a result, the axis of the cone of the main rotor is deflected aft, from its previous position, by an additional angle of β. (This movement is represented in Figure 13.4 by the dotted line.) Such a tilt of the coning axis leads to the development of a force F, which is in an opposite sense to the direction of flight. In other words, the main rotor has a tendency to slow down the helicopter. If the helicopter was flying straight and level, the speed has been reduced by an amount AV, the cone axis would then have been deflected forward, and the force, F, would have developed in the same sense as the direction of flight, thereby causing an increase in the forward speed of the helicopter. It can be concluded that with respect to changes in speed, the main rotor is statically stable. Angle of Attack In Figure 13.5 the helicopter is once more assumed to be flying straight and level with its main rotor at an angle of attack of (YMR*. The thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust must be zero. Under the influence of a vertical air current, say, the helicopter lowers its nose and, therefore, the angle of attack of the main rotor is reduced by an amount AA. (See the dashed line in Figure 13.5.) The vector of thrust is now displaced aft, and hence a moment, MT, given by eq. (13.72), is established which causes the value of the angle of attack of the main rotor to decrease. This moment is destabilizing. If the angle of attack of the main rotor is increased, however, the thrust vector will tilt afterwards and a nose-up moment, MT, will be established causing the angle of attack of the main rotor to increase further. The main rotor is statically unstable, therefore, with respect to changes in angle of attack. Fuselage Stability The greatest influence upon the static stability of a helicopter is that of its main rotor. The contribution of the fuselage to static stability is small in three areas of motion. A small tailplane is sometimes installed at the aft end of the fuselage to improve the static stability of longitudinal motion in straight and level flight. Its influence is practically nil at low speeds and at hover. However, the degree of instability in longitudinal motion can be reduced from the value at hover by increasing forward speed and by reducing the angle of attack until, at negative angles of attack, the fuselage plus tailplane possesses some static stability. This principle can be seen by referring to Figure 13.6. The longitudinal static stability is denoted by M. Curve 2 represents the change in M, with forward speed for a helicopter with an articulated rotor. The same characteristic is represented in curves 1 and 3 for a helicopter with a hinge- Helicopter Flight Control Systems Figure 13.6 Fuselage stability characteristic. less rotor and for the fuselage only of a helicopter, respectively. Note that in each of these curves dM,/dV is positive. The changes in static stability with forward speed corresponding to a tailplane is shown as curve 4; curve 5 represents the same characteristic for another tailplane, but with twice its area. Note that for these curves dM/dV is negative. The effect of having a helicopter with a hingeless rotor, the fuselage of curve 3 and the tailplane of curve 4, is shown in curve A. Curve B represents the results of combining curves 2, 3 and 4 and curve C is the result of the combination 1, 3 and 5. It is obvious that providing adequate static stability throughout the speed range of a helicopter is particularly difficult. If a helicopter is fitted with a tail rotor it has a profound effect on the fuselage's static stability, for if the directional equilibrium is disrupted and the helicopter turns to the right, say, the angle of attack of the blade elements of the tail rotor will increase and, consequently, the thrust from the tail rotor increases by some amount, AT. Therefore, the moment of this thrust must also increase thereby restoring equilibrium. The tail rotor, therefore, provides directional static stability. If the hub of the main rotor has offset horizontal (lagging) hinges (see Figure 13.2), the hinge moments associated with the offset have a considerable effect on both longitudinal and transverse static stability of the helicopter. The greater the offset of the hinge and the rotational speed of the rotor, the greater is the static stability possessed by the helicopter. These same factors also contribute to the increase in damping moment contributed by the main rotor. Dynamic Stability 13.4 465 DYNAMIC STABILITY Since the flying qualities of a helicopter are markedly different in forward flight and in hovering motion, these two flight regimes are dealt with separately. The subject of dynamic stability is further discussed in Johnson (1980), Nikolsky (1951) and Mil et al. (1967). 13.4.1 Longitudinal Motion Stick-fixed, forward flight The pilot's stick being assumed fixed, there are no control inputs. (δB, δC)=0. ST: the dynamic stability properties are determined solely from the coefficient matrix. For straight and level flight, V ≠ 0 (13.79) V (13.80) ... Hence, the corresponding coefficient matrix, A,Slong,can be expressed as: and it corresponding characteristic polynomial can be found by evaluating: ... (13.82) where: For complete dynamic stability it is necessary that every real root, or every real part of any complex roots, of that polynomial, should be negative. With the wide availability of computers it is now a simple matter to assess helicopter stability: simply read in the coefficient matrix, use one standard eigenvalue routine and obtain the real parts of the complex eigenvalues to determine if they are negative. Otherwise polynomial routines or the algebraic checks of Routh and Hurwitz (see Chapter 7) should be used. Hovering Motion When a helicopter hovers, V is zero and, usually, x,, z,, m, and m+ are negligible, i.e the equations of motion given by eqs (13.48)-(13.52) now become: ... (13.84) and hence, the characteristic polynomial can be shown to be: ... (13.85) which can be factorized into the form: The factor (λ + z,) corresponds to a stable, subsidence mode, whereas the quadratic factor corresponds to an unstable, oscillatory mode since 5 invariably lies in the range 0 to - 1.0. Consequently, the longitudinal dynamics of a helicopter at hover separate into two distinct motions: vertical and longitudinal. It is easy to show (from eqs (13.88)-(13.90) that: i.e. the vertical motion of a helicopter at hover is described by a first order linear differential equation, with a time constant given by: The time-to-half amplitude is typically about 2 s since the value of z, typically lies within the range - 0.01 to - 0.02. In many ways, this simplified representation of the vertical motion in response to collective input is misleading. The vertical damping, z,, is not a simple aerodynamic term but is composed of contributions from the fuselage and from the inflow created by the rotor. In hovering motion, the inflow contribution is predominant. The value of z,, however, which is speed dependent, does have a marked effect on the thrust-to-weight ratio required for hovering. For a particular height response is considerably affected by the response time of the engine(s) driving the rotor. Of considerable importance to any control in helicopters is the nature of the engine response. In terms of system design and analysis it is imperative to Dynamic Stability have a reasonable mathematical representation of the dynamics of the engine and the transmission system. Unfortunately, such reasonable and