



Flight Control from Namco has experienced notable success over on the iPhone and was recently released for the Windows Mobile platform. The concept is simply, you are in charge of flight control for a local airport. As planes come into play, you direct them to the appropriate runway while diverting other planes to a holding pattern and avoid midair collisions. The more successful landings, the higher your score. To add to the game's challenge, only certain planes can land on certain runways (color coded) and helicopters can only land on helipads. As the game progresses and you successfully land planes, the air traffic increases. Flight Control almost has the feel of a juggling act. See how many planes you can have circling about while you concentrate on landing one plane at a time. If the pace gets too fast, you can use the "Time Control" feature to slow things down temporarily. You control the aircraft's direction by touch. Tap/hold a plane or helicopter and drag your finger in the direction you want it to follow. A dotted line appears to show you the flight path. When you're ready for the plane to land, drag you finger to the runway. Planes at risk of colliding with other planes or flashing circle (risk is from an on-screen plane). Game play is simple but at times my finger got in the way of seeing other planes. Using a stylus made it easier to see more of the playing field. The stylus also help plot a more accurate course for the aircraft. A feeling of determination kicks in and makes ignoring the "try again" option all the more difficult. Flight Control's simple interface and fast pace come together to give the game a unique level of intensity that is hard to put down. Flight Control runs \$5.99 and is available through your wireless provider. Currently, the games are only available for T-Mobile and AT&T compatible Windows Phones. Stock Image McClean, D. Published by Prentice-Hall (1990) ISBN 10: 0130540080 ISBN 13: 9780130540089 Used Quantity: 1 Seller: Goldstone Books (Llandybie, United Kingdom) Rating Seller Rating: Book Description Textbook Binding. Condition: Very Good. All orders are dispatched the following working day from our UK warehouse. Established in 2004, we have over 500,000 books in stock. No quibble refund if not completely satisfied. Seller Inventory # mon0005846240 More information about this seller | Contact this seller 610 Pages • 142,997 Words • PDF • 16.4 MB Uploaded at 2021-09-26 07:25 This document was submitted by our user and they confirm that they have the consent to share it. Assuming that you are writer or own the copyright of this document, report to us by using this DMCA report button. i i \ - PRENTICE HALL INTERNATIONAL SERIES IN SYSTEMS AND CONTROL ENGINEERING SERIF^ 'DITOR: M.J. GRIMBLE DONALD MCLEAN Automatic Flight Control Systems Prentice Hall International Series in Systems and Control Engineering M. J. Grimble, Series Editor BANKS,S. P., Control Systems Engineering: Modelling and Simulation, Control Theory and Microprocessor Implementation BANKS,S. P., Mathematical Theories of Nonlinear Systems EUNZE,J., Robust Multivariable Feedback Control PATTON, R., CLARK, R. N., FRANK, P. M. (editors), Fault Diagnosis in Dynamic Systems SODERSTROM, T., and STOICA, P., System Identification WARWICK, K., Control Systems: An Introduction Automatic Flight Control Systems Donald McLean Westland Professor of Aeronautics University of Southampton, UK PRENTICE HALL New York . London . Toronto - Sydney. Tokyo. Singapore a First published 1990 by Prentice Hall International (UK) Ltd 66 Wood Lane End, Heme1 Hempstead Hertfordshire HP2 4RG A division of Simon & Schuster International Group @ 1990 Prentice Hall International (UK) Ltd All rights reserved. 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Title 629.135'2 ISBN CL13-054008-0 Contents Preface 1 Aircraft Flight Control 1.1 1.2 1.3 1.4 1.5 1.6 1.7 1.8 1.9 Introduction Control Surfaces Primary Flying Controls Flight Control Systems Brief History of Flight Control Systems Outline of the Book conclusions Note References 2 The Equations of Motion of a Rigid Body Aircraft Complete Linearized Equations of Motion Equations of Motion in Stability Axis System Equations of Motion for Steady Manoeuvring Flight Conditions Additional Motion Variables State and Output Equations of Motion of Thrust Effects Conclusions Exercises Notes References 3 Aircraft Stability and Dynamics 3.1 3.2 Introduction Longitudinal Stability vi Contents Static Stability Transfer Functions Related to Longitudinal Motion Transfer Functions Obtained from Phugoid Approximation Lateral Stability Transfer Functions Related to Lateral Motion Transfer Functions Two Degrees of Freedom Approximations Single Degree of Freedom Approximation State Equation Formulation to Emphasize Lateral/ Directional Effects of Structural Flexibility Upon the Motion of an Aircraft Introduction Bending Motion of the Wing Torsion of the Wing Coupled Motions The Dynamics of a Flexible Aircraft Mathematical Representation of the Dynamics of a Flexible Aircraft Motion Introduction Atmospheric Disturbances A flexible Aircraft Motion Exercises Notes References 5 Disturbances Affecting Aircraft Motion Science Continuous Gust Representations State Variable Models Angular Gust Equations The Effects of Gusts on Aircraft Motion Contents 5.9 Transient Analogue 5.10 Determination of the r.m.s. Value of Acceleration as a Result of Encountering Gusts 5.11 Wind Shear and Microbursts 5.12 Sensor Noise 5.13 Conclusions 5.14 Exercises 5.15 References 6 Flying and Handling Qualities 6.1 6.2 Introduction Some Definitions Required for Use with Flying Qualities 6.3 Conclusions 6.3 Longitudinal Flying Qualities 6.4 LateraVDirectional Flying Qualities 6.5 The C * Criterion 6.6 Ride Discomfort Index 6.7 Helicopter Control and Flying Qualities 6.8 Conclusions 6.9 Exercises 6.10 References 7 Control System Design Methods I AFCS as a Control Problem Generalized AFCS Conventional Control Methods Parameter Optimization Conclusions Exercises Note References 8 Control System Design Methods 11 Introduction The Meaning of Optimal Control Controllability, Observability and Stabilizability Theory of the Linear Quadratic Problem Optimal Output Regulator Problem State Regulators with a Prescribed Degree of Stability Explicit Model-Following Optimal Command Control System Use of Integral Feedback in LQP Contents viii 8.10 8.11 8.12 8.13 8.14 State Reconstruction Conclusions Exercises Notes References 9 Stability Augmentation Systems Introduction Actuator Dynamics Sensor Dynamics Longitudinal Control (Use of Elevator Only) Other Longitudinal Axis SASS Sensor Effects Scheduling Lateral Control Systems Roll Angle Control Systems Roll Angle Control Systems Notes Reference 10 Attitude Control Systems Notes Reference 10 Attitude Control Systems Roll Angle C Ground Roll Conclusions Exercises Notes References I 1 Flight Path Control Systems Introduction Height Control System Speed Control System VOR-Coupled Automatic Tracking System ILS Localizer-Coupled Control Systems ILS Glide-Path-Coupled Control System Contents 11.10 11.11 11.12 11.13 11.14 11.15 Automatic Landing System A Terrain-Following Control System for a Bomber Aircraft A Ride Control System for a Modern Fighter Aircraft Aircraft Positioning Control Systems Conclusions Exercises Note References 13 Helicopter Flight Control Systems Introduction Exercises Notes References 14 Digital Control Systems Introduction A Simple Discrete Control System A Data Hold Element The z-Transform Bilinear Transformations Discrete State Equation Stability of Digital Systems Optimal Discrete Control Use of Digital Computers in AFCSs Conclusions Contents 14.13 References 15 Adaptive Flight Control Systems Introduction Model Reference Systems The MIT Scheme Example System A Lyapunov Scheme Parameter Adaptation Scheme Conclusions Notes References Appendices A Actuators and Sensors A. 1 A.2 A.3 A.4 A.5 A.6 A.7 B Stability Derivatives for Several Representative Modern Aircraft B. 1 B.2 C Introduction Actuator Use in AFCSs Actuators Sensors Accelerometers Angle of Attack Sensor References Nomenclature Aircraft Data Mathematical Models of Human Pilots C. 1 C.2 C.3 Introduction Classical Models References Preface This is an introductory textbook on automatic flight control systems (AFCSs) for undergraduate aeronautical engineers. It is hoped that the material and the manner of its presentation will increase the student's understanding of the basic problems of controlling an aircraft's flight, and enhance his ability to assess the solutions to the problems which are generally proposed. Not every method or theory of control which can be used for designing a flight controller is dealt with in this book; however, if a reader should find that some favourite technique or approach has been omitted the fault lies entirely with the author upon whose judgement the selection depended. The method is not being impugned by its omission. Before understanding how an aircraft may be controlled automatically in flight it is essential to know how any aircraft will respond dynamically to a deliberate movement of its control surfaces, or to an encounter with unexpected and random disturbances of the air through
which it is flying. A sound knowledge of an aircraft's dynamic response, which can result in the aircraft's being considered by a pilot as satisfactory to fly, is also important. In this book the first six chapters are wholly concerned with material relevant to such important matters. There are now so many methods of designing control systems that it would require another book to deal with them alone. Instead, Chapters 7 and 8 have been included to provide a reasonably self-contained account of the most significant methods of designing linear control systems which find universal use in AFCSs. Emphasis has been placed upon what are spoken of as modern methods): it is most unlikely that today's students would not consider the use of a computer in arriving at the required solution. Being firmly based upon time-domain methods, modern control theory, particularly the use of state equations, is a natural and effective technique for use with the mathematical description of the aircraft dynamics which are most completely, and conveniently, expressed in terms of a state and an output equation. The form involved leads naturally to the use of eigenvalues and eigenvectors which make consideration of the stability properties of the aircraft simple and straightforward. Since computers are to be used, the need for normalizing the dynamic equations can be dispensed with and the differential equations can be solved to find the aircraft's motion in real time. The slight cost to be borne for this convenience is that the stability derivatives of the aircraft which are used in the analysis are dimensional; xii Preface however, since the aircraft dynamics of the flight controller, the control surface actuators, and the motion sensors can also be dealt with in real time, thereby avoiding the need for cumbersome and unnecessary transformations. Since dimensional stability derivatives were to be used, the American system of notation for the aircraft equations of motions was adopted: most papers and most data throughout the world now use this system. being concerned with stability augmentation systems, attitude and path coitrol systems. A particular AFCS may have some, or all, of these modes involved in its operation, some being active at all times in the flight, and others being switched in by the pilot only when required for a particular phase of flight. Although helicopter flight control systems do not differ in principle from those used with fixed wing aircraft, they are fitted for different purposes. Furthermore, both the dynamics and the means of controlling a helicopter's flight are radically different from fixed wing aircraft. problems that their use is intended to overcome. Active control systems are dealt with in Chapter 12 and only a brief treatment is given to indicate how structural motion. Ride controlling the aircraft's rigid body motion. Ride control and fuselage pointing are flight control modes dealt with in this chapter In the thousands of commercial airliners, the tens of thousands of military aircraft, and the hundreds of thousands of general aviation aircraft which are flying throughout the world today, examples of the types of AFCS discussed in this book can easily be found. But most modern AFCSs are digital, and to account for this trend Chapter 14 has been added to deal solely with digital control methods. The consequences for the dynamic response of the closed-loop system of implementing a continuous control law in a digital fashion is emphasized. Results complementary to those in Chapters 9 to 11, obtained using wholly digital system analysis, are also shown. The final chapter deals briefly with the subject of adaptive flight control systems, and three appendices provide a summary of information relating to actuators, sensors, aircraft stability data, and human operators. In writing a textbook, ideas and techniques which have been used effectively and easily by the author over the years are discussed and presented, but the original source is often forgotten. If others find their work used here but unacknowledged, please be assured that it was unintentional and has occurred mostly as a result of a middle-aged memory rather than malice, for I am conscious of having had many masters in this subject. At the risk of offending many mentors, I wish to acknowledge here only the special help of three people, for the list of acknowledgements would be impossibly long otherwise. Two are American scholars: Professors Jack d'Azzo and Dino Houpis, of the United States Air Force Institute of Technology, in Dayton, Ohio. They are nonpareil as teachers of control and taught me in a too-short association the importance of the student and Preface xiii his needs. The other is my secretary, Liz Tedder, who now knows, to her lasting regret, more about automatic flight control systems than she ever wished to know. D. McLEAN Southampton Aircraft Flight Control 1.I INTRODUCTION Whatever form a vehicle may take, its value to its user depends on how effectively it can be made to proceed in the time allowed on a precisely controllable path between its point of departure and its intended destination. That is why, for instance, kites and balloons find only limited application in modern warfare. When the motion of any type of vehicle is being studied it is possible to generalize so that the vehicle can be regarded as being fully characterized by its velocity vector. The time integral of that vector is the path of the vehicle through space (McRuer et al., 1973). The velocity vector, which may be denoted as ;6, is affected by the position, x, of the vehicle in space by whatever kind of control, u, can be used, by any disturbance, 6, and by time, t. Thus, the motion of the vehicle can be represented in the most general way by the vector differential equation: where f is some vector function. The means by which the path of any vehicle can be controlled only in its velocity; it cannot be steered, because its lateral direction is constrained by the contact of its wheel rims on the rails. Automobiles move over the surface of the earth, but with both speed and direction being controlled. Aircraft differ from locomotives and automobiles because they have six degrees of freedom: three associated with angular motion about the aircraft's centre of gravity and three associated with the translation of the centre of gravity.1 Because of this greater freedom of motion, aircraft which tend to make it resist any change of its velocity vector, either in its direction or its direction or its direction. magnitude, or in both, are what constitutes its stability. The ease with which the velocity vector may be changed is related to the aircraft's quality of control. It is stability which makes possible the maintenance of a steady, unaccelerated flight path; aircraft manoeuvres are effected by control. Of itself, the path of any aircraft is never stable; aircraft have only neutral stability in heading. Without control, aircraft tend to fly in a constant turn. In order to fly a straight and level course continuously-controlling corrections must be made, either through the agency of a human pilot, or by means of an automatic 2 Aircraft Flight Control flight control system (AFCS). In aircraft, such AFCSs employ feedback control to achieve the following benefits: 1.2.3. The speed of response is better than from the aircraft without closed loop control. The accuracy in following commands is better. The system is capable of suppressing, to some degree, unwanted effects which have arisen as a result of disturbances affecting the aircraft's flight. However, under certain conditions such feedback control systems have a tendency to oscillate; the AFCS then has poor stability. Although the use of high values of gain in the feedback loops can assist in the achievement of fast and accurate dynamic response, their use is invariably inimical to good stability. Hence, designers of AFCSs are obliged to strike an acceptable, but delicate, balance between the requirements for stability and for control. The early aeronautical experimenters hoped to make flying machines. What they tried to provide was a basic, self-restoring property of the airframe without the active use of any feedback. A number of them such as Cayley, Langley and Lilienthal, discovered how to achieve longitudinal static stability with respect to the relative wind, e.g. by setting the incidence of the tailplane at some appropriate value. Those experimenters also discovered how to use wing dihedral to achieve lateral static stability. However, as aviation has developed, it has become increasingly evident that the motion of an aircraft designed to be inherently very stable, is particularly susceptible to being affected by atmospheric turbulence. This characteristic is less acceptable to pilots than poor static stability. It was the great achievement of the Wright brothers that they ignored the attainment of inherent stability in their aircraft, but concentrated instead on making it controllable in moderate weather conditions with average flying skill. So far in this introduction, their imprecise sense being left to the reader to determine from the text. There is, however, only one dynamic property - stability which can be established by any of the theories of stability appropriate to the differential equations being considered. However, in aeronautical engineering, the two terms are still commonly used; they are given separate specifications for the flying qualities to be attained by any particular aircraft. When the term static stability is used, what is meant is that if a disturbance to an aircraft causes the resulting forces and moments acting on the aircraft to tend initially to return the aircraft can be said to be statically stable. Some modern aircraft are not capable of stable equilibrium they are statically unstable. Essentially, the function of static stability is to recover the original speed of
equilibrium flight. This does not mean that the initial flight path, but leave it of a disturbance, the resulting forces and moments do not tend initially to restore the aircraft to its former equilibrium flight, but leave it in its disturbed state, the aircraft is neutrally stable. If it tends initially to deviate 3 Control Surfaces further from its equilibrium flight path, it is statically unstable. When an aircraft is put in a state of equilibrium flight path, it is statically unstable. When an aircraft is put in a state of equilibrium flight path, it is statically unstable. When an aircraft is put in a state of equilibrium by the action' of the pilot adjusting the controls, it is said to be trimmed. If, as a result of a disturbance, the aircraft tends to return ibrium flight path, and remains at that position, for some time, the aircraft is said to be dynamically stable. Thus, dynamic stability governs how an aircraft recovers its equilibrium after a disturbance. It will be seen later how some aircraft may be statically stable, but are dynamically unstable, although aircraft unstable will be dynamically unstable. 1.2 CONTROL SURFACES Every aeronautical student knows that if a body is to be changed from its present state of motion. Every aircraft has control surfaces or other means which are used to generate the forces and moments required to produce the accelerations. A conventional aircraft is represented in Figure 1.1. It is shown with the usual control surfaces, namely elevator, ailerons, and rudder. Such conventional aircraft have a fourth control, the change in thrust, which can be obtained from the engines. Many modern aircraft, particularly combat aircraft, have considerably more control surfaces, which produce additional control forces or moments. and vertical canards, spoilers, variable cambered wings, reaction jets, differentially operating horizontal tails and movable fins. One characteristic of flight control surfaces to be used simultaneously. It is shown later in this book that the required motion often needs a number of control surfaces to be used simultaneously. It is shown later in this book that the required motion often needs a number of control surfaces to be used simultaneously. motion as well as the intended motion. When more than one control surface is deployed simultaneously, there often results Figure 1.1 Conventional aircraft Flight Control ading edge (LE) slats canard q 4 ' Figure 1.2 A proposed control configured vehicle. Considerable coupling and interaction between motion variables. It is this physical situation which makes AFCS design both fascinating and difficult. When these extra surfaces are added to the aircraft configured vehicle' (CCV). A sketch of a proposed CCV is illustrated in Figure 1.2 in which there are shown a number of extra and unconventional control surfaces. When such extra controls are provided it is not to be supposed that the pilot in the cockpit will have an equal number of extra levers, wheels, pedals, or whatever, to provide the appropriate commands. In a CCV such commands are obtained directly from an AFCS and the pilot has no direct control over the deployment of each individual surface. The AFCS involved in this activity are said to be active control technology systems. The surfaces, and how he commands the deflections, or changes, he requires from the controls is by means of what are called the primary flying controls. 1.3 PRIMARY FLYING CONTROLS In the UK, it is considered that what constitutes a flight control system is an arrangement of all those control elements which enable controlling forces and moments to be applied to the aircraft. These elements are considered to belong to three groups: pilot input elements, system output elements and intervening linkages and elements. The primary flying controls 5 operation of the control surfaces. The main primary flying controls are pitch control, roll control and yaw control. The use of these flight controls affects motion principally about the transverse, the longitudinal, and the normal axes respectively, although each may affect motion about the transverse, the longitudinal, and the normal axes respectively. primarily governed by considerations of engine management. Figure 1.3 represents the cockpit layout of a typical, twin engined, general aviation aircraft. The yoke is pulled towards, or pushed away from, the pilot the elevator is moved correspondingly. When the yoke is rotated to the left or the right, the ailerons of the aircraft are moved. Yaw control is effected by means of the pedals, which a pilot pushes left or right with his feet to move the rudder. In the kind of cockpit illustrated here, the link between these primary flying controls and the control surfaces is by means of cables and pulleys This means that the aerodynamic forces acting on the control surfaces have to be countered directly by the pilot. To maintain the required counterforce, which can be very difficult and fatiguing to sustain. Consequently, all aircraft have trim wheels (see Figure 1.3) which the pilot adjusts until the command, which he has set initially on his primary flying control, is set on the control surface and the pilot is then relieved of the need to sustain the force. There are trim wheels for pitch, roll and yaw (which is sometimes referred to as 'nose trim'). Dual r.p.m. gauge Magnetic compass Dual manifold pressure gaug Dual exhaust gas temperature Fuel pressure WR fuel quantity Avionics cluster ropellors blade pitch controls Instrument landing system (I Pitch trim wheel Figure 1.3 Cockpit layout. Aircraft Flight Control ..., m ~ i e v a t o r I\, Aileron Figure 1.4 Control surface deflection conventions. In large transport aircraft, or fast military aircraft, the aerodynamic forces acting on the control surfaces are moved by means of mechanical linkages driven by electrohydraulic actuators. A number of aircraft use electrical actuators are then used. Usually the control surfaces are moved by means of mechanical linkages driven by electrohydraulic actuators. but there are not many such types. The command signals to these electrohydraulic actuators are electrical voltages supplied from the primary flying control itself. By providing the pilot with power assistance, so that the only force he needs to produce is a tiny force, sufficient to move the transducer, it has been found necessary to provide artificial feel so that some force, representing what the aircraft successfully. The conventions adopted for the control surface deflections are shown in Figure 1.4. In the event of an electrical or hydraulic failure such a powered flying control system ceases to function, which would mean that the control surface could not be moved: the aircraft retain a direct, but parallel, mechanical connection from the primary flying control to the control surface which can be used in an emergency. When this is done the control system is said to have 'manual reversion'. Fly-by-wire (and fly-by-light) aircraft have essentially the same kind of flight control system, but are distinguished from conventional aircraft by having no manual reversion. To meet the emergency situation, when failures occur in the system, fly-by-wire (FBW) aircraft have flight control systems which are triplicated, to meet this stringent reliability requirement. With FBW aircraft and CCVs it has been realized that there is no longer a direct relationship between the pilot's command and the deflection, or even the use, of a particular control surface. What the pilot of such aircraft is commanding from the AFCS is a particular manoeuvre. When this was understood, and when Flight Control Systems Figure 1.5 Side arm controller. the increased complexity of flying was taken into account, it was found that the provision of a yoke or a stick to introduce commands was unnecessary and inconvenient. Modern aircraft are being provided with side arm controllers (see Figure 1.5) which provide signals corresponding to the force. By using such controllers a great deal, but respond to applied force. By using such controllers a great deal, but respond to applied force. avionics displays which modern aircraft require. 1.4 FLIGHT CONTROL SYSTEMS In addition to the control surfaces which are used for steering, every aircraft responds to the pilot's commands or as it encounters some disturbance. The signals from these sensors can be used to provide the pilot with a visual display, or they can be used as feedback signals for the AFCS. Thus, the general structure of an AFCS can be represented as the block schematic of Figure 1.6. The purpose of the controller is to compare the commanded motion with the measured motion and, if any discrepancy exists, to generate, in accordance with the required control law, the command signals to the actuator to produce the control surface deflections which will result in the correct control force or moment being applied. This, in turn, causes the aircraft to respond appropriately so that the measured motion and commanded motion are finally in correspondence How the required control law can be determined is one of the principal topics of this book. Whenever either the physical or abstract attributes of an aircraft, and its motion sensing and controlling elements, are considered in detail, their effects are so interrelated as almost to preclude discussion of any single aspect of the system, Aircraft Flight Control Pilot's direct command input Atmospheric disturbances controls actuators - dynamics Motion variables - I deflections - Flight controller (control law) sensors + Sensor noise Figure 1.6 General structure of an AFCS. broadly, the area of study upon which this book will concentrate. 1. 2. 3. The development of forces and moments for the purpose of restoring a disturbed aircraft to its equilibrium state, and regulating within specific limits the departure of the aircraft. response from the operating point, are regarded here as
constituting flight control. Regulating the aircraft's response is frequently referred to as stabilization. Guidance is taken to mean the action of determining the course and speed to be followed by the aircraft. guidance systems and the aircraft being guided in that the flight control system receives, as inputs from the guidance systems, correction commands, and provides, as outputs, appropriate deflections of the necessary control surfaces to cause the required change in the motion of the aircraft (Draper, 1981). For this control action to be effective, the flight control system must ensure that the whole system has adequate stability. If an aircraft is to execute commands properly, in relation to earth coordinates, it must be provided with information about the aircraft is to execute commands properly, in relation to earth coordinates, it must be provided with information about the aircraft is to execute commands properly. about sixty years, it has been common practice to provide aircraft with Flight Control Systems 9 reference coordinates for control and stabilization by means of gyroscopic instruments. The bank and climb indicator, for example, effectively provides a horizontal reference plane, with an accuracy of a few degrees, and is as satisfactory today for the purposes of control as when it was first introduced. Similarly, the turn indicator, which shows the aircraft's turning left or right, to about the same accuracy, is also a gyroscopic instrument and the use of signals from both these devices, as feedback signals for an AFCS, is still effective and valid. However, the use of conventional gyroscopic a the inherent accuracy of indication, which is to within a few degrees per hour. Such instruments are unsuitable for present-day navigation, which is to within a few degrees only, and also in the inherent accuracy of indication, which is to within a few degrees per hour.instruments in aircraft has fundamental limitations which lie in the inherent accuracy of indication, which is to within a few degrees per hour. Such instruments are unsuitable for present-day navigation, which requires that the accumulated error in distance for each hour of operation, after the accuracy of indication, which is to within a few degrees per hour. an inertial fix, be not greater than 1.5 km. An angle of one degree between local gravitational directions corresponds to a distance on the earth's surface of approximately 95 km. Consequently, special motion sensors, such as ring laser gyros, NMR gyros, strap-down, force-balance accelerometers, must be used in modern flight control systems Because this book is concerned with control, rather than guidance, it is more convenient to represent the motion of aircraft and moves with it. By doing this, the coordinate system of coordinate system, such as a system fixed in the earth, can be avoided. When the origin of such a body-fixed system of coordinates is fixed at the centre of gravity of the aircraft, which is in an equilibrium (or trimmed) state of motion along a nominal flight path, then, when only small perturbations of the aircraft small perturbations of of motion can be linearized. Since many flight control problems are of very short duration (5-20 seconds), the coefficients of these equations of motion can be regarded as constant, so that transfer functions can sometimes be conveniently used to describe the dynamics of the aircraft. However, it must be remembered that a notable feature of an aircraft's dynamic response is how it changes markedly with forward speed, height, and the aircraft's mass. Some of the most difficult problems of flight control occurred with the introduction of jet propulsion, the consequent expansion of the flight control occurred with the introduction of jet propulsion, the consequent expansion of the flight envelope of such aircraft, and the resulting changes in configuration, most notable of which were the use of swept wings, of very short span and greatly increased wing loading, and the concentrated mass of the aircraft being distributed in a long and slender fuselage. In aircraft being distributed in a long induction and greatly increased wing loading, and the concentrated mass of the aircraft's longitudinal motion, and the Dutch roll mode of its lateral motion. Other unknown, coupled Aircraft Flight Control modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender fuselages meant greater flexibility of the aircraft structure, and the modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender fuselages meant greater flexibility of the aircraft structure, and the modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender fuselages meant greater flexibility of the aircraft structure, and the modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender fuselages meant greater flexibility of the aircraft structure, and the modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender fuselages meant greater flexibility of the aircraft structure, and the modes also appeared, such as fuel sloshing and roll instability; the use of thinner wings and more slender flexibility of the aircraft structure, and the modes also appeared structure, and the mod modes of the aircraft's motion, caused further problems. One of the first solutions to these problems was the use of a stability augmentation system designed to increase the relative damping of a particular mode of the motion of the aircraft. or more of the coefficients of the equations of motion by imposing on the aircraft appropriate forces or moments as a result of actuating the control surfaces in response to feedback signals derived from appropriate forces or moments as a result of actuating the control surfaces in response to feedback signals derived from appropriate motion variables. (speed control system), much hold, height hold, and turn coordination systems. An integrated flight control system is a collection of such AFCS modes in a single comprehensive system, with particular modes being selected by the pilot to suit the task required for any particular phase of flight. In the past such functions were loosely referred to as an autopilot, but that name was a trademark registered by the German company Siemens in 1928. Today, AFCS not only augment the stability of an aircraft, but they can perform automatic take-off and landing; they can provide structural mode control, gust load alleviation, and active ride control. 1.5 BRIEF HISTORY OF FLIGHT CONTROL SYSTEMS The heavier-than-air machine designed and built by Hiram Maxim in 1891 was colossal for its time: it was 34 m long and weighed 3 600 kg. Even now, the largest propeller to be seen in the aviation collection of the Science Museum in London is one of the pair used by Maxim. It was obvious to Maxim, if to no-one else at the time, that when his aircraft flew, its longitudinal stability would be inadequate, for he installed in the machine a flight control system which used an actuator to deflect the elevator and employed a gyroscope to provide a feedback signal. It was identical, except in inconsequential detail, to a present-day pitch attitude control system. Two of the minor details were the system's weight, over 130 kg, and its power source, steam. The concept remains unique. Between 1910 and 1912 the American father-and-son team, the Sperrys, developed a two-axis stabilizer system in which the actuators were powered by compressed air and the gyroscopes were also air-driven. The system could maintain both pitch and bank angles simultaneously and, from a photographic Brief History 17 record of a celebrated demonstration flight, in which Sperry Snr is seen in the open cockpit, with his arms stretched up above his head, and a mechanic is standing on the upper surface of the upper wing at the starboard wing tip, maintaining level flight automatically was easily within its capacity. During World War I, aircraft design improved sufficiently to provide, by the sound choice of size, shape and location of the aerodynamic control surfaces, adequate stability for pilots' needs. Many aircraft were still unstable, but not dangerously so, or, to express that properly, the degree of damage was acceptable in terms of the loss rates of pilots and machines. In the 1920s, however, it was difficult to hold heading in poor visibility. Frequently, in such conditions, a pilot and his co-pilot had to divide the flying task between them. The pilot held the course by monitoring both the course and the pitch attitude indicator and by using the rudder; the co-pilot held the speed and the pitch attitude by using the elevator. From the need to alleviate this workload grew the need to control aircraft automatically. The most extensive period of development of early flight control systems took place between 1922 and 1937: in Great Britain, at the Royal Aircraft Establishment (RAE) at Farnborough; in Germany, in the industrial firms of Askania and Siemens; and in the USA, in Sperrys and at NACA (National Advisory Committee for Aeronautics - now NASA). Like all other flight control systems up to 1922, the RAE'S Mk I system was two-axis, controlling pitch attitude and heading. It was a pneumatic system, but its superior performance over its predecessors and competitors was due to the fact that it had been designed scientifically by applying the methods of dynamic stability analysis which had been developed in Great Britain by some very distinguished applied mathematicians and aerodynamicists (see McRuer et al., 1973; Draper, 1981; Hopkin and Dunn, 1947; McRuer and Graham, 1981; Oppelt, 1976). Such comprehensive theoretical analysis, in association with extensive experimental flight tests and trials carried out by the RAF, led to a clear
understanding of which particular motion variables were most effective for use as feedback signals in flight control systems. In 1927, in Germany, the firm of Askania developed a pneumatic system which controlled heading by means of the aircraft's rudder. It used an air-driven gyroscope, designed and manufactured by Sperrys of the USA. The first unit was flight tested on the Graf Zeppelin-LZ127; the system merits mention only because of its registered trade name, Autopilot. However, the Germans soon decided that as a drive medium, air, which is very compressible, gave inferior performance compared to oil, which was considered to be very nearly incompressible. Thus, in its two-axis 'autopilot' of 1935, the Siemens company successfully used hydraulic oil in preference to Maxim's steam. In 1950, the Bristol Aeroplane Company built a four-engined, turbo-prop 12 Aircraft Flight Control transport aircraft which used electric actuators, but it was not copied by other manufacturers. At present, NASA and the USAF are actively pursuing a programme of reasearch designed to lead to 'an all-electric airplane' by 1990. The reader should not infer from earlier statements that the RAE solved every flight control problem on the basis of having adequate theories. In 1934, the Mk IV system, which was a three-axis pneumatic system, was designed for installation in the Hawker Hart, a biplane in service with the RAF. In flight, a considerable number of stability problems were experienced and these were never solved. However, when the same system was subsequently fitted to the heavy bombers then entering RAF service (the Hampdens, Whitleys and Wellingtons) all the stability problems vanished and no satisfactory reasons for this improvement were ever adduced. (McRuer and Graham (1981) suggest that the increased inertia and the consequently slower response of the heavier aircraft were the major improving factors.) In 1940, the RAE had developed a new AFCS, the Mk VII, which was again two-axis and pneumatic, but, in the lateral axis, moved the ailerons in response to a combination of roll and yaw angles. At cruising speed in calm weather the system was adjudged by pilots to give the best automatic control yet devised. But, in some aircraft at low speeds, and in all aircraft in turbulence, the elevator motion caused such violent changes in the pitch attitude that the resultant vertical acceleration so affected the fuel supply that the engines stopped. It was only in 1943 that the problem was eventually solved by Neumark (see Neumark, 1943) who conducted an analysis of the problem entirely by timedomain methods. He used a formulation of the aircraft dynamics that control engineers now refer to as the state equation. German work did not keep pace with British efforts, since, until very late in World War 11, they concentrated on directional and lateral motion AFCSs, only providing a three-axis AFCS in 1944. The American developments had been essentially derived from the Sperry Automatic Pilot used in the Curtiss 'Condors' operated by Eastern Airlines in 1931. Subsequently, electric, three-axis autopilots were developed in the USA by firms such as Bendix, Honeywell and Sperry. The Minneapolis Honeywell C l was developed from the Royal Air Forces and the Royal Air Force. The development of automatic landing was due principally to the Blind Landing Experimental Unit of RAE, although in 1943 at the Flight Development Establishment at Rechlin in Germany, at least one aircraft had been landed automatically. The German efforts on flight control at this time were devoted to the systems required for the V1 and V2 missiles. On 23 September 1947 an American Douglas C-54 flew across the Atlantic completely under automatic control, from take-off at Stephenville, in Newfoundland, Canada, to landing at Brize Norton, in England. A considerable effort has been given to developing AFCSs since that time to become the ultra-reliable integrated flight control systems which form the subject of this book. The interested reader is referred to Outline of the Book 13 Hopkin and Dunn (1947), McRuer and Graham (1981), Oppelt (1976) and Howard (1973) for further discussions of the history of flight control systems. 1.6 OUTLINE OF THE BOOK Chapters 2 and 3 deal with the dynamic nature and characteristics of aircraft and, in so doing, it is hoped to establish the significance and appropriateness of the axis systems commonly used, and to derive mathematical models upon which it is convenient subsequently to base the designs of the AFCS. Chapters 4 and 5 have been included to provide the reader with a clear knowledge of those significant dynamic effects which greatly affect the nature of an aircraft's flight, but over which a designer had no control. The complexity, which inevitably arises in providing a consistent account of the structural flexibility effects in aircraft dynamics, has to be understood if the important development of active control technology is to make sense. The principal objective of Chapter 4 is to provide a reasonable and consistent development of the additional dynamical equations. representing the structural flexibility effects, to show how to incorporate them into the mathematical model of the aircraft, and to provide the reasons why aircraft require flight control systems is to achieve smooth flight in turbulent atmospheric conditions. An explanation of their physical significance. important forms of atmospheric turbulence is given in Chapter 5. How they can be represented mathematically, and how their effects can be properly introduced into the aircraft equations, are also covered there. Chapter 6 deals with the important subject of flying and handling qualities which are expressed mostly in terms of desirable dynamic properties which have been shown, from extensive flight and simulation experiments, to be most suited to pilots' skills and passengers' comfort. These qualities are the chief source of the performance criteria by which AFCS designs are assessed. In a subject as extensive as AFCSs many methods of control system design are tried, used and reported in the technical reports and journals. It is necessary for any student to be competent in some of these methods, and reasonably familiar with the general nature of them all. Although it is not intended to provide a text book in control theory, Chapters 7 and 8 have been included to provide students with a self-contained summary of the most commonly applied methods, together with some indication of the relative advantages and disadvantages of each from the viewpoint of a designer of AFCSs for aircraft. It is the objective of Chapters 9 to 11 to introduce students to the basic flight control modes which form the integrated flight control systems found in most modern aircraft. The nature of the dynamic response and the effects upon the performance of each subsystem of its inclusion as an inner loop in a larger system are both dealt with. Chapter 12 provides students with a clear account of the type of AFCS Aircraft Flight Control 14 which is now finding use in aircraft under current development, the so-called control configured vehicle. The flight control modes involved are specialist (except relaxed static stability, which can be handled by the methods outlined in Chapter 8) and, since the assessment of the search except relaxed static stability, which can be handled by the methods outlined in Chapter 8) and ealt with separately in this chapter. Rotary wing aircraft have quite distinctive methods of control and also have special dynamical problems. Although in forward flight, at all but the lowest speeds, they can be treated in the same manner as fixed-wing aircraft, the control problems are, in general, so distinctive that they are dealt with separately in Chapter 13, although the AFCSs employed in helicopters still involve stability augmentation and attitude and path control. Chapter 14 demonstrates how the control nethods, so that digital AFCSs, which are commonly fitted to modern aircraft, can be considered and also to provide an outline of the effects upon the AFCS7s performance in terms of the particular features of the digital method used. Modern fighter and interdiction aircraft are too great to be handled by control laws devised on the basis of the control methods dealt with earlier. For such situations, the use of adaptive control is advocated. Chapter 15 presents some information about the theories which are used to develop such systems. Since the dynamic equations of these systems are non-linear, special stability considerations apply and these are also dealt with. 1.7 CONCLUSIONS In considering the design of an AFCS an engineer will succeed only if he is able both to establish an adequate model representing the appropriate dynamical behaviour of the aircraft's motion, be familiar with their methods of solution, understand the characteristic responses associated with them, know what influence they have on the aircraft's flying qualities, appreciate how atmospheric disturbances can be characterized and know how such disturbances affect performance. Additionally, it is important to understand how primary flying controls can be improved, or their worst effects reduced, so that the match between a human pilot and the aircraft is optimized. In addition, the theory of control, with its attendant design techniques, must be thoroughly mastered so that it, and they, can be used to produce an AFCS based upon control surface actuators and motion sensors which are available, and whose dynamic behaviour is thoroughly known. References 15 The alternative methods of carrying out the required computation to produce the appropriate control laws have also to be completely understood, and the engineer is expected to be sound in his appreciation of the limitations of whatever particular method was chosen to perform the control design. Detailed engineering considerations of installing and testing