simple models are not easily found in the open literature: representation by a low order linear model is frequently misleading. Since no adequate empirical/transmission model was available to the author, it has been decided to omit such dynamic effects from the analyses presented in this chapter, but the reader who finds himself concerned with the practicality of helicopter flight control systems is reminded that such dynamics ought never to be ignored in practice. The instability of the longitudinal dynamics is a result of the coupling of the motion via the pitching moments which come about as a result of the change in longitudinal velocity, i.e. Mu (the so-called speed stability), and the longitudinal component of the gravitational force. For static stability, the requirement is that the constant term of the characteristic polynomial shall be positive, i.e. mgm, > 0 (13.95) The inequality (13.95) can be satisfied with a positive value of mu. The oscillation associated with the longitudinal dynamics is only mildly unstable with a typical period of 10-20 s. Time-to-double amplitude, td, is about 3-4 s. Both the period and the time-to-double amplitude are sufficiently long for the motion to be controllable by a pilot. Although the hub moments available in helicopters with hingeless rotors, or articulated rotors with offset hinges, are very much greater than in other types, thereby greatly increasing the capability of the rotor to produce moments about the helicopter's c.g., the character of the helicopter's dynamics are not radically altered, although there is a real improvement in the controllability. However, for hingeless rotors, the flap frequency is large enough to influence the dynamics. The moment derivatives, Mu, M,, M,, Ma,, and Ma, may be doubled (approximately) by using flap hinge offset; f8r a hingeless rotor, Mu and Ma, are increased by as much as three or four times.4 The pitch damping derivative, M,, is increased even more which results in an increase in the value of the real root, pl, of the vertical mode; it also increases somewhat the period and time-to-double amplitude of the oscillatory mode. Because a helicopter with a hingeless rotor has such large pitch damping (and consequently a less unstable oscillatory mode) than a comparable helicopter with an articulated rotor, and because it also has greater control power, the task of controlling such a helicopter is easier. In summary, for a hovering helicopter, the longitudinal dynamics are described by a stable, subsidence mode (a large negative real root due to pitch damping) and a mildly unstable, oscillatory mode (due to the speed stability M,). A pilot will have good control over the angular acceleration but poor direct control over translation. Because of low damping, in hover the control sensitivity is high. This combination of high sensitivity and only indirect control of translational velocity makes a hovering helicopter prone to pilotinduced oscillations (p.i.o.) thereby increasing the difficulty of the pilot's task. 468 Helicopter Flight Control Systems To approach matters, the lateral and longitudinal motions are not decoupled, as supposed, and, for many types of helicopter, a longitudinal cyclic input can result in large corresponding lateral motion. Furthermore, because of the speed stability of its rotor, a helicopter is susceptible to gusts whenever it is hovering and, as result, its position relative to the ground drifts considerably: this makes the task of station-keeping, for which helicopters are universally employed, particularly taxing. Forward Flight (with a Tailplane) In forward flight the unstable, oscillatory mode is made worse with increasing speed. At moderate and high speeds, the longitudinal dynamic stability of a helicopter can be improved by the use of a tailplane, which is particularly effective. It is usually fitted to helicopters particularly but at high speed and is generally inferior to that of a comparable helicopter which has an articulated rotor, with flapping hinges of small offset. Of course, its control power is generally increased and hence the pilot's SAS may be used to recover the required degree of stability. From Figure 13.7 it can be seen that, for a helicopter with an articulated rotor, the period of the unstable oscillatory mode increases with forward speed, and becomes stable at about 85 knots. This value of speed is influenced by the size of tailplane used. In Figure 13.7(b) it will be seen that for a hingeless rotor, the period does not change much with forward speed; if anything, there is a tendency to a much greater degree of instability at high speed, as a result of the large and unfavourable value of M a (see Figure 13.6). This tendency can be eliminated only if a tailplane can provide sufficient extra damping to result in the helicopter becoming stabilized. The longitudinal dynamic stability of a helicopter in forward flight is worse than that of a conventional fixed wing aircraft. From the open literature it can be concluded that ... Stability of the Lateral Motion Assume straight and level flight, i.e. : Y = A o then the coefficient matrix, A,lat, can be re-expressed as: Dynamic Stability 0 (a) 20 40 60 Forward speed (knots) 80 100 Figure 13.7 Forward flightstabilityparameters. (a)Articulatedrotor. (b)Hingeless rotor. The characteristic polynomial is given by ... (13.99) where ... (13.100) (13.101) (13.102) 470 Helicopter Flight Control Systems The single A term implies that A = 0 is a solution of the characteristic equation, and, consequently, a helicopter has neutral stability in heading. Hovering Motion In hovering motion the forward speed is zero. When longitudinal motion in hover is considered it is found that a number of stability derivatives are either zero, or negligible, which leads to a substantial simplification of the equations of motion. However, such simplifications do not occur in lateral motion studies, because the yawing (r) and rolling (p) motions are coupled by virtue of the stability derivatives, L and n,, which have significant values owing to the tail rotor. If, however, it is assumed that the shaft of the tail rotor is on the roll axis, then L, can be considered negligible. Then the characteristic polynomial becomes: The root (λ = n,) means that the yawing motion is stable (since nr is invariably negative) and independent of sideways and rolling motion. The cubic can be factored into: The first factor corresponds to a stable rolling, subsidence mode; the quadratic, to an unstable, oscillatory mode. Typically, for the rolling subsidence mode, l, is less than 0.5 s; the period of the oscillation is about 15-20 s, whereas the time-to-double amplitude is about 20-30 s. The time constant of the yawing mode is about 5 s. Forward Flight The quartic of eq. (13.98) has been solved for a range of values of the advance ratio, μ, and the values of the real and imaginary parts of the corresponding eigenvalues have been displayed on a root locus diagram in Figure 13.8. It is evident from this figure that as the forward speed of the helicopter increases the oscillatory mode becomes more stable. There is now a spiral mode, a rolling subsidence mode (still rapid) and a stable oscillatory mode corresponding to dutch roll oscillation, i.e. ; Hence, this helicopter will 'weathercock' with very little translation sideways. If a hingeless rotor is employed it can increase the hub moment by about a factor of five, in relation to an articulated rotor with a hinge offset by 4 per cent. Such an increase in hub moment considerably increases the stability derivatives, l, and l,, but not the stability derivatives, n, The quartic then becomes: ... (13.106) 13.4.3 Stability Augmentation Systems Figure 13.8 Root locus diagram. The root of the rolling subsidence mode, p2, has typically a value of about 10-15.0; t, reduces. The oscillatory mode, which is now neutrally stable, with a period of about 10-20 s. 13.5 STABILITY AUGMENTATION SYSTEMS In earlier chapters it can be learned how the application of feedback control of the proper kind can result in an unstable system becoming stable. For helicopter longitudinal dynamics the most common feedback control laws are: How such feedback control laws are implemented can depend on whether passive or active methods are to be used. There are a few passive techniques in current use; the best known is the stabilizer bar to be found on some Bell helicopters. (See, for example, the illustration of the Bell 212 in Jane's All The World's Aircraft (1983-1984).) 