such AFCSs, particularly in regard to certification procedures for airworthiness requirements, and the special studies beyond this book. The influence of these topics on the final form of the AFCS is profound and represents one of the most difficult aspects of flight control work. Any flight control work. Any flight control engineer will be obliged to master both subjects early in his professional career. 1.8 NOTE 1. Sometimes 'centre of mass' and 'centre of gravity' are used interchangeably. For any group of particles in a uniform gravitational field these centres coincide. For spacecraft, their separation is distinctive and this separation results in an appreciable moment due to gravity being exerted on the spacecraft. For aircraft flying in the atmosphere the centres are identically located. 1.9 REFERENCES 1981. Control, navigation and guidance. ZEEE Control Systems Magazine. 1(4): 4-17. HOPKIN, H.R. and R.W. DUNN. 1947. 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The Equations of Motion of an Aircraft 2.1 INTRODUCTION If the problems associated with designing an AFCS were solely concerned with large area navigation then an appropriate frame of reference, in which to express the equations of motion of an aircraft, would be inertial, with its centre in the fixed stars. But problems involving AFCSs are generally related to events which do not persist: the dynamic situation being considered rarely lasts for more than a few minutes. is a tropocentric coordinate system, i.e. one whose origin is regarded as being fixed at the centre of the Earth axis system. It is used primarily as a reference system to express gravitational effects, altitude, horizontal distance, and the orientation of the aircraft. A set of axes commonly used with the Earth axis system is shown in Figure 2.1; the axis, XE, is chosen to point north, the axis, YE, then pointing east with the orthogonal triad being completed when the axis, ZE, points down. If the Earth axis system is used as a basic frame of reference, to which any other axis frames employed in the study are referred, the aircraft itself XE (North) ZE Figure 2.1 Earth axis system. Introduction Lift (positive upwards) All directions shown are positive U, V, R are the forward, side and yaw moments P, Q, R are the angular velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yaw moments P, Q, R are the forward, side and yawing velocities L, M, Nare roll, pitch and yawing velocities L, M, Na a greater or lesser extent, in AFCS work. The choice of axis systems governs the form taken by the equations of motion. However, only body-fixed axis systems, i.e. only systems whose origins are located identically at an aircraft's centre of gravity, are considered in this book. For such a system, the axis, XB, points forward out of the aircraft's centre of gravity, are considered in this book. the axis, YB, points out through the starboard (right) wing, and the axis, ZB, points down (see Figure 2.2). Axes XB, YB and ZB emphasize that it is a body-fixed axis system which is being used. Forces, moments and velocities are also defined. By using a system of axes fixed in the aircraft the inertia terms, which appear in the equations of motion, may be considered to be constant. Furthermore, the aerodynamic forces and moments depend only upon the argular orientation of the body axis system with respect to the Earth axis system depends strictly upon the orientation sequence. This sequence of rotations is customarily taken as follows (see Thelander, 1965): 1. Rotate the Earth axes, XE, YE, and ZE, through some azimuthal angle, q, about the axis, XE, to reach some intermediate axes XI, Y1 and Z1. 2. 3. Next, rotate these axes XI, Y1 and Z1 through some angle of elevation, O, about the axis Y1 to reach a second, intermediate set of axes, X2, Y2, and Z2. Finally, the axes X2, Y2 and Zz are rotated through an angle of bank, @, about the axis, X2, to reach the body axes XB, YB and ZB. Three other special axis systems are considered here, because they can be found to have been used sufficiently often in AFCS studies. They are: the stability axis 78 Equations of Motion of an Aircraft system; the principal axis system; and the wind axis system. In AFCS work, the most commonly used system as system and the X-axis of the stability axis system and the X-axis of the stability axis system. In AFCS work, the most commonly used system and the X-axis of the stability axis system. In AFCS work, the most commonly used system and the X-axis of the stability axis system. body axis system, there is a trimmed angle of attack, a,. The equations of motion derived by using this axis system are a special subset of the set derived by using the body axes is special subset of the set derived by using this axis system. fact that in the equations of motion, all the product of inertia terms are zero, which greatly simplifies the equations. 2.2.3 The Wind Axis System Because this system is oriented with respect to the aircraft's flight path, timevarying terms which correspond to the moments and cross-products of inertia appear in the equations. considerably complicate the analysis of aircraft motion and, consequently, wind axes are not used in this text. They have appeared frequently, however, in American papers on the subject. 2.2.4 Sensor Signals Because an AFCS uses feedback signals from motion sensors, it is important to remember that such signals are relative to the axis system of the sensor and not to the body-fked axis system of the aircraft. This simple fact can sometimes cause the performance obtained from an AFCS to be modified and, in certain flight at cruise it is insignificant. Equations of Motion of a Rigid Body Aircraft 2.3 THE EQUATIONS OF MOTION OF A RIGID BODY AIRCRAFT 2.3.1 Introduction The treatment given here closely follows that of McRuer et al. (1953). It is assumed, first, that the aircraft do not change in flight. Special methods to take into account the flexible motion of the airframe are treated in Chapter 4 When the aircraft can be assumed to be a rigid body moving in space, its motion can be considered to have six degrees of freedom. By applying Newton's Second Law to that rigid body the equations of motion can be established in terms of the translational and angular accelerations which occur as a consequence of some forces and moments being applied to the aircraft. In the introduction to this chapter it was stated that the form of the equations of motion depends upon the choice of axis system were indicated there. In the development which follows, a body axis system is used with the change to the stability axis system being made at an appropriate point later in the text. In order to be specific about the atmosphere in which the aircraft is moving, it is also assumed that the inertial frame of reference does not itself accelerate, in other words, the Earth is taken to be fixed in space. 2.3.2 Translational Motion *~ewton's M Second Law it can be deduced that: d dt = - {H} where F represents the sum of all externally applied forces, M represents the sum of all applied torques, and H is the angular momentum. The sum of the propulsive (thrust) force is produced by expending some of the vehicle's mass. But it can easily be shown1 that if the mass, rn, of an aircraft is assumed to be constant, the thrust, which is a force equal to the relative velocity between the exhausted mass and the aircraft is assumed, the thrust, which is a force equal to the relative velocity between the exhausted mass and the change of the aircraft is assumed, the thrust is assumed to be constant, the thrust is assumed to be constant, the thrust is assumed to be constant. If it is assumed, and the change of the aircraft is assumed to be constant. for the present, that there will be no changes in the aircraft's state of motion from its equilibrium state can occur if and only if there are changes in the aircraft's state of motion of an Aircraft will be when dealing with airspeed control systems, for example) only a small extension of the method being outlined here is required. Details in relation to the stability axis system are given in section 2.2. For the present, however, the thrust force can be considered to be contained in the general applied force, F. When carrying out an analysis of an AFCS it is convenient to regard the sums of applied torque and force as consisting of an equilibrium and a perturbational component, A the component of perturbation. Since the axis system being used as an inertial reference system is the Earth axis system eqs (2.3) and (2.4) can be re-expressed as: By definition,
equilibrium flight must be unaccelerated flight along a straight path; during this flight the linear velocity vector relative to fixed space is invariant, and the angular velocity is zero. Thus, both Fo and Mo are zero. The rate of change of VT relative to the Earth axis system is given by: where w is the angular velocity of the aircraft with respect to XB, YB and ZB, as follows: VT = iU + jV + kW o=iP+jQ+kR and the cross-product, o x VT, is given by: (2.8) (2.9) 21 Equations of Motion of a Rigid Body Aircraft = i(QW - VR) + i(V - UQ) (2.11) In a similar fashion, the components of the perturbation force can be expressed as Hence, + + k(W + PV - UQ) + i(V - VR) + i(V - UQ) (2.11) In a similar fashion, the components of the perturbation force can be expressed as Hence, + + k(W + PV - UQ) + i(V - VR) + ~VP - U Q) m(U AFy = m AF, = m (2.14) Rather than continue the development using the cumbersome notation, AFi, to denote the ith component of the perturbational force, it is proposed to follow the American custom and use the following notation: It must be remembered that now X, Y and Z denote forces. With these substitutions in eqs (2.14)-(2.16), the equations of translational motion can be expressed as: + (+~UR - P W) A X = m (~VP + - UQ) 2.3.3 Rotational Motion For a rigid body, angular momentum may be defined as: H = Iw The inertia matrix, I, is defined as: where Iiidenotes a moment of inertia, and IVa product of inertia j # i. 22 Equations of Motion of an Aircraft Transforming from body axes to the Earth axis system (see Gaines and Hoffman, 1972) allows eq. (2.23) to be re-expressed as: However, cl,xobo and where h,, hy and h, are the components of H obtained from expanding eq. (2.21) thus: (2.28) h, = I_{XY}P - I_{XY}Q - I_{XY}P + I_{YY}Q - Zy, R - I_{Y}P + I_{Y}Q - Zy, R - I_{Y}P + I_{Y general, aircraft are symmetrical about the plane XZ, and consequently it is generally the case that: (2.31) I, = Zyz = 0 Therefore: h, = IxP - Ix, R (2.32) hy = IyyQ (2.33) h, = -I, P + PQ(Iyy - I,) + ZX, (p2 - R2) + PR(Z, -I,) + ZX, (p2 - R2) + ZX, (p2 - American usage: AM, = A L AM, = AM AM, = AN (2.38) where L, M and N are moments about the rolling, pitching and yawing axes respectively. Equations of Motion of a Rigid Body Aircraft AL = IXxP- $z_{,,}(R AM = I_{,,} Q AN 2.3.4 = Z_{,,}R + P Q) + (I_{,,} + I_{,,}(P' - Iyy)QR + (Ixx - I_{,,})PR - IX,P + P Q (Iyy - I_{,}) + Ix,QR - R2) Some Points Arising from the$ Derivation of the Equations It is worth emphasizing here that the form of equations arrived at, having used a body axis system, is not entirely convenient for flight simulation work (Fogarty and Howe, 1969). For example, suppose a fighter aircraft has a maximum velocity of 600 m s-' and a maximum angular velocity QB of 2.0 rad s-l. The term, UQ, in eq. (2.20) can have a value as large as 1200 m sK2, i.e. 120 g, whereas the term, AZ, the normal acceleration due to the external forces (primarily aerodynamic and gravitational) may have a maximum value in the range 10.0 to 20.0 m s-' (i.e. 1-2 g). It can be seen, therefore, how a (dynamic) acceleration of very large value, perhaps fifty times greater than the physical accelerations, can occur in the equations merely as a result of the high rate of rotation experienced by the body axis system. Furthermore, it can be seen from inspection of eqs (2.18)-(2.20) how angular motion has been coupled into translational motion. Moreover, on the right-hand side of eqs (2.39)-(2.41) the third term is a nonlinear, inertial coupling term. For large aircraft, such as transports, which cannot generate large angular rates, these terms are frequently neglected so that the moment equations become: + PQ) AL = ZP, AM = IYyQ + z ~, (P -~ R') AN = Z, R - IX, (R IX, (P - QR) A number of other assumptions are frequently invoked in relation to these equations: 1. 2. 3. Sometimes, for a particular aircraft, the product of inertia, I,,, is sufficiently small to allow of its being neglected. This often happens when the body axes, XB, YB, and ZB have been chosen to almost coincide with the principal axes. For aircraft whose maximum values of angular velocity are low, the terms PQ, QR, and p2 - R2 can be neglected. Since R2 is frequently very much smaller than p2, it is often neglected. It is emphasized, however, that the neglect of such terms can only be practised after very careful consideration of both the aircraft's characteristics and the AFCS problem being considered. Modern fighter aircraft, for example, may lose control as a result of rolYpitch inertial coupling. In such aircraft, pitch-up is sensed when a roll manoeuvre is being carried out. When an AFCS is fitted, such a sensor signal would cause an elevator deflection to be commanded to provide a 24 Equations of Motion of an Aircraft nose-down attitude until the elevator can be deflected no further and the aircraft cannot be controlled. Such a situation can happen whenever the term (Ixx - Z,,)PR is large enough to cause an uncontrollable pitching movement. 2.3.5 Contributions to the Equations of Motion of the Forces Due to Gravity The forces due to gravity are always present in an aircraft; however, by neglecting any consideration of gradients in the gravity field, which are

important only in extra-atmospheric flight if all other external forces are essentially non-existent, it can be properly assumed that gravity coincide in an aircraft, there is no external moment produced by gravity about the c.g. Hence, for the body axis system gravity contributes only to the external force vector, F. The gravitational force acting upon an aircraft is most obviously expressed in terms of the Earth axes. In Figure 2.3 shows the alignment of the gravity vector with respect to these axes. In Figure 2.3 O represents the angle between the gravity vector and the YBZBplane; the angle is positive when the right wing Is down. Direct resolution of the vector mg, into X, Y and Z components produces: SY = mg cos [- O] sin @ = mg cos O sin @ SZ = mg cos C s been introduced and it is necessary to relate them and their derivatives to the angular velocities, P, Q and R. How this is done depends upon whether it rotates relative to inertial space. Aircraft speeds being very low compared to orbital velocities, the vertical may be regarded as fixed. In very high speed flight the vertical will be seen as rotating and the treatment which is being presented here will then require some minor amendments. The manner in which the angular velocity of the body axes about the vector mg. This angular velocity is the azimuth rate, Zk; it is not normal to either 6 or 6, but its projection in the YBZB plane is normal to both (see Figure 2.4). By resolution, it is seen that: R = -6 sin Q, + Zk cos O cos Q, Also, 6 = Q cos @ - R sin Q, Ip = R c o s @ + QsinQ, cos 0 cos 0 Using substitution, it is seen that: Figure 2.4 Angular orientation and velocities of gravity vector, g, relative to body axis. Equations of Motion of an Aircraft 26 @ = P + R tan 0 cos @ + Q tan O sin @ @, O and l-lr are referred to as the Euler angles. 2.3.6 Axis Transformations The physical relationships established so far depend upon two frames of reference: the Earth axis system and the body axis system To orient these systems one to another requires the use of axis transformations. For each rotations. For each rotation a transformation matrix is applied to the variables. The total transformation array is obtained simply by taking the product of the three matrices, multiplied in the order of the rotations. In aircraft dynamics, the most common set of transformations is that between the Earth axis system which incorporates the gravity vector, g, as one axis, and the body-fixed axes, XB,YB and ZB. The rotations follow the usual order: azimuth 'Y, pitch 0, and roll @. The corresponding matrices are: 1 cos 'Y sin 'Y cos 0 0 sin 0 o - O 1 sin 0 1 cos 0 j The complete transformation matrix T is called the direction cosine array and is defined as: Before expressing the matrix T in full, a notational shorthand is proposed whereby a term such as cos 6 is written as ct and a term such as cos 6 is written as ct and a term such as cos 6 is written as cos 6 is written as cos 6 is written as ct and a term such as cos 6 is written as rotation is that which results in the least complicated resolution of the gravity vector g into the body axis system. It can easily be shown that: Another practical advantage is that the angles are those which are measured by a typically oriented vertical gyroscope. A two degree of freedom, gravity erected, vertical gyroscope, oriented such that the bearing axis of its outer gimbal lies along OXB, measures on its inner and outer gimbals the Euler angles O and @, respectively. 2.3.7 Linearization of the Inertial forces acting on the aircraft. Equation (2.45) represents the contribution of the forces due to gravity to those equations. All these forces are proportional to the mass of the aircraft. Consequently, these terms may be conveniently combined into components to represent the accelerations which would be measured by sensors located on the aircraft. axes XB, YB and ZB. The external forces acting on the aircraft can be re-expressed as: where 6X, SY and SZ are the gravitational terms and AX, AY and AZ represent the aerodynamic and thrust forces. For notational terms and AX, AY and AZ represent the aerodynamic and thrust forces of a section of the rigid body, for its six degrees of freedom, may be expressed as: x 4 m a X = m [\sim +Q W - R V + g s i n O] cg + U - P W - g cosO ] \sim k m aC%, = m [\sim R Z4maZ = \sim [] cg L = PIxx- z, (R + P Q) + (I, M = QZ, N = RZ, + I, (P* - R') - Iyy)QR + (Ixx - I, PR I, P + PQ(Iyy - Ixx) + Ix, QR The auxiliary equations of eq. (2.46) must also be used since they relate @ to R, Q and P. T,O and Equations of Motion of an Aircraft 28 The equations which constitute eq. (2.56) are non-linear since they contain terms which comprise the product of dependent variables, and some of the terms are transcendental. Solutions of such equations cannot be obtained analytically and would require the use of a computer. Some simplification is possible, however, by considering the aircraft to' comprise two components: a mean motion which accounts for the perturbations about the mean motion. In this form of analysis it is customary to assume that the perturbations are small. Thus, every motion variable is considered to have two components. For example: U&iJo+u R A R ~ + ~ + M A Mo + ml etc. Q A Qo q The trim, or equilibrium, values are denoted by a subscript 0 and the small perturbation values of a variable are denoted by the lower case letter. 3 In trim there can be no translational or rotational acceleration. Hence, the equations which represent the trim conditions can be expressed as: Xo = m [QoW o - RoV o + g sin 00] Y o = m [PoVo- QoUo - g cos O0 sin Oo] Zo = and yawing motion can occur in the trim condition; the equations which define Po, Q0 and Ro are given by eq. (2.56), expanding the terms and then subtracting eq. (2.57) into (2.56), expanding the terms and then subtracting eq. (2.58) from the result, or by differentiating both sides of eq. (2.56). When perturbations from the mean conditions are small, the sines and cosines can be approximated to the angles themselves and the value unity, respectively. Moreover, the products and squares of the perturbed quantities are negligible. Thus, the perturbed quantities are negligible. Rov dY = m [3 + Uor + g cos OoO] + Rou - Wop - Pow - (g cos 0 0 cos @o) + + (g sin sin @o)O] dZ = m [w + Vop + Pov - Uoq - Qou + (g cos 00sin @o) + + (g sin sin @o)O] dZ = m [w + Vop + Pov - Uoq - Qou + (g cos 00sin @o) + + (g sin Ol cos cPo)O] (2.59) Equations of Motion of a Rigid Body Aircraft where q o , O0 and Qo have been used to represent steady orientations, and Y, 0 and the perturbations in the Euler angles Equations (2.59) are now linear. Obviously, perturbation equations are required for the auxiliary set of equations given as eq. (2.46), because the gravitional forces must be perturbed, auxiliary equations is rarely used since it is complicated. But the components of angular velocity which represent the rotation of the body-fixed axes XB, YB and ZB relative to the Earth axes XB, YB
and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB and ZB relative to the Earth axes XB, YB QO- +(\$0 cos OOsin Qo + 7ko cos Qo) q = - 0 sin Qo - (2.60) sin B0 cos Qo) Although these equations are linear, they are still too cumbersome for general use owing to the completely general trim conditions which have been allowed. What is commonly done in AFCS studies is to consider flight cases with simpler trim conditions, a case of great interest being, for example, when an aircraft has been trimmed to fly straight in steady, symmetric flight, with its wings level. Steady flight is motion with the rates of change of the components of linear and angular velocity being zero. Possible steady flight is motion with the rates of change of the components of linear and angular velocity being zero. regarded as merely a 'quasi-steady' condition because u and w cannot both be zero for any appreciable time if Q is not zero. Straight flight is motion with the components of angular velocity being zero. Straight flight is motion with the components of angular velocity being zero. Straight flight is motion with the components of angular velocity being zero. Straight flight is motion with the components of angular velocity being zero. Straight flight is motion with the components of angular velocity being zero. Straight flight is motion with the components of angular velocity being zero. symmetry of the aircraft remains fixed in space throughout the manoeuvre taking place. Dives and climbs with wings level, and pull-ups without sideslipping, are examples of symmetric flight. Sideslip, rolls and turns are typical asymmetric flight conditions. The significance of the specified trim conditions may be judged when the following implications are understood: 1. That straight flight implies $\phi = B0 = 0$. 2. That symmetric flight implies $\phi = V_0 = 0$. 3. That flying with wings level implies $\phi = 0.2$. That symmetric flight impli the steady forward speed, Uo, must be greater than the stall speed if flight is to be sustained. However, certain rotary wing and V/STOL aircraft can achieve a flying state in which Uo, W o and O0 may be zero; when Uo and W o are simultaneously zero the aircraft is said to be hovering. Hence, for straight, symmetric flight with wings level, the equations which represent translational motion in eq. (2.59) become: z = m[w + Pov - Uoq - Qou + g sin Oo0] The equations (2.59) which represent rotational motion are unaffected. Equation (2.60), however, becomes: r = cos O0 From the same expression, for this trimmed flight state, it may be assumed that: (2.63) Qo = Po = Ro = 0 Therefore, it is possible to write eqs (2.59) and (2.61) in the new form: $x = m[u + Woq - g \cos OoO] ml = Iyyg n = Izz$? - Ixzp Consideration of eq. (2.64) indicates not only that the set can be separated into two distinct groups which are given below: $z = m[w - Uoq + g \sin OoO] ml = Iyyg n = Izz$? - Ixzp Consideration of eq. (2.64) indicates not only that the set can be separated into two distinct groups which are given below: $z = m[w - Uoq + g \sin OoO] ml = Iyyg n = Izz$? Complete Linearized Equations of Motion 37 In eq. (2.65) the dependent variables are u, w, q and 0 and these are confined to the plane XBZB. The set of equations is said to represented in eq. (2.66). Although it appears from this equation that the sideslip is not coupled to the rolling and yawing accelerations, the motion is, however, coupled (at least implicitly). In practice, a considerable amount of coupling can exist as a result of aerodynamic forces which are contained within the terms on the left-hand side of the equations. It is noteworthy that this separation of lateral and longitudinal equations is merely a separation of gravitational and inertial forces: this separation is possible only because of the assumed trim conditions. But 'in flight', the six degrees of freedom model may be coupled strongly by those forces and moments which are associated with propulsion or with the aerodynamics. 2.4 COMPLETE LINEARIZED EQUATIONS OF MOTION 2.4.1 Expansion of Aerodynamic Force and Moment Terms To expand the left-hand side of the equations of motion, a Taylor series is used about the trimmed flight condition. Thus, for example, Equation (2.67) supposes that the perturbed force z has a contribution from only one control surface, the elevator. However, if any other control surface on the aircraft being considered were involved, additional terms, accounting for their contribution to z, would be used. For example, if changes of thrust (T), and the deflection of flaps (F) and symmetrical spoilers (sp) were also used as controls for longitudinal motion, additional terms, such as az - &T, ST az az ass, - SF and -as, 3% would be added to eq. (2.67). Furthermore, some terms depending on other motion variables, such as 0, are omitted because they are generally insignificant. For the moment only longitudinal motion. Thus, it is now possible to write eq. (2.65) as: 32 Equations of Motion of an Aircraft + \$ = m [w - Uoq + g sin OoO] ~ S E aM aM aM dM aM aM aM u+-ti+-W+-w+-q+-q+au au dw aw 89 aq aM as E & I3 To simplify the notation it is customary to make the following substitutions: 1 ax =--" m ax x When this substitutions: 1 ax =--" m ax x When this substitutions: 1 ax =--" m ax x When this substitutions: 1 ax =--" m ax x When this substitutions: 1 ax =--" m ax x When this substitution is made the coefficients, such as M,, Z,, and X, are referred to as the stability derivatives. Equations of Longitudinal Motion 2.4.2 Equation (2.68) may now be rewritten in the following form: e, e + xSESE + Z*E6E q = Muu + M& + M, w + M, wof (2.62) is usually added to eq. (2.70a), i.e. 0 = q (2.70a) From studying the aerodynamic data of a large number of aircraft it becomes evident that not every stability derivative is significant and, frequently, a number of aircraft it becomes evident that not every stability derivatives. depend both upon the aircraft being considered and the. flight condition which applies. Thus, before ignored: X ~X,, X,+, X*,, ZC, Z,+, Mc, ZS, and Ms, . The stability derivative Z, is usually quite large but often ignored if the trimmed forward speed, Uo, is large. If the case being studied is hovering motion, then Z, ought not to be ignored. With these assumptions, the equations of perturbed longitudinal motion, for straight, symmetric flight, with wings level, can be expressed as: w = Zuu + Zww + Uoq - g sin OoO + Z6,SE q = M,u + M,w + M,+w + Mqq + M6E8E Notice that each term in the first three equations of (2.71) is an acceleration term, but since the motion and control variables, u, w, q, 0 and SB, have such units as m s-l, and s-I the stability derivatives appearing in these equations are dimensional. It is possible to write similar equations using non-dimensional stability derivatives, and this is frequently done in American literature and is always done in the British system; but when it is done, the resulting equations must be written in terms of 'dimensionless' time. The the reader requires details of the use of non-dimensional stability derivatives, Babister (1961) should be consulted. It has been decided in this book to use the form of equations given in (2.71) where time is real. Such a decision makes the design of AFCSs much easier and more direct for it allows direct simulation, and also makes the interpretation of the aircraft responses in terms of flying qualities more straightforward. 2.4.3 Equations of Lateral Motion From eqs (2.64) and (2.62) the following set of equations applies to lateral motion: y = m[Q + Uor - Wop]- g cos OO+] 1 = zxxp - ZXZ? Equations of Motion of an Aircraft P = Ijr cos o0 Expanding the left-hand side of the first three equations results in the following (subscripts A and R indicate aileron and rudder, respectively): = Zz2j.- zx2p Adopting the more convenient notation, namely: allows the eqs (2.73) to be written more simply as: For conventional aircraft, it can usually be assumed that the following stability derivatives are insignificant: Yc, Yp, Yj, Yr, Y1, YsA, Li,, Li, Nc, Ni. . Note, however, that Yr may be significant if Uois small. When this assumption is made the equations governing perturbed lateralldirectional motion of the aircraft are given by: + Uor - Wop- g cos Oo\$ + YaRSR 6 = YVv Equations of Motion in StabiEity Axis System p = \$-* i = Zfi. cos O0 EQUATIONS OF MOTION IN STABILITY AXIS SYSTEM 2.5 The aerodynamic forces which contribute to the x, y and z terms in eq. (2.65) are the components of lift and drag relative to the body-fixed axes are: the angle of attack, a, and the angle of sideslip, P. The angles are defined in Figure 2.5 where the subscript 'a' has been used to indicate that the velocity and its components are relative in the subscript 'a' can be dropped. The velocity components along the body axes are: V, = VT sin P W, = VT01 cos p sin a Earlier it was shown that if symmetric flight was assumed, Vo would be zero. Therefore, if the axis system is oriented such that Wo is zero, then both a. and Po are zero. This orientation results in the XB axis, in the steady state, pointing into the relative wind and the XB axis and the velocity vector being aligned such that: U" = VT (2.78) Such an orientation of relative wind with body axis system. Equations of Motion of an Aircraft 36 and a. is zero. This initial alignment does not affect the body-fixed character of the axis system with respect to the body axis system with respect to the body axis system with respect to the body axis system
changes as a function, the stability axes and the stability axis system with respect to the body axis system. rotate with the airframe and, consequently, the perturbed X, axis may or may not be parallel to the relative wind while the aircraft motion is being disturbed. The situation is being disturbed. The sinduced disturbed disturbed. The situation is MGw f M,q + M 6 E 8 ~ w = Z,u whereas eq. (2.76) may now be written as: 3 = Y,v + Uor - g cos yo+ + Y s R 8 ~ Relative On = Yn Figure 2.6 Direction of stability axes with respect to the relative wind. (a) Steady flight. (b) Perturbed flight. For Steady Manoeuvring Flight Conditions + = rlcos yo The cross-product inertia terms which appear in eq (2.81) can be eliminated by a simple mathematical procedure: the use of primed stability derivatives. By ignoring second order effects, the cross-product of inertia terms are taken into account in the following primed stability derivatives: LkA = LsA + ZBNaA NgA = NZiA + I A L s ~ LkR = LgR + zBNaR NkR = NsR + ZALFiR in which Then eq. (2.18) becomes + Uor - g cosyo\$ + YgRSR p = Lhv + Lip + Lir + LkASA + LbRSR i = Nhv + N i p + Nir + NkASA + NkRSR \$ = p + r tan yo d = YVv 2.6 EQUATIONS OF MOTION FOR STEADY MANOEUVRING FLIGHT CONDITIONS Steady flight conditions provide the reference values for many studies of aircraft motion. Once the relationships for steady flight are known, they are used subsequently to eliminate initial forces and moments from the equations of Motion of an Aircraft 38 2.6.1 Steady, Straight Flight This is the simplest case of steady flight. All time derivatives are zero and there is no angular velocity about the centre of gravity. Therefore, setting to zero all time derivatives, the angular velocities P, Q, R, and the time derivatives of angular position (attitude) reduces eq. (2.56) to: Xo = mg cos 0 (email protected) Zo = - mg cos 0 (cos @ These equations can be applied to a steady sideslip manoeuvre, for the velocity components V, W, and the bank angle, @, are not necessarily zero. However, if the motion is restricted to symmetric flight, the bank angle is zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In this case, the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In the equations become: Xo = mg cos O Again, all the moments are zero. 2.6.2 Steady Turns In the equations become the equations become the equations become the equations become the equations are zero. 2.6.2 Steady Turns In the equations become th Euler angles, @ and 0, are also zero; the rate of turn, q, is constant. Generally, such steady, turning manoeuvres are carried out for very small pitching angles, or for shallow climbing or diving turns. Hence, for small 0, the following relationships hold (see eq. (2.46)): Q = @ cos O sin @ = @ sin @ (2.88) For most manoeuvres of this type, q, although constant, is small so that the products of P, Q and R may be neglected. Furthermore, for co-ordinated shallow turns, the side force Y is zero (by definition) and the velocity components V and W are small. Therefore, for a steady, co-ordinated shallow turn, the equations become: X = mgO Z = - m (* ~ s i n @ + g c o s @) For Steady Manoeuvring Flight Conditions Again, all the moments are zero. 2.6.3 Steady Pitching Flight condition. In this case, U and W do vary with time but V, P, R, @ and Y are all zero. Therefore, the equations of motion for a rigid body aircraft reduces to: $X = m(\sim QW) + + mgsinOZ = m(\sim QU)$ -mgcosO (2.90) Equation (2.90) can be used to evaluate the initial conditions: $X_0 = m$ (QoWo + g sin 00) $Z_0 = m$ - m (QoUo + g cos 0 0) If the second equation is solved, a relationship is obtained between the initial pitch rate Q0 and the initial load factor n, , along the ZB axis: 0 Q - uo + amg- 1 COS 630 - (azo- cos 630) = u 0 where 2.6.4 Steady Rolling (Spinning) Flight Control to a relationship is obtained between the initial pitch rate Q0 and the initial pitch rate Q0 improperly describing the physical situation so that the results obtained are unrepresentative of the actual motion. Special methods of treatment are required and, consequently, no such simplified equations are developed here. See, for example, Thelander (1965) for such methods. Equations of Motion of an Aircraft 40 2.7 ADDITIONAL MOTION VARIABLES Even for the straightforward case of straight, steady, wings level, symmetric flight, the designer of AFCSs may be interested in motion variables other than the primary ones of change in forward speed u, in vertical velocity w, in pitch rate q, in pitch attitude 0, in sideslip velocity v, in roll rate p, in yaw rate r, in bank angle 4, and in yaw angle +. Other commonly used motion variables are treated here, with particular regard to the development of their relationship to the primary motion variables. Such additional motion 2.7.1 Normal acceleration, for perturbed motion, and measured at the c.g. of the aircraft, is defined as: a, = (w - Uoq) (2.94) cg For small angles of attack, a, In aircraft changes its attitude, the steady, normal acceleration due to gravity, g, also changes. In that case: a, = w - Uoq - g (2.97) CF, If it is required to know the acceleration at some point, x distant from the c.g. by l,, but still on the fuselage centre line, that acceleration is given by: a, = w - X Uoq - 1,q (2.98) The distance 1, from the c.g. is measured positive forwards. By definition: .. hcg = - a, (2.99) cg where h is the height of the aircraft's c.g. above the ground. Consequently: Additional Motion Variables 41 The variation of load factor with the angle of attack of an aircraft, n, , is an important aircraft parameter known as the acceleration sensitivity. It a ~ i l lbe shown in Chapter 3 how n, can be determined from the stability derivatives and the equations of motion; thz result obtained there is quoted here for convenience: Usually, for conventional aircraft, Ma E Z, B &, ME,; consequently: n, a = - 2, Uo/g For straight and level flight, at 1g, where CL is the lift curve slope and CL is the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know the lateral acceleration at the c.g. of the aircraft is defined by: A 6 - g + = + uor (2.106) If it is required to know t some point, XI,,, on the OX axis, distant from the c.g. and I, is measured positive forwards of the c.g. and I, is measured positive forwards. Heading angle, X, is defined as the sum of sideslip, P, and yaw angle, q. 2.8 THE STATE AND OUTPUT EQUATIONS 2.8.1 The State Equation A state equation is a first order, vector differential equation. It is a natural form in which to represent the equation of an Aircraft E Rn is the state vector, u E Rm is the control vector. The elements of the vector x are termed the state variables and the elements of the
vector u the control input variables. A is the state coefficient matrix and B the driving matrix; they are of order (n x n), respectively. From an inspection of eq. (2.108) it should be observed that the 1.h.s. terms involve only first derivatives of the state variables. vector x and the control vector u. Thus, the state equation is an attractive mathematical form for aircraft control and stability studies since its solution for known inputs can easily be obtained by means of integration. Furthermore, this same form of equation lends itself to simulation. In Chapter 1 it was stated that the flight of an aircraft can be affected as much by disturbances such as atmospheric turbulence as by deliberate control inputs, u. Such disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of disturbances can be taken into account by adding a term to the r.h.s. of eq. (2.108), i.e.: where d is a vector of dimension I which represents the I sources of the represents the represents the represents the represents the represent random, special methods are used to introduce the disturbances into the aircraft's state equation which is generally considered to be deterministic. These methods are dealt with separately in Chapter 5, and, consequently, for the remainder of this chapter 4 will be regarded as a null vector. Any set of first order, linear, constant coefficient, ordinary differential equations can be combined into the form of eq. (2.108). 2.8.2 The Output Equation is wanted. The output equation is merely an algebraic equation which depends solely upon the state vector, and, occasionally, upon the control vector form of expression is: v=Cx+Du (2.110)~ The output vector is v E R~ and its elements are referred to as the output variables. The matrices C and D, the output and direct matrix respectively, are generally rectangular and are of order (p x n) and (p x m), respectively. For AFCS work the sensors used to measure for use as feedback signals, are often subject to measurement noise. To incorporate these noise effects into an output Equations 43 The characterization of sensor noise and how it is modelled dynamically are dealt with in Chapter 5. For the rest of this present chapter 5 is assumed to be null. 2.8.3 Aircraft Equations of Longitudinal Motion If the state vector is defined as, say: and if an aircraft is being controlled only by means of elevator deflection, SE, such that its control vector is defined as: 4 SE (2.113) then, from eq. (2.80): xu x, A A 0 - g cosy0 - Z, Z, U0 - g sin yo Mu MW Mq - 0 0 1 k, 0 -The significance of the tilde in row 3 of eq. (2.114) is easily explained. In eq. (2.80) the equation, though, does not admit on its r.h.s. terms involving the first (or even higher) derivatives of any of the state or control variables. Fortunately, w, itself depends only upon x and u and, therefore, an easy substitution is possible. In eq. (2.80) the equation for w is given as: Substituting for w in the equation for q yields: Equations of Motion of an Aircraft 44 + (M, + M,Uo)q - gM, + sin yoO + (M8E + M G Z ~ ~) S E where hi, = (Mu + M, +ZW) M, = (-gMw + M, +ZW) M, = (-gWw + sin yo) If there were some other control inputs on the aircraft being considered, say, for example, a change of thrust, 6th, and a deflection of symmetrical spoilers, Ssp, then the order of the driving matrix, B, becomes (4 X 3) and the elements of the matrix become: It must be understood that the state equation is not an unique description of the aircraft dynamics. For example, if the state vector had been chosen to be rather than the choice of eq. (2.11), A and B must be changed to: - 0 0 1 Mu M A = M, M xu xw 0 - 0 - g cos yo 2, Z w Uo - g sin yo - State and Output Equations 45 When the state equation is solved, with either set of A and B, the responses obtained for the same control input SE, will be identical. In American work it is common to use as a primary motion variable the angle of attack, a, rather than the heave velocity, w. Since, for small angles: then: where 2: = Z/Uo and ZgE = ZSE/UO Frequently, again in American papers, a stability derivative Z, is quoted, and eq. (2.127) is written as: The reader is warned, however, that confusion can occur with this form. In eq. (2.128) Z, is identical to Z, in eq. (2.127), but, for consistency of notation, Z, ought to be identical to Z, and sometimes as equal to Z, is sometimes as equal to Z, and equation obtain the heave velocity w. If the angle of attack is required, then determine a from eq. (2.124). In this way, ambiguity and confusion can be avoided. If the output variable of interest was, say, a, , then eq. (2.98) can easily be shown (by substitution for w and q) to be given by: Hence: y aZx = [(Z, - \sim , M,)(z, -1, \sim ,) - IxMq O]x + [(Z S E I,M)]u which is the same form as eq. (2.110), where C = [(Z, - ~, M,) (Z~ IXMw)- IXMq 0] D = (z8E - IxMEiE) (2.131) Equations of Motion of an Aircraft 46 If the concern is with the height of an aircraft at its c.g., then: i.e. h = - Z,U - Z,W - Z8ESE To express this in terms of state variables let: h X, j = and let: Xg = .'. %, j = h x5 = - Z,U - Z W w - IXMq 0] D = (z8E - IxMEiE) (2.131) Equations of Motion of an Aircraft 46 If the concern is with the height of an aircraft at its c.g., then: i.e. h = - Z,U - Z,W - Z8ESE To express this in terms of state variables let: h X, j = and let: Xg = .'. %, j = h x5 = - Z,U - Z W w - IXMq 0] D = (z8E - IxMEiE) (2.131) Equations of Motion of an Aircraft 46 If the concern is with the height of an aircraft at its c.g., then: i.e. h = - Z,U - Z,W - Z8ESE To express this in terms of state variables let: h X, j = and let: Xg = .'. %, j = h x5 = - Z,U - Z W w - IXMq 0] D = (z8E - IxMEiE) (2.131) Equations of Motion of Aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of aircraft 46 If the concern is with the height of an aircraft 46 If the concern is with the height of aircraft 46 If the concern is with the h ZSEEE Hence: Then the state equation (2.108) is obtained once more, i.e.: zi: = A x Bu + but now: - xu xw Z, A = Z, 0 - g cos yo 0 0 Io - g sin yo 0 0 Io - g sin yo 0 0 Io - g sin yo 0 0 Ici, Mw Mq ni, 0 0 1 0 0 0 Z, - 2, 0 0 0 0 0 0 1 0 0 - 47 State and Output Equations If the motion variable being considered is the flight path angle y then it can be inferred from eq. (2.79) that: (2.141) y = 0 - a = 0 - (w/U0) Consequently, if y A y, then where x is defined as: If the state vector, x, is defined as: If the state vector, x, is defined as: then the state equation is given by: zi.=Ax+Bu where: A = I Y" 0 Uo - g cosy, 0 L: L;, L: 0 0 N: N;, N;, 0 0 1 tan yo 0 0 0 0 sec yo 0 0 0 48 Equations of Motion of an Aircraft The sideslip angle, P, is often used as a state variable, rather than the sideslip velocity, v. From eq. (2.77), for small angles: v = Uop (2.147) and consequently: which may be written as: where: Y,., = YSR/U0 If, now, the state vector is defined as: then eq. (2.108) obtains, but the coefficient matrix has become: - A = Yv 0 - 1 - c go s y 0 0 Lb L;, L: 0 0 NbN;, N: 0 0 I 0 uo 1 tan yo 0 0 0 secyo 0 O 0 I The driving matrix has become: The fifth column of A in both eqs (2.145) and (2.152) is composed entirely of State and Output Equations 49 zeros. The physical significance of this is explained in Chapter 3, but the presence of such a column of zeros can often be avoided by redefining the state vector, as in eq. (2.154) which has now dimension 4; i.e. let: then A becomes: - A = Yv 0 - Lb LA L: 0 N;, Nf, N; 0 - 0 1 glUo 1 tanyo 0 - and B becomes: It must be emphasized that in straight and
level flight (i.e. non-climbing or diving) yo is zero. Consequently, for this flight condition, those elements which appear in the various forms of A, and which depend upon yo, will take a value of zero if the element has the form sin yo or tan yo, or will take the value unity if the element involves cos yo or sec yo. Sometimes there is interest in the lateral acceleration of an aircraft at some point x, which is a distance I, from the c.g. (1, is positive forwards) and a distance I, off the axis OX (I, is positive when down from the c.g.). Hence: ayx = aycg+ lx? - lzp which can easily be shown to be: = (Yv + 1,N: - 1,L:)v + (1,N: - 1,L:)1,L~)(I,N~- 1,L:) O]X Equations of, Motion of an Aircraft 2.9 OBTAINING A TRANSFER FUNCTION FROM STATE AND OUTPUT EQUATIONS Whenever the variables of a linear system are expressed in the complex frequency domain, i.e. as functions of the Laplace variables of a linear system are expressed in the complex frequency domain, i.e. as functions of the Laplace variables of the Laplace variables of a linear system are expressed in the complex frequency domain, i.e. as functions of the Laplace variables of the Laplace variables of a linear system are expressed in the complex frequency domain, i.e. as functions of the Laplace variables of the Laplac the output variable to some particular input variables being considered identically zero) is the transfer function of the system. Given that the small perturbation of the form of eq. (2.108) and an output equation of the form of eq. (2.110), namely ri = Ax + B u and y = Cx + D u respectively, then, provided that y is scalar and that only those columns of matrices B and D are used which correspond to the particular control input uj being considered, then a transfer function relating y and uj can be found. If y is a vector and it is required to find the transfer function corresponding to some particular element, y, as a result of some control input, u,, the rows of the matrices C and D which correspond to yi are used in the calculation. To illustrate the procedure consider that y and u are scalars. Taking Laplace transforms, and assuming initial conditions are zero, results in eqs (2.108) and (2.110) being expressed as: sX(s) - AX(s) = BU(s) (2.1'59) In general, if: then: where Bj represents the column of matrix B which corresponding to yi and Dij is the ith row of the matrix D corresponding to yi and the jth column corresponding to yi and the jth column corresponding to yi. It is evident that transfer function relationships can be found for output motion caused by sensor noise or by atmospheric disturbances rather than manoeuvre commands acting through the control inputs, but these are not treated until Chapter 5. Important for flight control than others. This section treats only the latter type. A number of parameters appear frequently in the equations defining stability derivatives. They are listed here for convenience (note that all the stability derivatives presented are dimensional): S is the wing span. 2.1 0.1 Longitudinal Motion The non-dimensional pitching moment coefficient Cm is usually zero in trimmed flight, except in cases of thrust asymmetry. Mu represents the change in forward speed. Its magnitude can vary considerably and its sign can change with changes in Mach number and in dynamic pressure and also as a result of aeroelastic effects. In modern aircraft, the Mach number effects and the effects of aeroelasticity have become increasingly important. The change in angle of attack, CL, is often referred to as the lift curve slope. It is always positive for values of angle of attack, CL, is often referred to as the lift curve slope. It is always positive for values of angle of attack below the stall value. wing, the fuselage and the tail. For most conventional aircraft it has been found to be generally true that the wing contributes 85-90 per cent to the value of CL. Consequently, any aeroelastic distortion of the wing contributes 85-90 per cent to the value of CL. coefficient with angle of attack. It is re?erred to as the 'longitudinal static stability derivative'. Cm is very much affected by the location of the c.g. of the aircraft. Cm is proportional to the distance, xAc, between the c.g. and the aerodynamic centre (a.c.) of the whole aircraft. xAc is measured positive forwards. If xAc is zero, Cm is zero. If XAC < 0, Cmais negative and the aircraft is of the OL OL 52 Equations of Motion of an Aircraft a.c., XAC < 0 and Cm is positive, with the consequence that the aircraft is Oi statically unstable. In going from subsonic to supersonic flight the a.c. generally moves aft, and, therefore, if the c.g. remains fixed, Cm will tend to increase for a Oi statically static margin and M, is discussed in section 3.3 of Chapter 3, but it can be stated simply here that M, (or Ma) is the most important w longitudinal derivative. Although Cm, does not have a significant effect. Usually M; < 0; it increases the damping of the short period motion. rn For conventional aircraft, Mq contributes a substantial part of the damping of the short period motion. This damping comes mostly from changes in the angle of attack of the tail length, ZT. But IT is the lever arm through which the lift force on the horizontal tail is converted into a moment, 1.e.: M,O~Z? (2.170) Mq is a very significant stability derivative which has a primary effect on the rn handling qualities of the aircraft (see Chapter 6). Since CLg is usually very small, ZgE is normally unimportant except when an AFCS inv\$ving feedback of normal acceleration is used. Also, if a tailless aircraft is being considered, the effective lever arm for the elevator (or ailerons) is small, hence CL may be relatively large compared to C,,, . In these cases, ZgEcannot S~ &E rn safely be neglected in any analysis. Cm is termed the 'elevator control effectiveness'; it is very important in aircraft &E design and for AFCS work. When the elevator is located aft of the c.~.: the normal location, Cm is negative. Its value is determined chiefly by the maximum &E lift of the wing and also the range of c.g. travel which can occur during a flight. rn Important Stability Derivatives 2.10.2 Lateral Motion The sideslip motion, i.e. Cy < 0. But for P aircraft with a slender fuselage, at large values of the angles of attack the forces can be in an aiding direction. For certain (rare) configurations having a wing of low aspect ratio but required to operate at a large value of attack, this force on the fuselage can counter the resisting force of the fin which results in the stability derivative Cypbeing positive. Such positive values, even if very small, are undesirable because the reversed (or small) side force makes it difficult for a pilot to detect sideslip motion and consequently makes a co-ordinated turn difficult to achieve. Such values of Cy also reduce the damping ratio of the dutch roll mode, P whereas Cy normally makes a large contribution to this damping. In the normal P case Cy is not a derivative which causes great difficulty to AFCS designers. P Note that: The change in the value of the rolling moment coefficient with sideslip angle Cl is P called the 'effective dihedral'. This derivative is very important in studies concerned with lateral stability of an aircraft, particularly when lateral control is being exercised near stall by rudder action only. Usually small negative values of CI are wanted, as such values are rarely obtained without considerable aerodynamic difficulty. The change in the yawing moment coefficient with change in sideslip angle Cn is P referred to as the 'static directional' or 'weathercock' stability coefficient. It depends upon the area of the fin and the lever arm. The aerodynamic contribution from the fin is positive, but the contribution from the fin area of the fin and the lever arm. regarded as static directional stability; P a negative value signifies static directional instability (see Chapter 3). Cn priB marily establishes the natural frequency of the dutch roll mode and is an important factor in establishing the characteristics of the spiral mode stability. For good handling qualities Cn should be large, although such values magnify the disturbP ance effects from side gusts. At supersonic speeds C, is adversely affected P because the lift curve slope of the fin decreases. The change in rolling moment coefficient with change in rolling velocity, Cl is P referred to as the roll damping derivative. Its value is determined almost entirely by the geometry of the wing. In conjunction with Cl (q.v.), Cl establishes the 6~ P maximum rolling velocity which can be obtained from the aircraft: an important flying quality. CI is always negative, although it may become positive value is desirable. The more negative is Cn P the smaller is the damping ratio of the dutch roll mode and the greater is the sideslip motion which accompanies entry to, or exit from, a turn. The change in rolling moment coefficient with a change in rolling moment coefficient with a change in yawing velocity, Cl , r has a considerable effect on the spiral mode. For good spiral stability, Cl should be positive but as small as possible. r A major contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r