472 Helicopter Flight Control Systems 13.5.1 Stabilizing Bar This simple mechanical device is essentially a gyroscope: it is a bar pivoted to the rotor shaft and has a viscous damper provided. The bar is linked to the rotor blades so that if the bar is tilted relative to the shaft a change in the blade pitch is caused (see Figure 13.9). The angular displacement of the bar is denoted by θ. The equation motion of the bar is given by: where η is the angular pitching velocity of the rotor and η and the azimuthal angle of the blade, i.e., the angle between the blade span and the rear centre-line of the helicopter. In practice the term q cos is negligible, hence: ... Suppose the pitch angle, θ, of the rotor blade due to a displacement of the bar is proportional to that displacement ... Therefore, the cyclic pitch of the rotor can be defined as: ... (13.113) where SB represents the pitching of the lateral cyclic deflection, and SB the amplitude of the longitudinal cyclic deflection. ... Figure 13.9 Schematic representation of Bell stabilizing bar. Stability Augmentation Systems 473 If eq. (13.114) is substituted in eq. (13.111), and if the coefficients of the resulting sin and cos terms are equated then the following equations result: ... The prime denotes d/dt. The characteristic equation of these simultaneous differential equations is a cubic which can be factored into a first order and quadratic term. The quadratic factor corresponds to a high frequency, rotation mode which is of little practical use here, for the bar's frequency is Ω and the period is 2π/Ω. However, ... of the form of eq. (13.109). Thus, any change in the rotor's speed, or the pitch rate of the rotor shaft, will cause a change in the longitudinal cyclic pitch to oppose the causative change. Note that the settling time of the response of the system represented by eq. (13.117) depends upon K, the damper coefficient. Flight tests of this stabilizing bar have shown that it provides a longitudinal feedback control and has been observed to increase the stability derivative, M,, by a very effective factor of 3. Other examples of passive stabilizing devices are the Hiller bar and the gyro bar which is used on the Lockheed rigid rotor. There is a close resemblance to the Bell stabilizing bar except that damping is provided aerodynamically by means of small aerofoils mounted on the bar, rather than by using a viscous damper. Unlike the Bell stabilizer, a Hiller pilot controls the bar directly. Both the Bell and the Hiller bars are best suited to a two-bladed see-saw rotor. The Lockheed rigid rotor is a three-bladed rotor. The most serious disadvantage of stabilizer bars, apart from mechanical complexity, is that they add to the total drag of the rotor. 13.5.2 AFCSs for Helicopters In SAS in helicopters, the basis instability is such that the AFCS has to provide both restoring and damping moments. The control laws in general use tend to be either eq. (13.108) or (13.109). However, only limited control authority can ever be allowed, since control is through the rotor which provides the helicopter's sustaining lift and forward propulsion. 'Hands-off' operation of any helicopter, even with the most sophisticated of AFCS, is possible only for a few minutes; consequently, the pilot's stick system is very important in helicopter flight. A representative block diagram of such a stick feel system is shown in Figure 13.10. Note the presence of an input signal, [...], provided from the trim actuator, which reduces 474 Helicopter Flight Control Systems (From trim actuator) I - Acceleration Velocity Displacement (From pilot) Spring Fspring + Figure 13.10 Block diagram of a stick feel system. to zero the force required to be produced by a pilot for a constant manoeuvre demand. The feedback spring can be preloaded, and its stiffness can be altered as a function of speed to provide constant feel characteristics. The speed range of a helicopter is not very great, however, and such variability of the feedback spring force is often dispensed with. It should be appreciated, nevertheless, that the dynamics of the stick feel system act as a pre-filter, in the manner outlined in Section 10.8 of Chapter 10. The Simple SAS Represented by Figure 13.11 is robust and requires no electrical trim signal for varying c.g. margins, because there will be no input to the servomechanism when there is no rate of change of displacement from the datum. The system is an excellent regulator which maintains the helicopter at hover ... rate gyroscope, the effect of these parallel paths is identical to that of a phase-lag network. The feedback control law is: Tilt angle Helicopter Figure 13.11 Block diagram of an SAS. 4 Stability Augmentation Systems ... (13.121) where T1 > T2 (13.122) In response to a disturbance, the 'leaky integrator' path produces a signal proportional to the angle through which the helicopter has been displaced from its equilibrium value at the time of the disturbance. The control action applied through the swash plate tends to restore this angular disturbance to zero. If the helicopter does not respond to this corrective control action, or if it is held in this new attitude by the pilot, the quasi-integrated signal, o, decays to zero in about 5 T seconds, and the new angular position is considered as the equilibrium. ASE ASE is an attitude control system, represented in Figure 13.12. The rate signal required for the inner loop SAS is obtained by differentiating the output signal from the attitude gyroscope, the use of which implies a real attitude datum. Since a helicopter has to be flown at any speed or attitude, this datum must be variable. The input from the c.g. trim system centres the gyro for any given flight condition. However, since the attitude control system tends to hold the datum and therefore opposes even manoeuvre demands, a signal from the stick system is used to 'break off' the gyro signal produced as a result of a manoeuvre. When the stick is trimmed to some new datum position, this signal establishes a new datum for the gyro. When the ASE is being used to control bank angle, the resulting characteristic is unusual. If the stick is held over to maintain some desired bank angle, it is not centred once the angle has been reached; in any emergency, release of the stick re-establishes straight and level flight. An ASE can hold attitude indefinitely. Although it is more complicated than a SAS it performs the same function. However, it does offer the means for providing automatic trim, automatic flight control and, of course, of changing the control