located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing, but if the fin is r located either above or below the axis OX it also makes a substantial contribution to Cl is the lift force from the wing for the axis OX it also makes a substantial contribution to Cl is the lift force from the wing for the axis OX it also makes a substantial contribution to Cl is the lift force from the wing for the axis OX it also makes a substantial contribution to Cl is the lift force from the axis OX it also makes a substantial contribution to Cl is the lift force from the wing for the axis OX it also makes a substantial contribution to Cl is the lift force from the axis OX it also makes a substantial contribution to Cl is the lift force from the axis OX it also makes a substantial contribution to Cl is the axis OX it also makes a substantial contribution to Cl is the axis OX it also makes a substantial contribution to Cl is the axis OX it also makes a substantial contribution with a change in yawing velocity, Cn, is referred to as the 'yaw damping derivative'. It is proportional to 1%. Usually C: is r negative and is the main contributor to the damping of the dutch roll mode. It also contributes to the stability of the spiral mode. It also contributes to the stability of the spiral mode. Inclusion of Motion of Thrust Effects The change in side force coefficient with rudder deflection, Cy8, is unimportant R except when considering an AFCS using lateral acceleration as feedback. Cy is nearly always negligible. Because positive rudder deflection. It is usually negligible. Because the rudder is usually located above the axis OX, positive rudder deflection produces positive rolling motion, i.e. c, >O. The change in rolling moment coefficient with a deflection of the ailerons, Cl,A, is referred to as the aileron effectiveness. In lateral dynamics it is the most important control-related stability derivative. It is particularly important for low speed flight where adequate lateral control is needed to counter asymmetric gusts which tend to roll the aircraft. " The change in yawing moment coefficient which results from a rudder deflection, is referred to as the rudder effectiveness. When the rudder is deflected to Cn8R7 the left (i.e. SR > 0) a negative yawing moment is created on the aircraft, i.e. cn < 0., R The change in yawing moment coefficient which results from an aileron results in adverse yaw if Cn < 0, for when a pilot deflects the deflection, C, &A' 8A ailerons to produce a turn, the aircraft will yaw initially in a direction opposite to that expected. When Cn > 0 the yaw which results is favourable to that turning 8A manoeuvre, and this is referred to as proverse vaw. Whatever sign Cn takes, its SA value ought to be small for good lateral control. 2.1 1 THE INCLUSION OF THE EQUATIONS OF MOTION OF THRUST EFFECTS 1. Many of the stability derivatives which are used in the equations of motion are the result not only of aerodynamic forces but of forces arising from flows induced by the propulsion system. Such flows profoundly modify the derivatives but the effects are usually difficult to predict, Equations of Motion of an Aircraft wind 4 Figure 2.7 Thrust alignment geometry. requiring special wind tunnel tests for their resolution. But where slipstream interference is minimal, such being the case when a subsonic jet has a central exhaust aft of the tail, the forces and moments associated with direct thrust make considerable contributions to various derivatives. The number of forces acting on the inlet which result when the air mass entering the engine changes direction. (b) The moments caused by the angular velocity of a tube containing a mass of moving air. (c) The forces and moments resulting from the thrust itself. The angle which the thrust line makes with the relative wind is ET (see Figure 2.7) and is fixed at - ao). Hence: ZT = - T sin (ET - a,) MT = - T sin (ET - a, eTT where the thrust offset eT is positive downwards. 3. Of course, thrust is a function of density, throttle setting, and the relative speed of the aircraft (on rare occasions it is a function of Aco). Hence: dZT = sin (ET - ao) However: a T (COSET %= - au av cos2 a. + sin E, sin uo cos ao) (2.189) Inclusion of Motion of Thrust Effects axTaw av ax, - a T --38th (cos ET sin a. cos a. (cos ET cos a. + sin E, + sin ET sin2ao) sin ao) asth Z -- a - aT au av azT - - aT au av azT - - aT 88th 57 (sin E, cos 2 a 0 - cos ET sin a. cos a 0 - cos ET sin a. cos a 0 - cos ET sin ao) asth Z -- a - aT au av azT - - aT au av azT - - aT 88th 57 (sin E, cos 2 a 0 - cos ET sin a. cos a 0 - cos ET sin a. cos a 0 - cos ET sin ao) asth At the trim condition, however, the total moment must be balanced by an equal and opposite aerodynamic moment. Thus: From eq. (2.195), however: pu0sec, = . - 2ToeT uo aT dM = eT [(% - (2.198) 2) (Ucos a. + w sin ao) + It is evident that the perturbations in moment due to thrust are influenced by the trim condition term, To/Uo. 4. Thrust can be written as: However, Cth is not an aerodynamic coefficient so that eq (2.200) is misleading. The thrust contribution manifests itself chiefly in Xu and is expressed in the form: Equations of Motion of an Aircraft Figure 2.8 Resolution of thrust along the axis OX. The partial derivative BT,IaU is found from data on the power plant. The direct contribution of thrust to other stability derivatives is usually negligible. 5. When the throttle setting, Sth, is increased there is a corresponding increase in thrust. Figure 2.8: 2.1 2 CONCLUSIONS The form of the equations of motion of an aircraft depends upon the axis system which has been chosen. Once a particular axis system is adopted, it is helpful to expand the aerodynamic force and moment terms, and to linearize the inertial and gravitational terms so that when small perturbations are considered the resulting equations will be linear and can be separated into longitudinal and lateral motion. Using the stability axis system is the most convenient for AFCS work. Sometimes, small motion is not of concern, however, and it is essential instead to consider steady manoeuvring flight such as pitching or turning. Not every motion variables as flight path angle, height, heading, and normal and lateral accelerations, are related, however, to these equations and this chapter shows how these variables can be obtained from a knowledge of the equations, with the other variables being obtained from associated output equations. Once the state and output equations are known it is possible to determine any transfer function relating a particular control input. Not every stability derivative is significant in terms of its influence on the dynamics of the aircraft and only the most important need to be studied for their likely effects on the subsequent performance of an AFCS. Thrust changes do affect the motion of an aircraft, the origin of the stability axis system upon which the equations of motion are based. Consequently, special techniques are needed to introduced threse thrust effects into the equations of motion. 2.1 3 EXERCISES 2.1 Write down the state equation representing the small perturbation longitudinal motion of the aircraft CHARLIE-3. 2.2 Derive the transfer function relating the stability derivatives of aircraft BRAVO-4 calculate the state and output equations, if the output variable is defined as the normal acceleration of the aircraft at its c.g. 2.4 The stability derivatives for VTOL aircraft at its c.g. 2.4 The stability derivatives for VTOL aircraft at its c.g. 2.4 The stability derivatives for VTOL aircraft at its c.g. 2.4 The stability derivative not listed should be taken as zero. (a) aZcg, to elevator deflection, SE. (b) Sketch the response of aZcg to a step deflection of the elevator of 0.03846 radian. (c) If the aircraft is hovering at a height of 100 m, calculate the sinking speed at, and the time of, ground contact after the application of a step deflection of the elevator. speed obtained in part (c) excessive? Give a reason for your answer. 2.5 The lateral motion of the aircraft FOXTROT-:! is to be considered. Its rudder is not used at high Mach numbers. Derive the corresponding state and output equations, if the output variables of interest are heading angle, A, and change in roll angle, +. 2.6 For exercise 2.5 derive the corresponding transfer function relating, aycg, to aileron deflection, SA. 60 Equations of Motion of an Aircraft 2.7 An experimental VTOL aircraft 2.7 An experimental VTOL aircraft in hovering motion has the following stability derivatives: Y , = - 0.14 Nb = 0.001 Y8, = 0.0 Ni = 0.002 Y8, = 1.02 Nh = - 0.66 L6 = - 0.012 Lf, = - 0.273 Li = 0.083 LA, = 0.7 N&, = - 0.53 Uo = 0.3 m sf1 LA, = - 0.12 p, r, 6, and 4 have their usual meanings of roll rate, yaw rate, sideslip angle and roll attitude, respectively. SA denotes the rudder deflection. (a) Calculate the transfer function of 0.022 s, calculate by how much the heading of the aircraft will have changed some 10 s after the control deflection is applied. 2.8 A fighter aircraft, flying at 200m s-' and at a height of 104m has the following short period equations of motion: iu=-6a+q q = - 5.0~ -~ 0.69 - 12.0SE Derive the transfer function relating the pitch rate to the elevator deflection. If the aircraft's static stability is reduced to zero determine the pitch rate response of the modified aircraft to a step deflection of the elevator of - 1.0". Calculate the resulting steady state normal acceleration sensitivity) of the aircraft. If the angle of attack is changed by $5.73^{"}$, calculate by how much the load factor would change. (a) (b) 2.9 The linearized equations of perturbed longitudinal motion are given (in SI units) by: q = -0.659-0.2iuu = 225.0 Sth a - 1.2SE + 0.035 a -
9.810 - 0.18 u iu = $q - 0 \cdot 2 \sim -0 \cdot 6 \sim -0.035SE$ o=q (a) Determine the equilibrium flight speed of the aircraft. (b) Calculate the transfer function relating changes in forward speed to change in flight path angle, y, as a result of a deflection, SE, of the elevator can be evaluated. (b) Find a transfer function relating change in y to a change in pitch attitude using the equations found in part (a). (c) Comment on the validity of the transfer function found in part (b). (d) Find the corresponding value (if any) of the flight path angle. 2.11 A large jet cargo aircraft, DELTA, is powered by four engines, each having a thrust of 182 kN. The mass of the aircraft's motion is F/C#2. (a) Determine an appropriate state equation for the aircraft's motion. (b) Thence find the transfer function relating changes in forward of the aircraft's motion. c.g. and 2.5 m above it. (c) Calculate the steady normal acceleration experienced by the pilot if the angle of attack of the aircraft is changed suddenly by 2.85". 2.12 A high performance fighter is on approach at 165 knots. The linearized equations of perturbed lateral motion are given by: where Sdhtdenotes differential deflection of the horizontal tail. (a) Is it possible to find a combination of control surface deflections which will result in there being no lateral acceleration in the steady state, even though some of the steady surface deflections required. (c) Determine the transfer functions relating the lateral acceleration at the c.g. to each control surface independently, i.e. find: $ay_s(s) - 6.4(\sim)$ a Ycg(s - and - s ~ (~) & dht(~) Can you decide from these transfer functions which control surface is the most important for manoeuvring the aircraft on approach? 2.14 NOTES 1. 2. For example, see chapter 4 of McRuer et al. (1973). This depends upon the assumption of constant aircraft mass. Equations of Motion of an Aircraft mass, m. This form applies to linear, time-invariant systems only; when the system is nonlinear, the appropriate form is k = f(x, u, t). For linear, time invariant systems only; when the output relationship is non-linear the appropriate form is y = g(x, u, t). If the output equation is non-linear, the presence of measurement noise modifies y to become: y = g(x, u, t). If the output relationship is non-linear, the presence of measurement noise modifies y to become: y = g(x, u, t). This assumes that the matrix (sI - A) is non-singular, which can be proved by recalling that $3' \{ [sZ - A]' = g(x, u, t) \}$. eA'. Although Uo is used in these equations, the correct value to be used is the true airspeed. For small the errors are insignificant if Uo is used instead of VT. If the elevator is located forward of the c.g. it is renamed canard. This description is increasingly common, although canard referred origin liv to an aircraft confid the forward tail surface being called a foreplane. It is this foreplane which is now considered to be a canard. 2.1 5 REFERENCES BABISTER, A.W. 1961. Aircraft FOGARTY, L.E. and R.M. HOWE. Stability and Control. Oxford: Pergamon Press. 1969. Computer mechanization of six-degree-of-freedom flight equations. NASA CR-1344, May. GAINES, T.G. and S. HOFFMAN. 1972. Summary of transformation equations and equations of motion. NASA Sp-3070. McRUER, D.T., I.L. ASHKENAS and D.C. GRAHAM. 1973. Aircraft Dynamics of the airframe. Bur. Aero. Rpt. AE-61-4 (Vol. 11) USA. THELANDER, J.A. 1965. Aircraft motion analysis. FDL-TDR-67-70, WPAFB, Ohio, USA. March. Aircraft Stability and Dynamics 3.1 INTRODUCTION The equations of motion have been derived in some detail in Chapter 2. Only under a large number of assumptions about how an aircraft is being flown is it possible to arrive at a set of linear differential equations which can adequately represent the motion that results from the deflection of a control surface or from the aircraft's encountering atmospheric turbulence during its flight. This resulting motion is composed of small perturbations about the equilibrium (trim) values. To achieve such equilibrium values requires the use of certain steady deflections of the appropriate control surfaces. Consequently, the entire range of the angle of deflection of any particular control, since much of that range is required to trim the aircraft. What is meant, then, by small perturbation is that any angle be sufficiently small to guarantee that the assumptions concerning any trigonometrical functions involved remain valid. For practical purposes, a change of angle of 15" or more should then consider the likely effects of continuing to use the small perturbation theory whenever such angular values can occur. Similarly, translational velocity should always be small in relation to the steady speed; when the steady speed, such as Vo or Wo, is zero then changes of velocity of 5 m s-' should be regarded as being the limit of validity. However, it must be strongly emphasized that these are not firm rules but depend upon the type of aircraft being considered, its flight condition, and the manoeuvres in which it is involved. For the remainder of this chapter it is considered that all the assumptions of Chapter 2 hold, that any aircraft being considered is fixed wing and flying straight and level in a trimmed condition, and that its motion is properly characterized by eqs (2.109) and (2.110). For example, for longitudinal motion, eq. (2.112) is taken as the definition of the state vector x, i.e.: and the control vector u is defined as: 64 Aircraft Stability and Dynamics The state coefficient matrix B by: For lateral motion, the appropriate equations are (2.143) and (2.154), respectively where the coefficient matrix is: Y" 0 lglUo Lb Lf, L:. 0 Nb N f, N:. 0 and the driving matrix is: 3.2 LONGITUDINAL STABILITY 3.2.1 Short Period and Phugoid Modes The dynamic stability of perturbed longitudinal motion is most effectively established from a knowledge of the eigenvalues of the coefficient matrix A. They can be found by solving the linear equation: I X Z - Al = 0 (3.5) I is a 4 x 4 identity matrix. By expanding the determinant, the longitudinal 65 Longitudinal Stability stability stability stability stability stability stability evalues, hi, being real, have negative real parts. Zero, or positive, values of the real part of any complex eigenvalue means that the aircraft will be dynamically unstable.' Rather than solving the polynomial by numerical methods it is more effective to use a numerical routine to compute the four eigenvalues of A. It has been observed that for the majority of aircraft types, the quartic of eq. (3.6) invariably factorizes into two quadratic factors in the following manner: (1' + 2cphwphh + uEh)(h2 + 2cspwsph + wfp) (3.7) The first factor corresponds to a mode of motion which is characterized by an oscillation of long period. The damping of this mode is usually very low, and is sometimes negative, so that the mode is unstable and the oscillation grows with time. The low frequency associated with the long period motion is defined as the natural frequency, which he believed meant 'flight-like'. Unfortunately, +vyq implies flight as demonstrated by a fugitive, not a bird (Sutton, 1949). The second factor corresponds to a rapid, relatively well-damped motion associated with the short period mode whose frequency is w,, and damping ration is c,,. As an example, consider the passenger transport aircraft, referred to as aircraft DELTA in Appendix B. If flight condition 4 is considered, the aircraft is flying straight and level in its cruise phase, at Mach 0.8 and at a height of 13 000 m. From the values of the stability derivatives quoted in the appendix, A is found to be: [-0.033 0.0001 0 - 9.811 The eigenvalues corresponding to this matrix are found to be: h], h2 = +0.0033 f j0.0672 h3, A4 = -0.373 + j0.889(3.9)' (3. 10)' The eigenvalues of eq. (3.9) are seen to be those associated with the phugoid mode since the damping ratio, although positive, is very small (0.0489) and the frequency is very low (0.067 rad s-1), hence the period is long. Such an inference can be drawn because the solution of any quadratic equation of the form: 66 x2 Aircraft Stability and Dynamics + $2\cos + o2 = 0$ (3.11) is given by: whenever 5 < 1.0. Complex roots occur only when the damping ratio is 0.387. 3.2.2 Tuck Mode Supersonic aircraft, or aircraft which fly at speeds close to Mach 1.0, occasionally have a value of the stability derivative, Mu, such that Mu takes a large value which is sufficiently negative to result in the term wEh in the phugoid quadratic becoming negative to (see Section 3.6). When this happens, the roots of the quadratic equation are both real, with one being negative and the other positive. Hence the phugoid mode is no longer oscillatory but has become composed of two real modes; one being divergent, which corresponds to the negative real root, and the other being divergent, which corresponds to the negative real root. corresponding motion results in the nose of the aircraft dropping (tucking under) as airspeed increases. Aircraft DELTA in Appendix B will exhibit a divergent tuck mode in flight condition 3. 3.2.3 A Third Oscillatory Mode The c.g. of a modern combat aircraft is often designed to lie aft of the neutral point (n.p.) (see Section 3.3). When this is the case the stability derivative, M,, can take a value which will result in every root of the longitudinal stability quartic being real. As the c.g. is then moved further aft of the n.p., the value of M, changes so that one of the real roots of the short period mode, and one of the real roots of the short period mode, and one of the real roots of the short period mode, and one of the real roots of the short period mode, and one of the real roots of the short period mode of the real roots of the short period mode, and one of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the