characteristics. On helicopters the failure of SAS functions must not be critical, hence it is rare to provide redundant channels. CSAS When a helicopter has poor handling qualities, the performance of a specified mission without an AFCS can lead to levels of workload for the pilot which are unacceptably high, particularly if he is required continuously to monitor such Helicopter Flight Control Systems Stick displacement I Tilt angle of rotorswash plate I I I I---+u Helicopter dynamics I + *a, *O Pick off I C g. trim signal Figure 13.12 Differentiating network Block diagram of pitch attitude control system. quantities as rotor speed and control loads during certain manoeuvres to ensure that the flight is being carried out within the limits of safe operation. A CSAS is usually provided in such cases. It is similar to the ASE but involves the use of a Kalman filter, requiring signal inputs from the gyroscopes, an accelerometer and the vertical navigation system (INS) velocity sensor. The resulting estimated signal is compared with a reference value generated by a set of model dynamics (see Figure 13.13). A signal proportional to any error between these values is filtered and then added to the feedback signals from the attitude and rate gyros. Decoupling signals from other axes are added at the same point. Just as AFCS is larger and more complicated than any discussed earlier and its use is confined to large helicopters, often twin rotor types. Since the forces are correspondingly larger, a booster servo is usually added to amplify the stick displacement. Station-keeping System This AFCS is used to enable a helicopter to maintain its position fixed in space to keep its station - for quite long periods of time. (See Hall and Bryson (1973) for further discussion of this system.) Obviously, the situation requires that the flight is carried out at hover, or near hover, i.e. with forward speeds not greater than 1m s-1. For a Sikorsky S-61 helicopter, in which the blade dynamics are also taken into account, the state and control vectors are defined as: where OR denotes the pitch tilt angle of the rotor, ΦR the roll tilt angle of the rotor, qR the rate of pitch tilt angle, pR the rate of roll tilt angle, OF the pitch Stability Augmentation Systems 477 Velocity signal from INS Trim actuator displacement - Decoupling signal (from other axis) Figure 13.13 Block diagram of CSAS. attitude of fuselage, ΦF the roll angle of fuselage, q, the pitch rate of fuselage, p, the roll rate of fuselage, u the velocity along the x-axis of the fuselage, v the velocity along the y-axis of the fuselage, δB the longitudinal cyclic, and δA the lateral cyclic. The corresponding coefficient matrix, A, and driving matrix, B, are: ... Rotor dynamics ... Rigid body (fuselage) coupling effects ... (13.127) ... Rigid body (fuselage) dynamics 478 Helicopter Flight Control Systems Note that the rotor dynamics have been incorporated in the aircraft equations of motion. When the rotor and rotor disc ... It is obvious from an inspection of A that there are profound coupling effects caused by the inclusion of the rotor dynamics, which arise from the blade flapping motion. If it is assumed that the rotor disc can be tilted instantaneously (i.e. the rotor becomes: ... (13.128) ... Stability Augmentation Systems 479 is the lateral tilt of the rotor disc and OR is the longitudinal tilt. The corresponding matrices are: ... The eigenvalues corresponding to these two models are shown in Table 13.1. The response of the basic helicopter, with and without rotor dynamics, to an initial value of the pitch and roll attitude of the fuselage and change in the forward and side velocity, and to a unit impulse of lateral cyclic, are shown in Figure 13.14. The responses shown are the pitch and roll attitude of the fuselage and the lateral velocity of the helicopter. Similar impulse responses can also be found for the lateral cyclic. It should be noted from Table 13.1 and Figure 13.14 that when it is assumed that the rotor disc tilts instantaneously (i.e. that the rotor dynamics can be ignored) one of the rigid body modes of the fuselage is more lightly damped and slower. The responses shown in the figures can easily be found using the LQP method outlined in Section 8.3 of Chapter 8. For the models including the rotor dynamics, a choice of state weighting matrix of: 480 Helicopter Flight Control Systems Table 13.1 Eigenvalues of uncontrolled helicopter without rotor dynamics With rotor dynamics ... (a) Time (s) (b) Time (s) Figure 13.14 Response of uncontrolled helicopter. (a) With rotor dynamics. (b) Without rotor dynamics. placing emphasis on the dynamic behaviour of the pitch and roll angles of the fuselage and, to a much lesser extent, the changes in forward and side velocity, and a control weighting matrix of: G = diag[LO 1.0] (13.128) to penalize excessive controlled deflections of the longitudinal and lateral cyclic, results in an optimal gain matrix, K, which is found to be: Stability Augmentation Systems 481 Table 13.2 Eigenvalues of optimal closed loop system With rotor dynamics Without rotor dynamics Another optimal feedback control law was found for the model without rotor dynamics, using the same weighting matrices: The eigenvalues corresponding to the closed loop optimal system are given in Table 13.2. Table 13.2 Eigenvalues of optimal closed loop system With rotor dynamics Without rotor dynamics Optimal Helicopter Flight Control Systems 3 Time (s) Figure 13.15 Responses of the optimal closed loop system. Note that the constant initial conditions and the control variables were identical to those used earlier, i.e. The responses of the closed loop system are also given in Table 13.2. The responses of the pitch and roll attitudes of the fuselage and the changes in the forward and side velocities, to a unit impulse, in first the longitudinal cyclic and then the lateral cyclic, are shown in Figure 13.15 for the helicopter withwithout rotor dynamics. It should be noted from these responses how effectively the controlled helicopter keeps its station: compare particularly the responses of the translational velocities with those which arose in the uncontrolled case. Further discussion on stability augmentation systems can be found in Johnson (1980) and Mil et al. (1966, 1967). 482 Helicopter Flight Control Systems Velocity of (ft s-1) ... Time (s) Figure 13.15 Continued. 13.6 CONCLUSIONS This chapter opens with a brief introduction to the helicopter and its rotor system. Some distinctive features of helicopter flight which give rise to particular control problems were noted before proceeding to a development of appropriate equations of small perturbation motion for both longitudinal and lateral motion. Next, the particular qualities of static stability which obtain in Exercises 483 helicopters are dealt with, before turning to the special problems of dynamic stability for which the most effective means of cure is provision of effective SASs. The objectives of such SASs are discussed before a description and analysis of a rotor stabilizing bar, a very common helicopter SAS, are presented. Active SAS and CSASs are then discussed before dealing finally with optimal control of a station-keeping helicopter. A special feature of this analysis is the impact which the inclusion of the dynamics of the main rotor in the equations of motion can make to the response obtained from the controlled helicopter. 