real roots of the short period mode of the real roots of the short period mode of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the short period mode of the real roots of the real roots of the short period mode of the real roots of the real roots of the short period mode of the real roots of the root a new complex pair, corresponding to the third oscillatory mode. When this has occurred, that mode is the main influence upon the dynamic response of any AFCS which is used. The phugoid mode has now become a very slow aperiodic mode, and there also exists another extremely rapid real mode. Too positive a value of M, can result in dynamic instability, for one of these real eigenvalues can become positive (see Section 3.5.2). Longitudinal Stability 3.2.4 s-plane diagram (which is simply a special Argand diagram). In Figure 3.1 are shown the locations (denoted by x) of eigenvalues for a typical conventional aircraft with a third oscillatory mode the locations are denoted by 0 and for an aircraft with tuck mode A Aircraft with 3rd oscillating pair I - cr -5.0 I -4.0 n I -3.0 Figure 3.1 I -2.0 I -1.0 A - x -X - to I + IS s-plane diagram. A popular method of investigating how sensitive is an aircraft's stability to values of some particular stability derivative (and, consequently, some aerodynamic, inertial, or geometric parameter) is to illustrate how the eigenvalues travel around the s-plane as the values of the stability derivative are changed. This is a form of root locus diagram. Another effective way of determining to which stability derivatives the aircraft's dynamic response is most sensitive is to carry out a sensitivity analysis on coefficient matrix, A (Barnett and Storey, 1966). It is important to remember that when the aircraft dynamics can be assumed to be linear those stability derivatives associated with the control surfaces play no part in governing the stability properties of the aircraft. Their importance for achieving effective automatic flight control, including stability and Dynamics 3.3 STATIC STABILITY 3.3.1 Trim Condition In Chapter 2 where the derivation of the equations of motion for an aircraft was shown, a point was reached (in Section 2.6) where the set of equations governing the small perturbation motion about some equilibrium required a number of forces and moments to be balanced. The balance equations, for straight and level flight, were shown to be: Xo - mg sin O = 0 Zo + mg cos O = 0 Yo = Lo = Mo = No = 0 For the linearized equations, the control required to achieve trim is: where [BITis the generalized inverse of matrix B. When reference is made to the stability of these static components what is meant is the inherent tendency of an aircraft to develop forces or moments (or both) which directly oppose any deviation of this motion from equilibrium flight. The only forces which can change significantly as a result of disturbances are sideforce, lift and drag, the values of which depend upon the orientation of the aircraft relative to the oncoming airstream. Obviously, each motion variable can be considered from a stability viewpoint. Only the most significant criteria of aircraft stability are considered here. 3.3.2 Forward Speed Stability An aircraft is considered to be statically stable for any disturbance, u, in its forward speed if the value of the stability derivative. This requirement may be understood from considering Figure 3.2. Suppose the aircraft in the figure is in a steady, trimmed flight condition corresponding to point A, i.e. the throttle setting is fixed and constant thrust is being produced which results in the aircraft's flying at some speed, VA. If, for any reason, the aircraft speed increases to, say, VA U, a drag force is generated which opposes the increase in speed, i.e.: + Another way of interpreting Figure 3.2 is to see that, at point A, if it is wished to increase the speed of the aircraft, the thrust has to be increased; to reduce its speed of the aircraft, the thrust has to be increased; to reduce its speed of the aircraft at point A, if it is wished to increase the speed of the aircraft at point A. Speed stability diagram. many aircraft would typically fly on approach). There it is seen that any decrease in speed leads to an increase in the drag force which will result in a further reduction in speed. At point B, Xu > 0 and the situation is regarded as unstable. If the difference between the thrust available from the engines and what is required to sustain flight in a particular manoeuvre is small (sometimes this is expressed by saving that the thrust margin is small), or if the change in throttle setting, is slow, then it is possible for an aircraft operating at point B to be in a position where recovery of the required airspeed is possible only by diving the aircraft. During the approach phase of flight, if this was not regarded as undesirable, it would certainly be regarded as unseemly. At speeds lower than VB the aircraft will tend to stall. If the unstable portion of the curve corresponding to maximum thrust intersects the line for which F, is zero at a value of speed higher than VStal1, the aircraft's speed will diverge, which will result in a stall, unless the pilot is able and willing to dive the aircraft. It is principally delta wing aircraft, such as the F-106, B-58 and Concorde, which tend to have positive values of Xu on approach. 3.3.3 Vertical speed, w, if the value of the stability derivative, Z, is negative. This means that if, somehow, there is generated a positive velocity increment along the axis OZ, a force is generated which tends to oppose the initial disturbance in w. For this to be true, the lift curve slope of the wing must be positive for all values of angle of attack, an aerodynamic condition which is always satisfied. However, for wings of high aspect ratio (when span2/surface area is large) and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and hence Z, is reduced. On delta wings, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and hence Z, is reduced. On delta wings, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and hence Z, is reduced. On delta wings, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, and which are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, are highly swept, aeroelastic effects generally cause the wing to distort so that the lift curve slope, are highly swept, aeroelastic effects generally cause the wing to distort so the lift curve slope, are highly swept, aeroelastic effects generally cause the wing to distort so the lift curve slope, are highly swept, aeroelastic effects generally cause the wing to distort so the lift curve slope, are highly swept, aeroelastic effects generally stability requirement that the value of Yp be negative is unimportant. However, although Yp < 0 is the usual condition, it is of advantage to the pilot. Sideslip angle P is not easily detected by pilots when Yp < 0 because the condition at very low height, and there is a restriction on the bank angle which can be commanded, because of the proximity of the terrain, a negative value of Yp will allow a skidding turn to be performed. It can assist the side-step manoeuvre, which is sometimes performed when an aircraft on its final approach is not correctly aligned with the runway centre line. 3.3.5 Static Directional Stability An aircraft is said to have static directional stability derivative. This means that the yawing moment N will increase as a result of a positive (sideslip) velocity v and the aircraft aligns itself with the relative are of the stability derivative. 'weathercock stability7. A large part of Cn is contribute8 by the volume of P the vertical tail.4 For supersonic transport aircraft, such as Concorde, the high Mach numbers and high values of the angle of attack, which commonly occur in operational flight, can cause considerable deterioration in the value of Nb. 3.3.6 Lateral Static Stability If there is a positive change in the sideslip angle then the aircraft's right wing drops and the aircraft slides to the right. Lb must be negative, the spiral mode (see Section 3.4) will be convergent. 3.3.7 Longitudinal Static Stability; the 'dihedral effect' results in the right wing drops and the
aircraft slides to the right. Chapter 1 that in the XZ plane, an intentional change of the aircraft's orientation can be achieved by deflecting the elevator, or flaps. Deflection of either surface produces a small, unbalanced force which, because of the distance of the distance of the distance of the distance of the aircraft's c.g., can result in a large pitching moment. Such a moment causes an aircraft to rotate about its c.g. until the steady moments adjust themselves to come into balance. How fast an aircraft will rotate, i.e. its angular acceleration, depends upon the size of the moment of inertia about the axis OY, namely Iyy. Since the centre of pressure moves with changes in the angle of attack, any change in lift causes a change in the moment produced by the lift force Static Stability * IT Zero lift line + ZT #- f. IT Ma, Figure 3.3 Geometry of wingltail. about the c.g. Thus, an aircraft rotates to a new orientation when disturbed and, as a result, the moments due to drag, to the lift from the wing and from the tail, etc. must all change. If they change in a way that increases the extent of the rotation, this is an unstable condition. As a result, it is customary to take as a criterion of longitudinal static stability the sign of the stability derivative M,, for when M, < 0 any increase in the angle of attack. Once the configuration of an aircraft has been fixed, then, for any particular flight condition, the stability derivative M, depends principally upon the aircraft's c.g. where xac is the distance from the a.c. of the aircraft and its c.g. and E is the chord length (the chord measured along the zero lift line of the wing see Figure 3.3). If the mean a.c. and the c.g. are coincident, that is, if the lift force acts through the c.g. is condition, which corresponds to this condition, is called the stick-fixed neutral point (n.p.). If the c.g. is

located aft of the n.p., then M, has a positive value and the aircraft is statically unstable. If the c.g. is then located even further aft of the n.p., a condition is reached where an 'infinite' normal acceleration, a, is produced with no force being applied to the control stick. This particular locatyon of the c.g. is called the stick fixed manoeuvre point (m.p.) By varying the location of the c.g. the manoeuvre stability can be made zero, positive, or negative. The m.p. is called the manoeuvre margin. To be statically stable the c.g. of an aircraft must be located forward of its n.p. Its m.p. Its m.p. Its m.p. for this condition must be aft of the c.g., but for conventional aircraft to be as manoeuvrable as possible, i.e. to produce as much acceleration as possible in response to a given control surface deflection, the c.g. should be only just forward of the n.p. Its m.p. for this static stability is relaxed and the c.g. is often deliberately located aft of the n.p., thereby reducing the manoeuvre margin but, as a result, increasing the manoeuvrability of the aircraft 72 Aircraft Stability and Dynamics successfully, dynamic stability is required, which has to be provided by a stability augmentation system specially fitted for the purpose. It is a relatively simple matter to show that the static margin can be expressed by Cm -= CL Cm - a (-h CL~ - h,) Xac = E where Cmis the coefficient. h, h,, xac and E are defined in Figure 3.3. Cm and CL denote dCm/aa and dCLlaa respectively. The stability derivative M: represe in pitching moment which occurs as a result of a change in the vertical velocity, w. If moments being defined as positive), L, D and Mac, which is the moment about the c.g. of the aircraft (nose-up moments being defined as positive). c.g., i.e.: + L cos a (hE - h,E) + D sin a (hC - h,E) Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently, Mcg = Mac + (L + Da)(h - h,)E + (L a - D)Z (3.19) Cg When the c.g. is closely located (3.17) Consequently (3.17) Consequ near the zero lift line, z is negligible. Moreover, a is usually a small radian quantity and CD < CL, consequently CDa may be neglected. Hence eq. (3.19) can be written as: Cm = Cm ac Cg + CL a(h - h,) - xac = - E C ~ a because: If the static margin is positive, the aircraft is stable, i.e. Cma is negative. (3.15) Transfer Functions Related to Longitudinal Motion 3.4 TRANSFER FUNCTIONS RELATED TO LONGITUDINAL MOTION 3.4.1 Relationship Between Transfer Function and State Equation The theory relating to deriving transfer functions from the linearized equations of motion is given in Section 2.9 of Chapter 2. In this present section, some of the more commonly used transfer functions for longitudinal motion will be derived, but the reader should be aware that a number of computer programs are available (see for example, Systems Control Technology, Inc., 1986; Larimer, 1978) for the automatic determination of appropriate transfer functions from a knowledge of the stability derivatives. These programs are usually based on the Leverrier algorithm (Faddeeva, 1959). The purpose of deriving analytically a number of transfer functions in this present section is to arrive at their final forms, to see which parameters and terms are significant, and to note possible simplifications which can lead to useful approximations. It has been shown in Chapter 2 that if only a single control, 8B, is considered, the linearized, small perturbation equations of longitudinal motion relating output variable, yi, to control input, ui, is given by: 74 Aircraft Stability and Dynamics Thus, every transfer function depends upon the variable. But it must always be remembered that when the control deflection is used to change some particular motion variable that same control deflection changes other motion variables simultaneously. It is this simple fact which results in so many systems, designed by means of the conventional theory of control for single input, single output, linear systems, producing aircraft performance which is unacceptable to pilots. Although transfer functions are useful, their use is limited, particularly for AFCS design for modern aircraft where many control surfaces are employed simultaneously. However, from eq. (2.164) it is evident that every transfer function relating to the motion of the aircraft must depend on the inherent characteristics of the aircraft through the resolvent matrix, [sI - A]-'. 3.4.2 Use of Output Matrix, C, to Select a Particular Motion Variable For the present, normal acceleration, and those motion variables such as h which are directly related to it, are not being considered. Thus: and, for further simplicity, since transfer functions are being considered, only a single output variable will be dealt with at a time. Consequently, eq. (3.28) now becomes: where C is a 1 x4 rectangular matrix. C contains only one non-zero element and that element has the output variable of concern. For example, if the output variable is chosen to be u, then: y b [I 0 0 O]x The other three relationships are: Thus, the unit element can be looked upon as a kind of pointer indicating which state variable has been chosen as the output variable. Quite often, the output variable. rad s-' but is required to work with pitch rate in degree s-' as an output variable results in y = [0 0 57.3 O]x. Transfer Functions Related to Longitudinal Motion 3.4.3 Transfer Functions is identical: G (s) = N(s)ID (s) (3.34) The denominator polynomial is the characteristic polynomial of the aircraft, namely det[XI - A] which was dealt with in Section 3.2. When the roots of the polynomial are known, i.e. those values of s are known which result in: it will be seen that they are identical to the eigenvalues of A. The polynomial det[sI - A] is often called the stability quartic. Every transfer function for longitudinal motion has the same denominator, because every transfer functions can differ for a particular motion, longitudinal or lateral, of an aircraft, is in their numerator polynomials. These numerator polynomials are direct functions of the output variable and the control input, and to emphasize this fact, they are often denoted, in American reports especially, as N\$(s). The superscript yi denotes the particular output variable, and uj denotes the control input. Thus, for the four transfer functions considered up to this point, the corresponding denotations would be: N&(s), N 1) and in problems where yaw rate feedback to the rudder is involved (wN/wD< 1). The same phenomenon also occurs in the dynamics of any electrohydraulic actuator when the oil Control System Design Methods I 206 compressibility effects are significant, or when structural compliance exists. For these AFCS problems, particular care must be exercised if the design is based upon transfer functions obtained from a mathematical representation of the aircraft's dynamics which approximated the degrees of freedom involved. 7.4 PARAMETER OPTIMIZATION 7.4.1 Introduction It was shown in Chapter 6 how the flying qualities are specified in terms of parameters such as short period damping, natural frequency of yawing motion, roll subsidence time constant, and so on. These parameters refer to idealized, low order models of the aircraft dynamics and have been specified because the settling time, or peak overshoot, or time-to-first crossover of the time response, produced by the corresponding low order model, is close to what is required from the aircraft motion when the aircraft motion when the aircraft motion when the aircraft has been subjected to some similar forcing function. The conventional design methods for s.i.s.0. control systems, dealt with in the earlier sections of this chapter, provide adequate means of achieving these figures of merit but require considerable experience, skill and judgement to produce acceptable designs. It would be helpful to have a design method which does this depends upon a performance measure which is a member of a class of performance indices. 7.4.2 Performance Indices For any control system, its output, y, is required to follow its input signal, r, as closely as possible. Any difference between the input and output is an error, e. If e(t) is transient, by which it is meant that e gradually reduces to zero as time goes on, i.e. e + 0 as t + w, it is appropriate to adopt as a performance index the scalar, J, where: . l = lom j(e)dt (7.92) in which j(e) is a non-negative, single valued function of error. Time, t, is measured from zero, the instant at which an input is applied. If it is assumed that j(e) is of the form: the performance index, J, can be denoted as Parameter
Optimization 207 where u is a constant. When u = 1, J, is the integral of absolute error (i.a.e.); when u = 2, J2 is the integral of squared error (i.s.e.). The performance index, when u = 0, is defined as a special case, meaning that Jo= [[1 i m v-30 1 lv]dt where If it is supposed that e(t) is non-zero throughout the interval 0-t, except possibly at a finite number of points, and is uniformly zero for t r t, then t, is the settling time of the system. Then it follows, from eqs (7.95) and (7.96) that: Jo is, therefore, the settling time of the system. 1.s.e. (Jz) is a much favoured performance index because it is easy to work with analytically, but the time response which results from an AFCS, designed on the basis of minimizing J2, is often unsatisfactory and, consequently, alternative performance indices, such as i.a.e. (J1) and i.t.a.e., (integral of the product of time and absolute error) are used, because they penalize large and persistent errors. Neither i.a.e. (integral of the product of time and absolute error) are used, because they penalize large and persistent errors. Neither i.a.e. (J1) and i.t.a.e. (J2) indices, the interested reader should consult Fuller (1967). 7.4.3 Parseval's Theorem and Definite Integral Table Integrals of the form: often need to be solved in AFCS work. By means of Parseval's theorem, a solution to eq. (7.98) can be found by evaluating I in the domain of the complex frequency, s a procedure which is easier than solving for I directly in the time domain. Parseval's theorem states that the integral defined in eq. (7.98) can be reexpressed as: 208 Control System Design Methods / Of particular interest to AFCS designers is the case when: fl(t) = f2G = f(t) for then: Of course, eq. (7.101) is still the Parseval equivalent to: provided that, for t < 0, f(t) = 0. Suppose there is a variable, x (t), and it is necessary to evaluate its integral-squared value, I, where: Table 7.1 Phillips' integrals Let: where the subscript n refers to the degree of the denominator polynomial, d (s) = 2 j = 0 n cjd and d be a rational function of s of the form: If all the poles of the function in eq. (7.104) have negative values or, if any poles are complex with negative, real parts, then I exits, and can be expressed as an algebraic function of the coefficients cj, where j = 1, 2, ..., n. The results are summarized in Table 7.1. 7.4.4 Design of Optimal s.i.s.0. Linear Systems The method of achieving a design is procedural: a structure is assumed for example, series compensation elements. Or it may possibly be as simple a problem as setting some gain to that value which results in the lowest value of i.s.e. The procedure and method are illustrated by means of two simple examples. Example 7.10 A simple system is represented by the block diagram for Example 7.10. 1. In response to a unit step input, r(t) 6 IY-(~), find the value of K which will minimize the i.s.e. if A is a constant. 2. In response to the same input find a which minimizes the i.s.e. if K is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the values of K and T which minimize the i.s.e. if A is a constant. 3. If the forward transfer functions is re-expressed as Kls(1 + ST), find the value is a constant. 3. If the forward transfer functions Figure 7.17 it can be deduced that: Control System Design Methods 1 210 and so for a unit step input R(s) = 11s: Thus, for E(s), referring to eq. (7.104): co = a, cl = 1, do = K, dl = a, d2 = 1, and from Table 7.1: Now, firstly: Therefore K -+ w yields minimum i.s.e. Secondly: For minimum i.s.e. a = hence 5 = W: VK and the optimum transfer function is K, giving: = 0.5. Thirdly: $E(s) = (Ts + 1) s 2 \sim + s K + Thus: co = 1$, cl = T, do = K, dl = 1, d2 = T, and: Therefore K = w for minimum i.s.e. and T = 0. Example 7.11 A unity feedback system has the closed loop transfer function: Parameter Optimization Find the values of a, and a 2 which minimize i.s.e. for a unit step input. Now, Referring to eq. (7.104): Therefore: co = al, cl = a2, c2 = 1, do = 1, dl = a1, d2 = a2, and d3 = 1. From Table 7.1: and I3 is a minimum with respect to a when: 7.4.5 Lagrange Multipliers Suppose that the variables, x(t) and u(t), are related by some differential equation, and let it be assumed that u(t) is to be chosen to minimize: 6 m I = x2dt (7.105) 2 12 Control System Design Methods I subject to a constraint on u (t), the difference between the right and the left sides of (7.106) is then defined as Z, i.e.: A new problem has now to be solved: to find that u(t) which minimizes the scalar, K, where KJ + XZ, i.e.: 9 K = xZdt + h (low u2dt - C] (7.108) Note that in the new problem there is no constraint on u(t). The control, u(t) which minimizes the scalar, K where KJ + XZ, i.e.: 9 K = xZdt + h (low u2dt - C] (7.108) Note that in the new problem there is no constraint on u(t). X, i.e.: u0 = u0(X, t) (7.109) With this special value of u, Z also depends upon X. Thus, If X is chosen such that Z is zero, then Z(X) = 0 and, of course, K=J Therefore, if K is minimized too. Moreover, it will have been minimized too. Moreover, it will have been minimized too. constraint, owing to its 'squared' nature. Therefore, rather than finding some control, u (t), from a class of control functions which satisfy the constraint, to minimize eq. (7.105), for some value of X and which satisfies the constraint eq. (7.106). The procedure which is used is illustrated in the three examples which follow. Parameter Optimization Example 7.18 Block diagram for a unit step input, the values of K and T which minimize the i.s.e. subject to the constraint that - I 2 I Figure 7.18 Block diagram for Example 7.12. For the specified input: R (s) = 11s : Therefore, for the error, e: co = 0, c1 = 1, c2 = T, do = K, dl = lOKT, d2 = 1, and d3 = T. Therefore: and for the variable, q: co = K, c1 = 1, c2 = T, do = K, dl = lOKT, d2 = 1, and d3 = T. Therefore: and for the variable, q: co = K, c1 = 1, c2 = T, do = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, c1 = 1, c2 = T, do = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, c1 = 1, c2 = T, do = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, dl = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, d1 = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, d1 = lOKT, d2 = 1, d3 = T. Therefore: and for the