13.7 EXERCISES 13.1 The state vector of the Black Hawk helicopter (UH-60) can be defined as: and the control vector as: u' = [SB &A ST soo ihtl where iht represents the change in incidence of the horizontal tail. The corresponding matrices A and B are: 484 Helicopter Flight Control Systems (a) Find the eigenvalues of the helicopter at forward speed: (i) 1ft s-' (ii) 150 ft s-' Comment on the stability of the helicopter at each of these flight conditions, and compare. (b) Find the following transfer functions for the hovering condition: p(s) (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of the longitudinal cyclic control; (ii) lateral acceleration at the c.g. to a deflection of lateral cyclic control; (d) What is the peak normal acceleration to a unit step deflection of the longitudinal cyclic, and how long does it take after the application of the step function to attain the peak? (e) Is the acceleration response concave downwards within 2.0s of the application of the control? 13.2 A helicopter, which is perturbed by a wind gust, is to be maintained at zero ground speed by means of controlling its motion about the pitch axis. If the deflection angle of the rotor swash plate is denoted by q and a feedback control law, q = K,O + K,u + K,q is used, find values of KO, K,, and K , which will optimize the helicopter's hovering flight, in the sense that its use minimizes the ISE where u is taken to be the error, since u represents any small variation in horizontal speed from the derived ground speed of 0.0. The helicopter being used is a Sikorsky S-58 and the appropriate, but approximate, equations of motion are given by: 13.3 The Iroquois (UH-1H) helicopter has its state vector defined as: x'=[uwqOvp+r] and its control vector as: u' = [SB Seo SA 81-1 The appropriate matrices A and B for the hovering case, i.e. Uo A 0 m s-', are: Exercises (a) Find the eigenvalues of the uncontrolled helicopter. (b) Determine, by any appropriate means, a state feedback control law which will completely stabilize the helicopter. (c) Does the controlled helicopter have acceptable flying qualities? 13.4 (a) Show that, for hovering motion, the vertical velocity of a helicopter can be expressed as a simple, first order differential equation: (b) How significant was your assumption in part (a) that the rotor speed was constant? (c) If the rotor speed is not fixed, design a height control system to allow the helicopter to hover without the pilot's attention. (d) If w, is represented by a (1 - cos) gust (see Chapter 5), what would be the corresponding height change if Z , = - 0.015? The scale length of the gust is 10.0 m. 13.5 A helicopter has the capability of making a landing by means of maintained autorotation in which the lift force being generated by the rotor is maintained, even though there is a loss of power due to engine failure, and the helicopter descends at a steady rate. (Warning: even for forward flight, this autorotation descent rate is quite large, so autorotational descents are used only in emergencies.) 486 Helicopter Flight Control Systems Assume that the collective pitch is unchanged. The equation of motion for the vertical acceleration of a helicopter is given by: (~ 1 g) h= W - T where h is the height of the helicopter above the ground, W is the gross weight and T the thrust developed by the rotor. The equation of motion for the rotor speed is: N Z = ~- Q ~ NZb is the total moment of inertia of the rotor, Cl is the rotor speed, and Q the decelerating torque on the rotor. If Qo and Clo represent the torque needed in level flight and the initial rotor speed, respectively, show that: (a) The descent velocity (sinking speed) of the helicopter can be expressed as: h = @4(t+ 7) where the time constant T (b) The rotor speed becomes: = NZbfidQO aOT/(t+ 7) If NZb = 3 000 kg m2, Qo = 7 500 N-m, and Clo = 36 rad s-l, sketch the response of h and Cl with time. (d) If the aircraft is at a height of 100 ft when autorotation begins, calculate the aircraft's sinking rate and its rotor speed at the moment of ground contact. (c) 13.6 A helicopter at near hover flight condition, with constant rotor speed, has the following state equation: where: x' 5 [OR $R PR 9~ OF $F PF q~ U V] OR denotes the pitch tilt angle of the rotor, $R the roll tilt angle of the rotor, pR the rate of roll tilt angle of the rotor, q~ the rate of pitch tilt angle of the rotor, OF the pitch attitude of the fuselage, $F the roll attitude of the fuselage, pF the roll rate of the fuselage, q, the pitch rate attitude of the fuselage, u a small change in the velocity of the c.g. along the X-axis of the fuselage, v a small change in the velocity of the c.g. along the Y-axis of the fuselage, SB the longitudinal cyclic pitch, and SA the lateral cyclic pitch. The corresponding matrices A and B are shown below. Rotor dynamics 1 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 0 0 1 0 0 0 0 0 0 - 30.3 - 42.6 0 0 0.14 - 0.26 - 30.4 - 50.6 600.0 -54.3 41.4 -27.3 0 0 50.2 - 30.3 - 0.28 - 0.12 0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 0 0 0 1 0 0 5.0 - 0.92 0 0 - 0.05 0.03 0.001 - 0.002 3.5 18.4 0 0 - 0.1 - 0.2 - 32.2 0 4.37 0 32.2 - 15.0 21.6 22.0 15.0 - 0.04 0.004 - 0.01 1.0 - 0.17 - 0.05 - 0.05 - 1.0 Rotor coupling effects 1.4 - 0.0008 - 0.06 1.44 - 0.017 0.007 - 4.4 - 0.007 - 0.017 Rigid body (fuselage) dynamics 488 Helicopter Flight Control Systems Note that the rotor dynamics have been incorporated in the aircraft equations of motion. When the helicopter is represented simply as a rigid body, with the rotor dynamics being represented as a solid disc, the state vector is then defined as: xi = [OF +F PF 9~ U V] and the control vector becomes: u' = [- + R OR1 where +R denotes the lateral tilt angle of the rotor disc, and OR denotes the longitudinal tilt angle of the rotor disc. The corresponding matrices are Al and B1: Al = - 0 0 1 0 0 0 0 0 0 1 0 0 0 0 - 0.042 0.32 0.003 0.001 0 0 32.2 0 0 - 1.23 - 1.6 0.004 4.7 - 0.012 - 1.0 - 0.02 - 0.005 32.2 - 1.0 - 4.7 0.005 - 0.02 - Evaluate the eigenvalues for the helicopter with blade flapping motion and then with the solid rotor disc. (b) What is significant about the eigenvalues obtained for the two cases? (c) For the following weighting matrices, Q1 and GI, find an optimal feedback control law for the helicopter without blade flapping motion: (a) Ql = diag[l0.0 2.0 2.0 5.0 1.0 10.01 (d) Using the feedback control law obtained in part (c), determine the pitch rate response to an initial value of the fuselage pitch rate of - 3" s-l of the solid rotor disc helicopter. (e) Find the same response for the helicopter with blade flapping motion, still using the control law found in part (c). Compare the response with that of part (d) and comment upon any significant differences. (f) Using weighting matrices Q and G References 489 determine a feedback control law for the helicopter with state vector, x. (g) Use the control law found in part (f) to determine the pitch rate response to an initial value of the fuselage pitch rate of - 3" s-l. Compare the response with that determined in parts (d) and (e). Comment upon these results. 13.7 A small experimental helicopter, with a single main rotor and a NOTAR system, is flying at a near hover condition i.e. Uo = 0.3 m s-I. In the course of the flight the NOTAR system ceases to operate causing the stability derivatives Li, NL and N; to have values of almost zero. The stability derivatives of the helicopter in this flight condition are: (a) If an impulse of 0.035rad-s is applied to the lateral cyclic, what is the resulting steady state value of the helicopter's yaw angle? (b) How long does it take the helicopter to reach its new heading? (c) Is the helicopter stable? Can it be maintained in its near hover state with the NOTAR system failure? Could you design an AFCS to permit automatic hovering without using the NOTAR system? 