variable, q: co = K, d1 = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, d1 = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, d1 = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K, d1 = lOKT, d2 = 1, d3 = T. Therefore: and for the variable, q: co = K. $T \sim r = -0 \sim I O O A K \sim = T 1 \sim results$ in: 1:. 0 0 + 1 $\sim -0 \sim -1 \sim -AK \sim = -0 \sim = 10 T 2$ hence : 1. 100 = + IUU-.d l o x - V A - + 10 - 1 dio 1.8 % A Example 7.13 For the system represented in Figure 7.19 choose Kl and K2 such that, in response to a unit step inupt, yD, the performance index, J, is minimized. If J is to be finite, there has to be zero steady state error in response to a unit step input. Hence Kl must equal unity. Parameter Optimization Figure 7.19: - 26s4 + 8.4 + 6.82 XX lo5s4 + (2.76 X 1061C2 - 1.35 denom (-s) X lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K; denom (-s) K lo7)s2 lo5 + 2.43 x106K2 + 2.25 x 106K2 + 2.25 solutions of K2 in aJIaK2 = 0. From a solution of the quartic, a positive real solution is found: K2 = 1.93. Note that the response of this system to a step input is very sluggish; it can be changed by increasing the weighting factor of 25 on the (y - yD)2 term. Control System Design Methods I Example 7.14 For the system represented in Figure 7.20 its bandwidth, BW, is defined as: Figure 7.20 Block diagram for Example 7.14. Determine the gain which will minimize the i.s.e., subject to the constraint that B W S 1. Note that: For i.s.e. co = 1, cl = 1, do = K, dl = 1. For BW: co = K, cl = 0, do = K, dl = 1. For BW: co = K, cl = 0, do = K, dl = 1. For BW: co = K, cl = 1. For BW: co = K, cl = 1. For BW: co = K, cl = 0, do = K, dl = 1. For BW: co = K, cl = 1. For BW: co = K, cl = 0, do = K, dl = 1. For BW: co =0 = 1. Exercises 7.5 CONCLUSIONS The chapter introduces the AFCS as a control problem and shows that, with linear feedback control, it is never possible to satisfy simultaneously the requirements for good stability, good tracking performance and good disturbance or noise rejection. The use of feedforward was discussed and the equations for a generalized AFCS, which allowed the linear feedback control to contain dynamic elements, were developed. The chapter concentrates on the use of the conventional control methods, such as pole placement, model-following, root locus and frequency response, to design AFCSs which are essentially s.i.s.o., linear and time-invariant. This work is extended by considering parameter optimization techniques to achieve feedback controllers which optimize a performance index, usually the integral of error squared and subject to constraints on the control surface, some rate of change of a motion variable, or the bandwidth of the closed loop system. These methods are illustrated by a number of examples. 7.6 EXERCISES 7.1 Design a closed loop system using linear state variable feedback for the open loop system must have a damping ratio of not less than 0.45. And in response to a unit step input the peak overshoot of the response of the closed loop system must not exceed 20 per cent and must not occur later than 0.15 s after the step has been applied. The complete response must have settled in 0.4 s. Aircraft dynamics Amplifier Rudder servo Figure 7.21 I A \ Open loop system for Exercise 7.1 (a) Draw a root locus diagram for the aircraft system of Figure 7.21. (b) If A = 0.04 calculate the values of the poles of the system with positive unity feedback. (c) Use a pole placement method to obtain a feedback control law which will result in the required closed loop response. 7.2 The short period dynamics of an aircraft can be represented by the equations: + q + Z8, SE Q = Mae + M&&+ Mqq + Ma, & = Z, a (a) Plot the root locus diagram for the transfer function q(s)/SE(s) for the aircraft BRAVO-2. Control System Design Methods 1 (b) (c) (d) (e) 7.3 Repeat part (a) for the transfer functions. Generate the corresponding Nichols diagrams. Calculate the gain and phase margins for each transfer function. Suppose that the elevator of the aircraft BRAVO is driven by an actuator which is characterized by the first order differential equation: iE = - 7.0SE + 7.0SEc where SEc represents the commanded step of 6E of - 2.0". 7.4 For the aircraft DELTA-3 plot the Nyquist diagram corresponding to the transfer function A(s)/~~(s), where A denotes the aircraft's heading, and ijR the rudder deflection. 7.5 Plot the Nyquist diagram for the transfer function y(s)/SE(s) for GOLF-2. Plot its Bode diagram for O(s)/SE(s). Hence plot the Nyquist diagram for the transfer function y(s)/SE(s) for GOLF-2. Plot its Bode diagram for O(s)/SE(s). y(s)/O(s). Is this diagram valid? 7.6 The lateral dynamics of a large cargo aircraft, DELTA, for flight condition 3, are augmented by the addition of the second with a washout filter which operates on the yaw rate. The state vector is given by: and the control vector by: U' = [SA SR] The output vector is defined as: Y' = [P P ewo \$1 Use an eigenvalue assignment technique to obtain a feedback control law which will result in the closed loop system having the following eigenvalues: - 1.5 + j1.5 A, ,,,,, = - 2.0 + jl.O AdirlZZ = 7.7 A simple yaw damper is represented by the block diagram of Figure 7.22 The required transfer function r(s)/SR(s) can be found by using the two degrees of freedom approximation discussed in Section 3.10 of Chapter 3 and the stability i Exercises 2 19 derivatives corresponding to GOLF-4. Determine an appropriate value of Kc which will result in the integral of error squared being minimized, subject to the constraint SR 5 1 5 O, for a unit step command. Controller Rudder servo 6, (~) 4 (s + 4) Aircraft dynamics 6~(s) r(s) ~R(s) 4s) F Figure 7.22 Yaw damper system for CHARLIE-2is shown in Figure 7.23. The gain, Kc, of the controller is to be found such that the integral of error squared is minimized subject to the constraint Aw 5 4.0 where the system bandwidth Ao is defined as: Aircraft dynamics Controller Compensation Figure 7.23 Roll attitude control system which is used to control the pitch rate of an aircraft is shown in Figure 9.10(b) in Chapter 9. The transfer function is known to be: of the compensation network used with aircraft FOXTROT-3 and the sensitivity, K,, of the rate gyro is unity, and K,, = 2.0. (a) Show that the closed loop pitch rate control system can be represented by: where 2' A [x xc] (b) Evaluate the matrices K and A 7.10 The rate of change of height in response to a change in the collective of the compensation network used with aircraft FOXTROT-3 and the sensitivity, K, of the rate gyro is unity, and K, = 2.0. (a) Show that the closed loop pitch rate control system can be represented by: where 2' A [x xc] (b) Evaluate the matrices K and A 7.10 The rate of change of height in response to a change in the collective of the compensation network used with aircraft FOXTROT-3 and the sensitivity of the rate gyro is unity, and K, = 2.0. (a) Show that the closed loop pitch rate control system can be represented by: where 2' A [x xc] (b) Evaluate the matrices K and A 7.10 The rate of change in the collective of the compensation network used with aircraft FOXTROT-3 and the sensitivity of the rate gyro is unity, and K, = 2.0. (a) Show that the closed loop pitch rate control system can be represented by: where 2' A [x xc] (b) Evaluate the matrices K and A 7.10 The rate of change of height in response to a change in the collective of the compensation of th deflection of a hovering helicopter can easily be shown (see Chapter 13) to be governed by a transfer function: Control System Design Methods I For a small helicopter with a single main rotor the
values of the stability derivatives have been found to be: Z g00 = 4.0 and Zw = 1.4 When a simple height control system with a proportional controller, with a value = 6.0, and with unity feedback is used it is found that the dynamic response of the closed loop system is unacceptable. (a) Determine a feedback control system will have a closed loop transfer function: K (b) Compare the natural frequency and damping ratio of this system with the values which resulted with the simple height control system. 7.7 NOTE 1. y = max(a, b) means that y will take either the value a or b, whichever is the larger. 7.8 REFERENCES BOWER, J.L. and P.M. SCHULTHEISS. 1958. Introduction to the Design of Servomechanisms. New York: Wiley. 1978. Linear Control System Analysis and Design. New York: McGraw-Hill. ERZBERGER, H. 1968. On the use of algebraic methods in the analysis and design of modelfollowing control systems. NASA TN-D4663, July. FULLER, A.T. 1967. 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Control System Design Methods II 8.1 INTRODUCTION Whenever a set of specifications has been laid down for the dynamic behaviour of an aircraft, and when those specifications cannot be met, and the met and the met and the dynamic behaviour of an aircraft and when those specifications cannot be met, and the dynamic behaviour of an aircraft. AFCS problem exists. If the required dynamic performance has to be achieved then additional equipment must be used in conjunction with the basic aircraft, in all but the most trivial cases. Those conventional control techniques, outlined in Chapter 7, all depend upon an interpretation of the system's dynamic response, in terms of such parameters as the system's dynamic response. settling time, frequency of oscillation of the transient, value of the peak overshoot, time-to-half amplitude, gain margin, and so on. Inevitably, the design which results from using such methods is obtained as a consequence of some compromise, and it may not be unique. By using the modern theory of optimal control, a specified performance criterion is met exactly and the corresponding control design is unique. How this unique solution may be found, and how the performance criteria, are the subjects of this chapter. THE MEANING OF OPTIMAL CONTROL 8.2 An optimal control system is one which provides the best possible performance from its class when it responds to some particular input. To judge whether the system's performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by which the quality of the performance is optimal requires some means by the performance to have been optimized if some control input, uO, has been used such that the value of J is least over the period from to, when the response ceases. (In some cases J may be chosen so that it can be maximized over the chosen period: for example, this could be achieved by choosing the cost functional, Y{.), in eq. (8.1) to be negative). Equation (8.1) is not the only type of performance index of use in aeronautical studies, of course, but it is the most common. Y{x, u, t} is 223 Meaning of Optimal Control known as the cost, or pay-off, functional: it represents the cost of a system's having been at a particular point in the state space, particular control inputs, for the entire period of time (T - to). Posing an optimal control problem in this way has considerable merit. because it includes in its statement most of the important problems relating to flight control, namely stability, the dynamic response of the closed loop system, and the determined control law.' One of the chief problems in setting up an optimal control problem is the particular choice of performance index. The significance of performance index. The sig of a control surface, or a limit on the rate of change of the position of a control surface actuator, it is often convenient to replace actuator. vector, e, is defined as the difference between the actual state vector and the commanded value: e 4 (x - xcomm) (8.3) and A is a Lagrange multiplier. If a different value of weighting is required on each of the elements o and x,, are equal. It is important sometimes to place different weighting penalties on the cost of using each control input, uj. In such a case, a square matrix, G, is associated with the control vector u and is used in the performance index, so that eq. (8.2) becomes: J = It: (elQe + ulGu)dt (8.4) For each choice of Q and G, minimization of J corresponds to a unique choice of u A Y, Ornrn(t) - Y (t) and let it be assumed that, as t + m: e(t) + 0 t + m Further, let: where: $11 \sim (-nP)$ Tx ij = where T is non-singular: & = TAT-'^ + TBu = Fg + H u ... where: Thus: + F12rl + H1u li = FZIY + F22rl + HZU 9 = FIIY Now, if the following: 4 i~ (as before) u2 = - Y However: +j = FIIr + F12q + HIi. F11u2- F12u3 + H2V u3 = i j = FZ1f FZ2q H2fi Hence: 2 = @H + rv and also the following: 4 i~ (as before) u2 = - Y However: +j = FIIr + F12q + HIi. F11u2- F12u3 + H2V u3 = i j = FZ1f FZ2q H2fi Hence: 2 = @H + rv and also the following: 4 i~ (as before) u2 = - Y However: +j = FIIr + F12q + HIi. where: O Letting: J =i I 0 low+ {H'QH vfGv)dr allows the determination of the optimal feedback control law: where P is the solution of the corresponding ARE eq. (8.192): + @'P - prG-lrrp +Q=o Control System Design Methods I1 252 The optimal feedback control law: where P is the solution of the corresponding ARE eq. (8.192): + @'P - prG-lrrp +Q=o Control System Design Methods I1 252 The optimal feedback control law: where P is the solution of the corresponding ARE eq. (8.192): + @'P - prG-lrrp +Q=o Control System Design Methods I1 252 The optimal feedback control law: where P is the solution of the corresponding ARE eq. (8.192): + @'P - prG-lrrp +Q=o Control System Design Methods I1 252 The optimal feedback control law: where P is the solution of the corresponding ARE eq. (8.192): +
@'P - prG-lrrp +Q=o Control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The optimal feedback control System Design Methods I1 252 The o second order system shown in Figure 8.6. 4s) * rl(4 1 - L 1 Y 6) / (s + 2) (s + 1) d Figure 8.6 Simple second order system. ~(\$1 = 1 ~ comrn(s) (s 2 + 3 s + 3) If yCornrn(t) is a unit step input, y, 0.666. Choose: then: hence: I" 0-2-1 -O11 = 0.333 and there is a steady state error of State Reconstruction If: then it can be shown that: + v = 1 0 0 ~ $4.150 \sim 2.050 \sim$ and the optimal system is shown in Figure 8.7. Therefore: Figure 8.7 Optimal control system with integral action. Y(S) = y O m m () s3 :. 8.10 10 + 4.15s + 5.05s2 + 10.25s + chapter involve full state variable feedback. Two situations can arise which makes it difficult to implement a feedback control law once it has been determined: the first is the case where only p output variables can be measured, and not the full n state variables, i.e.: 254 Control System Design Methods 11 ykcx (8.196) where y E RP, x E Rn and p < n The second is the case where there are measurements of the state variables which are corrupted by noise. This situation can be represented by: where n is a vector E RP and represented by: where n is a used, the difference between which depends upon what is known a priori about the probability characteristics of the signals involved. Readers are referred to Curry (1970) for further discussion. 8.10.2 Weighted Least Squares Method In this method the only assumptions are that, on the average, n is zero and x is near the equilibrium flight value, x, What is taken as the best estimate is the value of the state vector x which minimizes the performance index: Q and G are selected to be symmetric and p.d. matrices. In eq. (8.198), y represents a constant vector, as it represents the measurement. What is significant about choosing eq. (8.198) as the performance index is that the weighting can be arranged so that the situations can be avoided where x is close to XE, but n has large values, or n is near zero, but the difference between x and xE is substantial. The value of the state vector, xO, which minimizes eq. (8.198) is the weighted least squares estimate of x. Using the chain rule of differentiation for vectors it can be shown that: x0 = (Q Let: + C'GC) - \sim C'G + \sim (Q + CrGC) - 1 \sim x, 255 State Reconstruction (QL + C'Or) z-'zA-I = (CL - G - 1 o') (QN (CQ-1 + C'M) -] G-IM) Hence: QL + C'GCL = I from which: L = (Q + C'GC) - However, from eq. (8.205): Equation (8.213) is a matrix inversion lemma; when substituted in eq. (8.201) it yields: from eq. (8.205): Equation (8.213) is a matrix inversion lemma; when substituted in eq. (8.201) it yields: from eq. (8.205): Equation (8.213) is a matrix inversion lemma; when substituted in eq. (8.201) it yields: from eq. (8.205): Equation (8.213) is a matrix inversion lemma; when substituted in eq. (8.201) it yields: from eq. (8.205): Equation (8.213) is a matrix inversion lemma; when substituted in eq. (8.201) it yields: from eq. (8.201) it yields which it can be shown that: x0 = XE + H(Y - C x E) where: H - Q-~c'(cQ-~c~ + ~ -1) - 1 256 Control System Design Methods I1 Equation (8.216) indicates that the estimate of the state vector, xO, is given by the equilibrium state vector plus a linear combination of the difference of the measured values from their nominal values, CxE. The matrix H is an indication of how important the measurements are relative to the quality of the estimated value. For example, suppose it is known for an AFCS that the resulting measurements are poor. G should be chosen so that its norm is small; hence, its inverse will be large. As a consequence, H will be small and the contribution of the measurements to the estimate in eq. (8.216) will be small. 8.10.3 Optimal Linear Estimation Given eq. (8.197), assume that for the random vectors x and n the following first and second order probability characteristics are known: $F = C(x) (8.218) J = C\{(x - f)(x - X)'\}$ (8.219) C(n) (8.220) = 0 N = e{nn1} where (() is expectation (or eq. (8.197), assume that for the random vectors x and n the following first and second order probability characteristics are known: $F = C(x) (8.218) J = C\{(x - f)(x - X)'\}$ (8.219) C(n) (8.220) = 0 N = e{nn1} where (() is expectation (or eq. (8.197), assume that for the random vectors x and n the following first and second order probability characteristics are known: $F = C(x) (8.218) J = C\{(x - f)(x - X)'\}$ (8.219) C(n) (8.220) = 0 N = e{nn1} where (() is expectation (or eq. (8.197), assume that for the random vectors x and n the following first and second order probability characteristics are known: $F = C(x) (8.218) J = C\{(x - f)(x - X)'\}$ (8.219) C(n) (8.220) = 0 N = e{nn1} where (() is expectation (or eq. (8.197), assume that for the random vectors x and n the following first and second order probability characteristics are known: $F = C(x) (8.218) J = C\{(x - f)(x - X)'\}$ (8.219) C(n) (8.220) = 0 N = e{nn1} where (() is expectation (or eq. (8.197)) (8.218) J = C{(x - f)(x - X)'} (8.218) $F = C(x) (8.218) J = C{(x - f)(x - X)'}$ (8.218) $F = C(x) (8.218) J = C{(x - f)(x - X)'}$ (8.218) $F = C{($ averaging) operator, f is the mean of the vector x and J is a covariance matrix. The sensor noise has zero mean and its covariance is N. The optimum linear estimated vector, based upon the sensor measurements, y. The gain matrix K is chosen to minimize the mean square error in the estimate. It is easy to show that the correct choice of K for this criterion is: Comparing eqs (8.217) and (8.222), the gain matrices K and L must be different unless the matrices K
and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different unless the matrices K and L must be different criteria and the basic assumptions are wholly different. 8.10.4 State Estimation - Observers is due to Luenberger (1966); it is used where the available measurements are not heavily corrupted by noise, which is the usual State Reconstruction 257 situation prevailing in AFCSs. Its merit is that the state estimator which results is a dynamic system with a lower order than the system whose state vector is being reconstructed. For a system defined by eqs (8.225) and (8.226): where x E Rn, u E Rm and y E RP. Luenberger showed that an observer of order (n - p) can be constructed with a state vector, z, such that the observer state vector is related to the true state vector by: where z E Rn - P and S is a matrix of order [(n - p) x n]. The observer is defined by: where E is a matrix of order [(n - p) x m]. Suppose a transformation matrix S can be found which satisfies: SA - E S = FC (8.229) and the matrix J is arranged to be: J = SB If: then: i - SH + Fy + Ju - SAX - SBu = Ez + Fy - SAX = Ez But, substituting for S A from eq. (8.229), yields: i - SH = E (z - S x) (8.234) which has a solution: If E is chosen such that the eigenvalues of the observer are more negative than those of the aircraft dynamics, the observer are more negative than those of the aircraft dynamics. available for solving for S and F, thus completing the design of the observer. Control System Design Methods 11 258 The required estimate P of the aircraft state vector, y, and the observer state z, i.e.: + D2z (8.236) + D2S = I (8.237) f = Dly where D,C A block diagram representation of the observed aircraft is given in Figure 8.8. i Figure 8.8. observer system. 8.10.5 Optimal Observer system. 8.10.5 Optimal Observer system. 8.238) and (8.239) It is intended to design an observer to provide an estimated state vector xE which will be close to the original state vector x, but requires as its inputs only the control vector u and another vector which is related to the output vector y of the aircraft, i.e.: State Reconstruction 259 The forcing vector, w, is chosen to be: AK (-~ YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + G U + KCX However, from eq. (8.238): Bu=%-Ax and if: GAB then: riE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE fiE = (F - KC)xE + S - (A - YE) where: YE . A CXE f KC)X By choosing the coefficient matrix F of the observer to be identical to that of the aircraft, namely: FAA (8.248) and by defining any difference between the estimated and actual state vector, e, it can easily be shown that: Provided that X(A - KC) < 0, then as t -+ a, the error vector, e, it can easily be shown that: Provided that X(A - KC) < 0, then as t -+ a, the error vector, e, it can easily be shown that: Provided that X(A - KC) < 0, then as t -+ a, the error vector, e, it can easily be shown that: Provided that X(A - KC) < 0. will correspond to the state vector x of the aircraft. To secure this desirable condition requires only that K be determined. As a first step, let K be chosen to be (A - KC), and: eAx-XE then: C = (A - KC)e = Fe 260 Control System Design Methods I1 Suppose that we have a system defined by: C = Me + Nv then if we choose as a performance index: then minimizing eq. (8.256) Hence: C = (M + NH)e If it can be arranged that: X(A - KC) = X(M + NH)e. required observer provided that: M ' = A, N' = C and H' = - K (8.259) A block diagram representing the aircraft DELTA, at flight condition 2, the equation of motion representing the aircraft because, including the flexibility effects, is given by % = A x + Bu where X' and A [W Q iL iZ i3 i4 is hl 44 4 As h6 6A ijE 6E) State Reconstruction SE, and ?iE denote the respective deflections of the inboard and outboard sections of the inboard and inboard and inboard and inboard and out that C results because the output is assumed to be solely the vertical velocity w. It is from this solitary measurement that the state vector is to be reconstructed. The resulting optimal gain matrix, A, for DELTA-2 (with flexibility effects). C = ~ 0 0 0 0 0 0 0 = = 0 0 0 0 1 0 i) and G of: that every state variable is available for measurement, are shown in Figure 8.12. Also shown there are the results of applying the same control law, but with the state vector having been entirely reconstructed in the observer from the solitary, continuous measurement of w. 8.10.6 The Kalman-Bucy Filter In situations where noise contaminated measurements must be used (where for example, radio or radar receivers are used as sensors) then a Kalman-Bucy filter Optimal control with reconstructed state vector Figure 8.12 Response of optimally controlled DELTA-2to initial angle of attack. 263 State Reconstruction may be used to reconstruct the state vector from the noisy output signals. Although its use results in optimal rejection of the aircraft dynamics. The aircraft dynamics which corrupt the measurement, it requires a dynamic system of the same order (usually) as the aircraft dynamics. The output signals, y, are affected by sensor noise, n. Hence: where 6() is a unit impulse function. where $.\&\{$) is the expectation operator. The solution is obtained when a control u(t) has been found to minimize the performance index: $+J = E \{x(t)Q(t)x(t) + u(t)Q(t)x(t) + u(t)Q(t)$ (1964) allows the optimal control law to be found first. It can be shown to be: where P(t) is the time-varying solution of the matrix Riccati equation: ~ (t =) P(~) A+ A' P(~) BG-'B'P(~) (8.268) where: Equation (8.267) depends upon the best estimate of x which is obtained from the Kalman-Bucy filter defined by: The notation B(tlt) means the estimate of x(t) based upon measurements up to and including y(t). ~ (tis)the gain matrix of the Kalman-Bucy filter and is given by: where the error covariance, W(t/t)(t) + W(t/t)A' Control System Design Methods I/ + ES(t)E1- W(t/t)C1T'(t)CW(t/t) (8.278) with: Example 8.8 An integrated flight control system has a height hold mode which can be represented by the transfer function: i.e. wph = 0.055 rad s-I and = 0 hcommo+ ahcorn,. The statistical characteristics of the noise in the command channel are defined by: NCommis defined as 400 ft2 s. Height is measurements contain white noise, i.e.: where S{Sm> = 0 S{sm(t)sm(v>) =