13.8 NOTES 1. Some newer types of helicopter are called NOTARs (no tail rotors), yet even though the method of generating the countering moment has been changed in these types, the same function as the tail rotor performance is maintained. That is OX parallel to the flight path, O Z pointing vertically downwards, and OY pointing starboard. This assumes that the direction of rotation of the main rotor is counter-clockwise (viewed from above), the usual direction for USA and UK manufactured helicopters. These comparisons are made in relation to an articulated rotor of the same type, but with no flap hinge offset. This supposes that xi is q, the pitch rate. The same system operates on p , the roll rate, provided that a properly oriented rate gyroscope is fitted. 2. 3. 4. 5. 13.9 REFERENCES URAMWELL, A.R.S. 1976. 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Digital Control Systems 14.1 INTRODUCTION In the preceding chapters it is assumed that any AFCS being considered had a feedback control law which was synthesized as a continuous function of time. Most current and new types of AFCS now depend (and all future types of AFCS will depend) upon digital synthesis of the control law, that is to say, that the AFCS is now, or is going to be, digital. The objectives of this chapter are, therefore: 1. To introduce several essential concepts of digital control. 2. To discuss a few effective methods of designing suitable digital controllers for AFCSs. 3. To note the effects upon the dynamic performance of an AFCS of using such a means of control. The distinguishing feature of digital control is that some of the signals within the system cannot be known at every instant of time, but are known only at particular instants. For example, imagine that a switch is being used to control some electrical signal x (t), in the way shown in Figure 14.1. If that switch be closed at some instant of time, say t,, its output will be as shown in Figure 14.2(b). (x(t) is shown in Figure 14.2(a) for comparison.) If the switch remains closed for only a brief period, At, and is then opened once again, the output signal will be as represented in Figure 14.2(c). If some longer period of time, T, passes, and then the switch is once again rapidly closed and opened, the resulting output is as represented in Figure 14.2(d). If it is arranged to rapidly close and open the switch in a regular cycle, of period T, the output which results has the form represented in Figure 14.3. Whenever the duration of switch closure is extremely brief, the period At + 0; the output from the switch (as it periodically closes and opens) can be represented in the manner shown in Figure 14.4. This signal has been denoted as y*, and it is said to be the sampled version of the input signal, x(t). The discrete signal, y*, is known accurately only at the sampling intervals (ktl + T), where k = 0, 1, 2, ... When the sampling period, T, is constant, the sampling is uniform; much of the work in this chapter assumes uniform sampling. When At -+ 0 and the switch is closed periodically every T seconds, the switch is referred to as a sampler (or sampling device) and can be represented as Digital Control Systems x(0 P S Figure 14.1 - ~(t) Sampling switch. t (4 O tI Time Figure 14.2 Switch output signals. Figure 14.3 Periodically sampled signal. A Simple Discrete Control System Figure 14.4 Sampled signal. Figure 14.5 Idealized sampler. in Figure 14.5. A sampler, then, is a device for transforming a continuous into a discrete signal. Whenever the amplitude of a discrete signal is quantized, that signal is regarded as being digital.' 14.2 A SIMPLE DISCRETE CONTROL SYSTEM A simple system, in which the variables are continuous, is represented by Figure 14.6. The system is merely an integrator whose output is also used as a negative feedback signal. The transfer function of this simple system can easily be shown to be: Y(s)/R(s) = Kl(s + K) (14.1) Figure 14.6 Simple first order system. If the input, r(t), is a unit step function, then: y (t) = 1 - ePk' (14.2) The output response for this system is shown in Figure 14.7. Now if it is supposed that the error signal, e, is sampled in the manner shown in Figure 14.8, the output signal, y, will change at a constant rate, being proportional to the error during the sampling period T. Let the error signal at an Digital Control Systems Time (s) Figure 14.7 Step response of simple first order system. device Figure 14.8 Sampled first order system. instant kT be denoted by ek. The rate of change of the output signal can then be expressed as: dy ldt = Kek (14.3) For any arbitrary input, r(t), the situation may be represented by Figure 14.9, from which it can be deduced that: I 0 I I I I I T 2T 3T 4T 5T Time Figure 14.9 Error signals at sampling instants. Let: A ~ K T A Simple Discrete Control System 495 (sometimes referred to as the specific control step), then: yk + I + (A - l)yk = Ark (14.6) If only the transient motion is considered, i.e. r(t) is assumed to be zero, then: Yk + 1 = (1 - A) Y ~ (14.7) 1.e.: Yk = - k - 1 h) Yl (14.8) The output response is heavily dependent upon the particular value of A. For example, suppose both A and yl are unity. Then y2, y3, y4, etc. are all zero. If A is 0.5, then for y, = 1.0, the output sequence becomes: y2 = 0.5, y3 = 0.25, y4 = 0.125, y5 = 0.0625, etc. If A is increased to 1.5 the output sequence becomes: y2 = - 0.5, y3 = 0.25, y4 = - 0.125, y5 = 0.0625, y6 = - 0.03126, etc. When A is increased to 2.0, however, the output sequence alternates between - 1.0 and 1.0. The responses corresponding to these sequences are shown in Figure 14.10, having been obtained from a digital simulation of the system shown in Figure 14.8. The dynamic situation can be summarized thus: If 0 < A < 1, y(t) tends to zero without oscillation. If A = 1, y(t) reaches zero in a single sampling interval. If 1 < A < 2, y(t) tends to zero with oscillation (at a frequency equal to sampling rate, 1IT). If A < 0, y(t) is unstable. If A > 2.0, y(t) is oscillatory and unstable. If the input r(t) is constant and non-zero, then, in the steady state: Figure 14.10 Responses of sampled first order system. 496 Digital Control Systems Yk+ 1=r (14.9) It should be noted that, whereas the continuous system of Figure 14.6 can never oscillate and can never be unstable (unless K is negative), the digital version may oscillate and can even be unstable merely as a consequence of the value chosen for the sampling period, T, in relation to K. Such an oscillatory response is referred to as a 'hidden oscillation' since it occurs solely as a result of having used digital control with a particular value of sampling rate. The condition should be avoided in every digital AFCS by a proper choice of T. Example 14.1 Find a discrete equation which will represent the transfer function of a phase advance network: + G(s) = U(s)lE(s) = (1 + ~ s) / (l a m) 0 5 a5 1 The corresponding differential equation is given by: e - ~t = u + a ~ u . Let en - u, - en and u, be the value of the continuous signal at the (n - 1)th and nth sampling instants respectively. Now: t b (en - en - l)lT u b (u, - u, - l)lT T is the sampling period. Then: U, = en = + e n - Ten clen - 1 a7 7 - - (u, - U, + c2en - 1 + c3u, T - - where: CI = (T + 7)/(T + a ~ Appendix A 554 It is evident from inspection of (A.62) that some stability has to be provided by the control amplifier and, consequently, it has a transfer function of the form: (A.63) (A.64) Usually T2 is negligible, therefore: (A. 6.5) where: (A.66) (A.67) (A.68) A.6 ANGLE OF ATTACK SENSOR There are a number of methods of sensing angle of attack, but essentially only two types are in general use. In low speed flight, the moving vane type is preferred. It is a small vane protruding into the airstream, close to the body of the aircraft. The vane can rotate over a limited angular range. It is mounted on the shaft of a position transducer which provides an electrical signal proportional to the angle of attack. The use of this type in AFCS work is restricted since its accuracy is much affected by local flow conditions. The other method employs a stationary pressure-ratio sensing probe, a sketch of which is shown in Figure A.18. The angle of attack is usually obtained from the pressures measured at two (often more) suitable positioned orifices, i.e.: (A.69) where K is a constant, M is the Mach number of the moving air, P is the sideslip angle, (PT - P,) is a measure of the dynamic pressure (I, and $I and F are nonlinear functions, in general. Both dynamic pressure and Mach number change slowly compared to changes in the angle of attack, which is governed by the short Actuators and Sensors PT W i n d y f p2 / Probe Figure A.18 Pressure-ratio angle of attack sensor. period motion of the aircraft. The sideslip effect cannot be easily ignored. With the pressures from the probe being fed to bellows via pneumatic lines, the motion of the bellows, which results from volume changes caused by the changes in pressure, is used to drive position transducers to provide an electrical output signal. There is, however, an unavoidable time delay involved in pneumatic line transmission, and there is also a time lag associated with any bellows transducer. Thus, the transfer function of the angle of attack sensor becomes: where T represents the transport time delay, TL the time constant of the interconnecting pneumatic lines, and TB the time constant of the bellows. A.7 REFERENCES AHRENDT, W.R. and C.J. SAVANT. 1960. Servomechanism Practice. New York: McGraw-Hill. COLLEWE, J.G.R. 1970. Sensor and actuator dynamics. In A.L. Greensite, Analysis and GREEN, Design of Space Vehicle Flight Control Systems, Vol. 11, Chapter 3. New York: Spartan Books. W.L. 1985. Aircraft Hydraulic System. Chichester: Wiley. Appendix B Stability Derivatives for Several Representative Modern Aircraft NOMENCLATURE B.l Some stability data for seven aircraft are presented here. These aircraft are generic types and are referred to as follows: ALPHA BRAVO CHARLIE DELTA ECHO FOXTROT GOLF a a a a a a a four-engined, executive jet aircraft twin-engined, jet fighter aircraft very large, four-engined, passenger jet aircraft very large, four-engined, cargo jet aircraft single-engined, CCV, jet fighter aircraft twin-engined, jet fighterlbomber aircraft twin-piston engined, general aviation aircraft When referring to an aircraft and its particular flight condition, the aircraft name is given first followed by a number corresponding to the flight condition. For example, FOXTROT-3 means flight condition 3 for the aircraft, FOXTROT. B.2 AIRCRAFT DATA B.2.1 ALPHA - A four-engined, executive jet aircraft General Parameters Wing area (m2) Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) I, : I, P : P 50.4 5.325 3.33 59.2 0.25 E - 6.77 0.73 Stability Derivatives 557 Weight (kg): Approach 10 635 All other flight conditions 17 000 57 000 171 500 218 500 7 500 162 000 185 000 330 000 6 900 Inertias (kg m2) Zxx ZYY : Zzz : 1x2 1 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N m2) a. (degrees) Yo (degrees) 1 2 S.L. 0.2 67.7 2 844.0 6.5 0 6 100 0.35 110.6 4 000 + 9.9 0 + Flight condition 3 4 6 100 0.75 237.1 18 338 2.6 0 12 200 0.8 236.0 8 475 4.2 0 + + Stability Derivatives Longitudinal Motion Stability derivative 1 2 Flight condition 3 4 Appendix B 558 Lateral Motion Stability derivative 1 B.2.2 - A twin-engined, jet fighter aircraft BRAVO 2 Flight condition 3 4 General Parameters Wing area (m2): Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 56.5 3.0 4.86 210 (no reheat) 0.255 L or 0.311 L Weight (kg) : Approach 15 x lo3 Inertias (kg m2): Ixx : All other flight conditions 16 x lo3 559 Stability Derivatives Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) q (N m-2) a. (degrees) Yo (degrees) c.g. Flight condition 3 1 2 S.L. 0.4 136 11348 3.5 0 0.311 6 100 0.6 190 11760 8.5 0 0.255 + + 6 100 0.6 190 11760 8.5 0 0.311 + 4 9 150 0.8 240 10 700 2.5 0 0.311 + Stability Derivatives Longitudinal Motion only Stability derivative B.2.3 1 CHARLIE 2 Flight condition 3 4 - A very large, four-engined, passenger jet aircraft General Parameters Wing area (m2): Aspect ratio: Chord, C (m): Total related thrust (kN): C.g.: 510 7.0 8.3 900 0.25 E Appendix B Pilot's location (m) (relative to c.g.) Weight (kg): Approach 250 000 All other flight conditions 290 000 Inertias (kg m2): zxx Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) q (N mP2) a0 (degrees) Yo (degrees) Flight condition 3 1 2 S.L. 0.198 67 2 810 8.5 0 6 100 0.5 158 8 667 6.8 0 4 6 100 0.8 250 24 420 0 0 12 200 0.8 250 9 911 4.6 0 Stability Derivatives Longitudinal Motion Stability derivative 1 2 Flight condition 3 4 56 1 Stability Derivatives Longitudinal Motion Cont'd Stability derivative 4 1 Flight condition 3 2 - 0.357 - 0.378 0.7 x 1 0 - ~ 4 - 0.421 - 0.668 - 0.339 1.09 - 2.08 - 1.16 0.67 x 1 0 - ~ 0.67 x 1 0 - ~ 0.67 x 1 0 - ~ - Lateral Motion -- Stability derivative B.2.4 DELTA 1 2 Flight condition 3 4 - A very large, four-engined, cargo jet aircraft General Parameters Wing area (m2) Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1, : 1, P : P Weight (kg): 576 7.75 9.17 730 0.3C 25.0 + 2.5 Approach 264 000 All other flight conditions 300 000 Appendix B Inertias (kg m2) zXx 2.6 4.25 6.37 3.4 IYY: IZZ: 1x2 : x 107 3.77 4.31 7.62 3.35 x lo7 x lo7 x lo6 x x x 107 lo7 lo7 lo6 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) (N m2) a. (degrees) Yo (degrees) a Flight condition 3 1 2 S.L. 0.22 75 3 460 2.7 0 6 100 0.6 190 11730 2.2 0 + + 4 6 100 0.8 253 20 900 0.1 0 12 200 0.875 260 10100 + 4.9 0 + Stability Derivatives Longitudinal Motion Stability derivative 1 2 Flight condition 3 4 Stability Derivatives 563 Lateral Motion Stability derivative 8.2.5 1 ECHO Flight condition 3 2 - A single-engined, CCV, jet fighter aircraft General Parameters Wing area (m2): Aspect ratio: Chord, t (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1, : 1, P : P Weight (kg) : Inertias (kg m2): zxx: IYY: IZZ: I~~: 4 26 3.0 3.33 11 0.35 E - 3.9 0.326 84.52 11 x 6.38 x 7.24 x 4.7 x 103 lo4 lo4 104 Appendix B 564 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) q (N mP2) cro (degrees) Yo (degrees) Flight condition 3 1 2 S.L. 0.6 207 26 245 1.92 0 4 600 0.8 258 25 860 2.17 0 + + 9 100 0.95 288 17 362 4.25 0 + 4 15 250 1.7 502 23 400 + 1.6 0 Stability Derivatives Longitudinal Motion only Stability derivative B.2.6 1 FOXTROT 2 4 - A twin-engined, jet fighterlbomber aircraft General Parameters Wing area (m2): Aspect ratio: Chord, C (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1, : zz;: Flight condition 3 49.24 4.0 48.88 160 0.29 E - 5.32 1.0 Stability Derivatives 565 Weight (kg): Approach 148 All other flight conditions 173 32 100 16 000 181 400 2 100 33 900 166 000 190 000 3 000 Inertias (kg m2): L X : IYY: Zzz : 1x2 : Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight condition 3 1 2 S.L. 0.206 70 2 997 11.7 0 10 650 0.9 265 13 550 2.6 0 10 650 1.2 350 24 090 1.6 0 4 13 700 2.15 650 48 070 1.4 0 Stability Derivatives Longitudinal Motion Stability derivative 1 2 Flight condition 3 4 Appendix B 566 Lateral Motion -- Stability derivative B.2.7 GOLF 1 2 Flight condition 3 4 - A twin-piston engined, general aviation aircraft General Parameters Wing area (m2) Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1, : 1, P : P Weight (kg): Approach 20 Inertias (kg m2) zxx: IYY: Zzz : 1x2: 13 470 20 450 27 200 2 150 All other flight conditions 27.75 Stability Derivatives 567 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N m2) a. (degrees) Yo (degrees) Flight condition 3 1 2 S.L. 0.143 50.0 1530 S.L. 0.19 65 2 590 1600 0.207 70 1960 6 500 0.345 105 3 440 - - - - 4 Stability Derivatives Longitudinal Motion Stability derivative 1 2 Flight condition 3 4 Appendix B 568 Lateral Motion Stability derivative 1 2 Flight condition 3 4 Appendix C Mathematical Models of Human Pilots C.1 INTRODUCTION Notwithstanding the extent to which flight control is being made automatic, it remains essential for the designers of flight control systems to remember that a human pilot acts as the 'outer loop' of a complete flight control system. As AFCSs have been improved and developed, the need to represent human pilots by appropriate mathematical models has become more pressing, although the need for such representation has been recognized for a considerable time. It has been the cause of a great amount of research which is recorded in a most extensive literature. Chief among the workers researching in this field have been McRuer, Krendel and Graham, and it is their work (see the various references at the end of this appendix) which provides the basis for those models dealt with briefly below. More extensive models exist, such as Paper Pilot (Dillow, 1971), but they are beyond the scope of an introductory textbook such as this. There are several reasons for using a mathematical model in studies relating to the performance of closed loop flight control systems being operated by a human pilot; the include the following: 1. The prediction of what may be possible from some given arrangement. 2. The evolution and, perhaps, development of critical flight or simulator experiments. 3. The interpretation of flight tests or simulator results. 4. The determination of the limitations of validity of any experimental results. From examining the nature of a pilot's behaviour when flying it becomes clear that he normally demonstrates those characteristics commonly described as adaptive and multimodal. Even when carrying out familiar tasks, the pilot is also capable of learning. This knowledge suggests that the construction of any appropriate mathematical model may incorporate some of the following features: 1. The differential equations involved should be invariant , or time-varying. 2. The model may be multi- or single-variable. 3. The equations may be linear or non-linear. 4. The data may be continuous or sampled. Appendix C 570 The model should represent adequately the pilot's actions when carrying out a pursuit task or controlling the aircraft using a compensatory display. From extensive experiments on human operators it has been learned that one appropriate form of model was a describing function which represents the linear response of the operator whose actual response can only be accurately described by non-linear equations. But these describing functions represent very good approximations for most pilot actions. The validity of the describing function model does depend upon the addition of a remnant term, but, for simplicity, only the linear models represented by describing functions are used here. A remnant term can be considered to be a bias term to ensure that the describing function corresponds to the appropriate operating point. One example of how such a term can be included in the model is given in paragraph 4 below. C.2 CLASSICAL MODELS 1. The pilot's response is denoted by v,; his command is taken as pcomm. Basically, the model assumes that the response is linear and proportional to the command, with some prediction, but with a pure time delay caused by the finite reaction time of the pilot. The model is represented in Figure C . l from which it can be deduced that Figure C.l Block diagram of pilot model - lead term and pure time delay. The transfer function representing the pure time delay, namely: is a transcendental function and can only be completely represented by an infinite series. Consequently, a suitable approximation is needed. One of the most accepted is the first order Padé approximation: Let: R1 = d p +d Mathematical Models of Human Pilots then: fl=dp+3= 2 2 7 7 -v--vp - 4 v 2 - -XI 7 7 However, V = .'. KpT~Ijcomm+ Kppcomm x1 = 4 K p T L Pcomm 7 Vp = XI - 2. 4K 2 +P Pcomm - XI 7 7 Kppcomm - K p T ~ I j c o m m Refer to Figure C.2. Figure C.2 Block diagram of pilot model - phase advance and pure time delay. (C. 10) K T ... v. = - - 1 v + K Pcomm + &pcomm (C.ll) Tl Tl Tl Using the first order Padé approximation of eq. (C.3) and choosing the state variables for this model to be: XI = v, Pcomm = v +v (C. 12) (C.13) the following equations are obtained: (C. 15) Appendix 572 3. C Refer to Figure C.3. The term: o;/(s2 + 250,s + w;) represents the addition of a neuromuscular lag to the model. The transfer can easily be represented by the following state function V(s)lpComm(s) equation: [:I = [- - 21n] Finally, if we define x3 as v (C.3) then: - + 0 [I] [f 1 + + v,, and use the Padé approximation of (C. 19) - 1 (C. 17) pcomm Pcomm 0 - 4. a Refer to Figure C.4. Using a more comprehensive model relating to hovering motion in which a remnant term and phase advance compensation are added, the following equations are obtained: 1 el=- Kpl KPTL1 el - -pcomm - -Pcomm Tl Tl Tl (c.21) Mathematical Models of Human Pilots Figure C.4 Block diagram of pilot model - phase advance, pure time delay, lead term neuromuscular lag and remnant term. (C.22) e3 = e2 + e4 (C. 23) i4 = - m ~ e '+ , uRv (2 w (C. 24) R) ~ Also: (C.26) N o w let: XI x2 = e4 = el x5 = v + vp (C.27) then the following state equation can be obtained: -ir+ E i = A x + B z + M n (C .28) v, = Cx (C.29) where: x1 = [XI x2 z1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M= (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ ~ w ~ (C.36)

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PRENTICE HALL INTERNATIONAL SERIES IN SYSTEMS AND CONTROL ENGINEERING SERIES EDITOR: M.J. GRIMBLE B I I Automatic Flight Control Systems is an introductory text, coverisg in detail the subjects of stability and control, flight dynamics and modern control theory, culminating in the examination of particular, important AFCSs. The book begins by discussing the dynamic responses of aircraft to atmospheric turbulence and structural flexibility, an understanding of which is essential for the successful design of any AFCS. Several fundamental AFCS modes are described, including stability augmentation systems, attitude and flight path control systems. Key features include: Coverage of both fixed-wing and rotary-wing aircraft AFCSs. Two self-contained chapters on relevant modern control theory. End-of-chapterexercises, references and summaries. Suitable for undergraduate and professional aeronautical engineers alike, this book will prove invaluable to those requiring an introduction to modern flighi control systems. DONALD McLEAN is currently Westland Professor of Aeronautics at the University of Southampton, UK. PRENTICE HALL

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