Nma(t - v) Nm is taken as 900ft2 S. The requirement is to design a system which will provide an estimate of height with the least possible variance. The height hold system can be represented in state variable form as: Let hcOmmo x3. Then, since h, representation becomes: = hcommo+ ShComm, the complete state State Reconstruction and: hm A y = [0.003 0.3 O]x + 6, then: + W C ' T - ' ~ C X] ir = A x where W is the solution from: 0 = AW + W A ' + ESE' - WC'T-'CW From the problem statement: S = Ncomm = 400 T = Nm = 900 Now: I wll wl2 w13 w = ' wl2 w22 w23 wl3 w23 w33 WC' = Hence: I + $0.3Wz1 \ 0.003W12 + 0.3Wz1 \ 0.003W$ Steady state starting conditions are usually assumed, such as: 3.3 x lo6 8.11 CONCLUSIONS This chapter introduces the important topics of linear optimal control, controllability, which are very important topics of linear optimal control, controllability, which are very important properties of the mathematical models systems as AFCSs with analytical redundancy (a topic which is not covered in this book) these subjects are of considerable importance and need to be thoroughly understood by the control system designer. The solution of the linear quadratic optimal problem by means of the algebraic Riccati equation (ARE) is presented, with particular reference to effective methods of obtaining the required feedback control law. Based upon this work, methods of designing an optimal output regulator, or a system with a prescribed degree of stability, or one which explicitly follows a model response are also presented. In all the methods, the result depends upon solving an ARE. Furthermore, an optimal command control system was presented, which is also based on the work of the LQP. The use of a dynamic feedback controller is also dealt with, before concluding the chapter with a study of a few, or even a single, output variable. 8.12 EXERCISES 8.1 For a system defined by the state and output equations S y = Cx D u the following matrices apply: + 0 1 0 l = [1 0 11 - 20-2 B' = [- 1 2 01 D = [0] Is the system has two components can be connected in any one of three possible ways: 1. Cascade: Gl(s) G ~ (s) 2. Parallel: Gl(s) 3. In a closed loop configuration: + G~(s) Discuss the controllability and observability for each connection. 8.3 Write down an appropriate state equation for the lateral acceleration at the pilot's station for the lateral acceleration at the pilot's station for the aircraft completely controllable? Is it completely observable? What effect would losing rudder action have on the controllability? 8.4 Write down the state equation corresponding to the lateral motion of DELTA-1.If a performance index, J, is chosen to be: Establish: (a) Whether the feedback control law obtained as a solution to this linear quadratic problem can stabilize the aircraft. (b) The gains of the optimal feedback control law. (c) The eigenvalues of the closed loop flight control system. 8.5 The executive jet, ALPHA, is cruising at an altitude of 6100 m and a forward speed of 237 m s-l. The aircraft is controlled by means of its elevator and by changing its thrust. For the longitudinal motion the output vector has a set of the closed loop flight control system. its elements the pitch rate and pitch attitude, i.e. y' [q 01 (a) Compute the steady state response to a unit step change in thrust. (b) For Q = diag[1 1 11 and G = diag[1 1 11 and G = diag[1 1 001 find the optimal feedback gain matrix. 8.6 For the aircraft detailed in Exercise 2.5 determine a feedback control law which will result in the acceleration response of the controlled aircraft being identical to that obtained from an aircraft which has been idealized and modelled by the equation: a - - 10ay,o, + 10sa + s* Show that the feedback control law found does indeed provide model matching. 8.7 The aircraft BRAVO-3 is represented by the following state equation H = Ax where: x' 4 [U w q 01 u' 4 [&,I (a) Using weighting matrices: + Bu Control System Design Methods I1 Q = diag[1 1 11 G = [1] determine the optimal flight control law. (b) Find the eigenvalues of the optimal flight control system and compare them with those of the optimal flight control system. of attack of 1". 8.8 A pitch rate damper is represented by the block diagram of Figure 8.13. It is of 10" s-I there is a steady observed that in response to a step command of q,,,, state error of 8.88". Controller 0.3 Actuator 6 ~ (s) c 10 st10 Aircraft dynamics 6~(s) Figure 8.13. It is of 10" s-I there is a steady observed that in response to a step command of q,,,, state error of 8.88". Section 8.9 to obtain an optimal control law which will both minimize the performance index: and will result in there being zero steady state error in the optimal pitch control system for which the feedback gain matrix, K, is given by: K = [0.0184 - 0.0855 - 2.905 - 14.0351 The actuator dynamics have been ignored. It is found, however, that only the pitch rate and pitch attitude can be measured on the aircraft. (a) Show how the motion variables u and w may be reconstructed if the elevator deflection can be measured also. (b) Draw a block diagram of this complete flight control system Include all the gains involved in your scheme. 8.13 NOTES 1. The pay-off functional is introduced as a postulate. The reader may like to be reminded of the view of the philosopher Bertrand Russell: it has a great many advantages, all of which coincide exactly with those of theft in comparison with honest labour. References 2. 3. 4. 8.14 269 A necessary condition that a matrix Z be positive definite is det Z > 0; hence Z is non-singular. This restriction is required only when the upper limit of the integral is m then c?= [(clc)] 1 C 1]. REFERENCES ANDERSON, B.D.O. and J.B. MOORE. 1971. Linear Optimal Control. Englewood Cliffs, NJ: Prentice Hall. ATHANS, M. and P.L. FALB. 1966. Optimal Control. New York: Wiley. CURRY, R.E. 1970. A brief introduction to estimation for dynamic systems. 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PORTER, W.A. 1966. Modern Foundations of Systems Engineering. New York: McGraw-Hill. Stability Augmentation Systems 9.1 INTRODUCTION The term 'stability augmentation system' came into use in the USA. about 1950. At that time, Northrop, an American manufacturer, was famous for its 'flying wing' aircraft, being the leading exponent of such designs. It was known from the outset, however, that such designs. It was known from the outset, being provided by an AFCS. What Northrop called the kind of AFCS it proposed to use to remedy the poor, inherent flying qualities of its YB-49, a flying wing bomber aircraft, was a 'stability derivative augmentor'. However, on the standard form for installation drawings, the title block was insufficiently wide, so that the name was reduced by a draughtsman to 'stability augmentor' to fit the available space (McRuer and Graham, 1981). All similar systems have been called stability derivatives of a number of specific stability derivatives could stability derivatives could stability derivatives could stability derivatives of a number of specific stability derivatives could stability derivatives could stability derivatives of a number of specific stability derivatives of an aircraft are to be increased by means of negative feedback control. Although all stability derivatives could stability derivatives could stability derivatives of a number of specific stabi be so altered, only a few are usable candidates, since it is only by means of their alteration that any required change in the flying qualities of an aircraft can be effected. In general, SASs are concerned with the control of a single mode of an aircraft's motion. The general structure of such an SAS is shown in the block diagram of Figure 9.1, in which it can be seen there are four principal elements: aircraft dynamics, actuator dynamics, actuator dynamics, sensor dynamics, sensor dynamics, sensor dynamics, sensor dynamics, sensor dynamics, sensor dynamics and flight controller. These elements are essential and are always present in any SAS. When the SAS is switched off, the aircraft can be controlled directly by the pilot moving the appropriate control surface(s) through his cockpit controls. The flight controller, of course, is not then active. When the SAS is switched on, the control surface is driven by its actuator which is controlled by the flying qualities of the aircraft are enhanced by the control action of the feedback control system in such a manner that the effects of atmospheric, or other disturbances upon the aircraft's motion are suppressed. Sensor noise also affects the quality of control. However, many types of SAS cannot be switched off by a pilot, but always remain active, from the moment the master electrical switch is on. In the event of any failure of the SAS, the aircraft has then to be controlled solely by means of inputs from the pilot's cockpit controls. If an aircraft motion motion SAS oOn command Y c o ~ ~ Actuator dynamics -..-. Pilots stick motion motion SAS oOn command Y c o ~ ~
Actuator dynamics -..-. Figure 9.1 Stability augmentation system. direct link between some particular cockpit control and its corresponding control surface, then the aircraft is said to possess, in the event of an AFCS failure, the property of manual reversion. In some cases, the forces which need to be supplied by the pilot are beyond the limits of human performance. In some modern aircraft, particularly CCVs, which depend upon active control technology for their successful operation, no such manual reversion is provided, reliance being placed upon some form of redundancy in the AFCS to ensure continuous operation, no such manual reversion is provided, reliance being placed upon some form of redundancy in the AFCS to ensure continuous operation of the SAS. system may be referred to as a command and stability augmentation system (CSAS). The principal SAS functions which are found on modern aircraft are: pitch rate SAS, yaw damper, roll rate damper, and relaxed static stability SAS. Essentially, certain desirable values of the non-dimensional stability Cn, Cn, and Cl, can be obtained more effectively derivatives, C, , C, , C n~' P using automatic"cont\$ol than b i physical sizing of aerodynamic surfaces. 9.2 ACTUATOR DYNAMICS Actuators used in combat and transport aircraft sometimes use electric actuators. Such actuator systems have their own dynamic characteristics which affect the performance of the closed loop SAS. In Appendix A, some information on the nature of the dynamic characteristics of such actuator, in the design of an SAS (or any AFCS mode) is initially being considered it may be assumed, as a first 272 Stability Augmentation Systems approximation, that the dynamic response of the actuator, in comparison with that of the mode of flight of, the aircraft which is being controlled, is so rapid that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S,(s) = K (9.1) K is taken as the gain of the actuator: it is dimensional, usually that it can be represented by a very simple transfer function: S(s)/S, (s) = K (9.1) K is taken as the gain of the actuator is the gain of the actuator is taken as the gain of the actu degrees per volt. If the aircraft is large, then it is known that the response can then be instantaneous and, consequently, the transfer function usually assumed in this situation is: S(s)/S,(s) = KXI(s + X) (9.2) Typically, X lies in the range 5-10.0s-l; 1 is the inverse time constant of the actuator. Even when it is known that the SAS design is likely to be much affected by the nature of the actuator's performance, it is customary to proceed with the design on the basis of either representation eqs (9.1) or (9.2) and then to simulate the final design, including the complete known description of the dynamic characteristics of the actuator, noting any loss of SAS performance as a result of including the more representative actuator dynamics. If a significant change is noted then some adjustment in the control law is normally tried in order to minimize the loss of dynamic performance. It is more important to realize that this final test must always be made particularly to confirm that the existence of the higher order terms in the actuator dynamics, which can sometimes have very grave effects on the SAS performance, is the existence of non-linearities. Such effects should be accounted for in the simulation study. There is a semantic difference which occurs between British and American usage; in British work, the actuator is usually taken to mean the device which converts electrical signals, of low power, but of sufficient power to drive the hydraulic value which converts electrical signals to mechanical signals. control surface. In American work, 'actuator' covers the whole system from the command voltage, S,, to the control surface deflection, 6. This is the usage followed in this book. When an electric actuator's characterization. It should also be remembered that the hinge moment is not a linear function; large deflections cause considerable loading of the actuator which affects the dynamic performance. For flight critical conditions, such effects should be studied by means of the simulation of the system, before committing to a final design. Finally, it must be remembered that no AFCS is generally allowed to Longitudinal Control 273 make use of the full range deflection is not allowed at the limits of the control surfaces: it is usually limited to deflection). It is then said that the SAS has 10 per cent control authority. Such limits on control authority were set by aviation authorities to ensure the safety of the aircraft which could result was, therefore, strictly limited. This safety measure is inimical of good manoeuvring performance, however, and on military combat aircraft and CCVs considerably greater authority - sometimes 100 per cent - is now allowed, and is, in a few cases, essential. 9.3 SENSOR DYNAMICS Every sensor used in an AFCS is a transducer. In modern aircraft, its purpose is to measure motion variables and to produce output voltages or currents which correspond to these motion variables. Some of the electronic sensors, such as radar altimeters, or radar, process the information so quickly in comparison with the aircraft's response that it is customary to regard their transducing action as instantaneous. time constant of such filters is often considered as representing that of the sensor. The inertial instruments, such as gyroscopes and accelerometers, do have well defined dynamics characteristics (see Appendix A), but the sensors employed in AFCS are chosen to have bandwidths and damping such that they can be considered to be instantaneous in their action. A sensor is frequently represented, then, by its sensitivity, i.e. : K, can have units, such as V rad-' for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate gyro, and Vm-' sP2 for an attitude gyro, Vrad-' s-' for a rate g and the full representation should be used. For SASs, the most common sensors are gyroscopes and accelerometers. What usually affects the performance of the SAS more strongly than a sensor's dynamics is its location on the fuselage. 9.4 LONGITUDINAL CONTROL (Use of Elevator Only) 9.4.1 Introduction In Section 2.8 of Chapter 2 the state and output equations used with aircraft dynamics are presented. In SASs, the controller output is the command voltage to 274 Stability Augmentation Systems the control surface actuator dynamics are presented by, say, eq. (9.1) no change to the form of the state equation is required but, if eq. (9.2) is used to represent the actuator dynamics, there is an additional differential equation to be accounted for, namely: sj = - AS_j + KAS, I (9.4) where S j is, of course, uj, one of the control surface deflections. This additional equation is usually made to augment the state equation by choosing x, + to be uj. For example, consider the state equation for longitudinal motion with a single control input, SE - i.e. eq. (2.108). Let: then: kg = - AX^ + KXU hence: 0 0 B = 0 0 - KX, When the control function; when it depends upon time, we speak of it as the control law, namely: U = f(~) (9.13) The control law of eq. (9.13) means that the control is based on output feedback; whether the control law is linear or non-linear depends upon the nature of the functional, f(). The customary forms of feedback control for AFCSs, and hence an SAS, are linear, i.e. the control takes the form: 275 Longitudinal Control When: then full state variable feedback is involved. If: applying a control law such as eq. (9.14) to the aircraft dynamics represented by eq. (2.108) results in the control law has resulted in a change in the dynamic response of the control law such as eq. (9.14) to the aircraft dynamics represented by eq. (2.108) results in the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a
change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in a change in the dynamic response of the control law has resulted in the dynamic response of the control law has resulted in the dynamic response of the control law has resulted in the dynamic response of the control law has resulted in the dynamic response of the control law has resulted that only a limited number of forms of linear control laws, involving such motion variables as pitch attitude, 0, change in forward speed u, height h, and flight path angle y, are dealt with in Chapter 10. It should be noted here that control designs involving full state variable feedback are a mixture of SAS and attitude control; they are dealt with, therefore, in the next chapter. There are two methods excepted: pole placement and modelfollowing, for, even though the resulting control law in each case is one involving full state variable feedback (FSVF), the design intention is to improve the basic stability of the aircraft dynamics. It should be noted also that the control law of eq. (9.19) involves the feedback of more than a single motion variable: = a, cg w- Uoq + Z,W + K, ZGESE S, = (1 - K, Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' :: (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U + K, Z,W(1 (9.21) = Z,U .' : (9.22) (9.23) z -K, ~Z ~ ~) - 'KZ,U measured at the aircraft's c.g. but at some other station, XA, then: 276 Stability Augmentation Systems = (z, xAMU)u + (Z, - xAMQ)u + (Z, stability of the controlled aircraft. 9.4.2 Pitch Rate SAS The stability derivative which such systems try to augment is M,, and, thereby, the damping ratio of the short period motion is increased. The block diagram of a typical, conventional pitch rate SAS is shown in Figure 9.2. The feedback signal is obtained from the rate gyro used to measure pitch rate, q. Since there is a sign change present inherently in the aircraft dynamics associated with the relationship of pitch rate to elevator deflection, the feedback signal is added to the A number of books and papers show a sign change command signal, q,,, between the signal representing commanded elevator deflection, SEc, and the actual angular displacement of the elevator, SE. In such cases, the feedback voltage, vf, is shown to be subtracted from the command signal. Controller Actuator Aircraft dynamics Rate gyro - - K vf(4 4 * K,,,, = 0.01745 rad V-' (Io V-') K, = 5.73 V rad-' s-' Figure 9.2 Pitch rate stability augmentation system. From Appendix A, it can be seen that a typical value for the sensitivity, K,,, of a rate gyro is 100 mV degree-' s-' (5.73 V rad-' s-l). The problem is solved when some suitable choice of Kc,,, is made to cause the damping of the short period motion to be increased. The usual assumption involved (but it remains no more than an assumption) is that a control system, typified by Figure 9.2, affects only the short period motion of the aircraft. Its phugoid motion is assumed to be unaffected by the control and its is also assumed that the phugoid motion does not affect the operation of the SAS. As a consequence, only the short period approximation needs to be used to represent the aircraft dynamics. dynamics in Figure 9.2 corresponds to aircraft FOXTROT at flight condition 3. The actuator dynamics a value of are assumed. It can be deduced from the short period dynamics that Longitudinal Control - 15 - 10 - 5. - 2.0 2.5 3.0 3.5 I 4.0 I 4.5 5.0 Time (s) Figure 9.3 (a) Response of FOXTROT-3 to a (0) = 1'. (b) Pitch rate response of FOXTROT-3 to a (0) = 1'. (b) Pitch rate response of FOXTROT-3 to a (0) = 1'. is shown in Figure 9.3. Note how the phugoid mode is evident chiefly in the speed response. The aircraft's rating is shown as point Z. This point corresponds to a need to Stability Augmentation Systems Figure 9.4 Handling qualities diagram. increase the damping ratio to 0.6 and to ensure that the short period frequency is not less than 6.0 rad s-l. These increases must be achieved by an appropriate choice of Kcon, (which effectively results in Mq being augmented). The control law of the SAS is, therefore: A root locus diagram corresponding to the system represented by Figure 9.2 is shown in Figure 9.5. It can be calculated from that diagram that the required system gain K, is + 0.36. Hence: Figure 9.5 Root locus diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode
diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s) for FOXTROT-3. 279 Longitudinal Control The Bode diagram for q(s)/SE(s)/SE(s) for F dynamics there is a 180" shift of phase introduced at all frequencies, corresponding to the numerator of the transfer function. Consequently, the change in phase angle due to the dynamic terms is about + 130" to - 90" over the range of frequencies. Figure 9.6(b) shows the Nichols diagram corresponding to Figure 9.6(a). The gain margin, being infinite, means that any value of Kcon, will obtain, w is in rad s-' 0.01 1 -300 (b) I -250 I I I -200 -150 -100 Phase angle (degrees) I -50 Figure 9.6 (a) Bode diagram for FOXTROT-3. (b) Nichols diagram for FOXTROT-3. (c) Nichols diagram for FOXT handling gualities specification (point Z) only a single value will suit. Any of the other methods of Chapter 8 can be used, but these methods require FSVF for their synthesis. Hence, the basic uncontrolled system is better represented as in Figure 9.7, and the problem is merely to determine the feedback gain matrix, K, in the control law: Actuator Figure 9.7 Aircraft dynamics Open loop control system. For example, using the pole placement technique of Section 7.3 of Chapter 7, if the desired closed loop poles are chosen to be: phugoid: XI, h2 = -0.004 + j0.04 short period: h3, h4 = -5.0+ j8.0 it is easy to determine the required matrix of feedback gains, namely: From an examination of the relative magnitude of the elements of K i t is tempting to consider that the feedback control law can be represented by: as before, but now K, has the value of 0.3824. That this approximation is inappropriate can be seen from considering the closed loop poles, defined in eq. (9.32), which are achieved when the control law of eq. (9.31) is used, instead of eq. (9.30), which produces the desired closed loop poles of eq. (9.29). phugoid: XI, h2 = -0.0075 f j 0.04 short period; h3, h4 = -4.9 f j 3.3 If, however, eq. (9.30) is approximated to: the closed loop poles corresponding to the short period mode are not greatly affected being: phugoid: hl, h2 = -0.0075 f j 0.025 short period; h3, X4 = -5.0 f j 8.1537Equation (9.33) should be compared with the desired closed loop poles of eq. (9.29). 28 1 Longitudinal Control law defined in eqs (9.28) and (9.30), is shown by the dashed line in Figure 9.8; the solid line represents the same response for the control law defined by eqs (9.28) and (9.31). I1 -0.08 Y 0 I 1 I I 2 I I I 3 I I 4 I I 5 Time (s) Figure 9.8 Pitch rate response for pole placement system. Using the LQP solution is relatively straightforward but depends, of course, upon the choice of eqs (9.34) and (9.33, namely: Q = diag[O.Ol 0.01 0.5 0.21 (9.34) the resulting feedback matrix, obtained from solving the ARE, is: K = [-0.056' 0.046 2.4 18.1441 (9.36) Using the optimal control law results in the controlled aircraft having roots of: phugoid: XI, $h^2 = -0.0934 + j^27.556$ The closed loop response to an initial change of angle of attack of lois shown in Figure 9.9. Note how large changes of pitch rate have been penalized: the peak value at 0.2 s is only - 0.013" compared to a peak value of - 0.07 at 0.3 s for the uncontrolled aircraft. However, the long, drooping response, which has arisen because of the dominant effect of the pitch attitude feedback, can only be reduced by penalizing the use of the elevator less heavily and allowing greater peak values of q. Other choices of Q and G matrices are needed. The dotted curve in Figure 9.9 shows the closed loop response which obtained for an arbitrary choice of weighting matrices. Stability Augmentation Systems Time (s) Figure 9.9 LOP response for FOXTROT-3. The corresponding feedback gain matrix, K, was: 9.4.3 Phase Advance Compensation One common form of an SAS for pitch rate is to use a dynamic control law defined by: Either form (a) or (b) of Figure 9.10 may be used: their characteristic equations will be identical. Consequently, they have identical responses to initial conditions and to atmospheric turbulence. The SAS function is identical whichever structure is adopted. However, the command function will be affected. Figure 9.10(b) is preferred whenever the aircraft FOXTROT at flight conditions 3 (see Appendix B) the corresponding Bode diagram is shown as Figure 9.11, from which it can be seen that the gain and phase margins are both infinite. A phase advance network can safely be introduced; its transfer function is chosen to be: $G_{s}(s) = (1 + s)/(1 + s0.1)$ (9.42) Longitudinal Control Compensation network Elevator Aircraft dynamics - Rate gyro - Kg AFCS operational Actuator Compensation network Figure 9.10 Rate gyro Pitch rate SAS. (a) Series Compensation. (b) Compensation in feedback. Figure 9.11 Bode plot of FOXTROT-3. 284 Stability Augmentation Systems When Kg is chosen to be 0.5 the modified Bode diagram that is also shown in Figure 9.11 applies. The dynamic response of the closed-loop system to an initial change of attack of + lo is shown in Figure 9.12. The use of this phase advance networks tend to make the closed loop system perform less well in the presence of sensor noise. It has been shown in Section 7.2 of Chapter 7 how dynamic feedback controllers may be represented in state variable form. The procedure can be used with the phase advance compensation. Longitudinal Control 285 Let: x A x5 (Note q = x3) then: . SB = Kfi where: K = Afi + Bs, 9.4.4 Additional Feedback Terms Sometimes, to achieve required handling qualities, an additional feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon the normal acceleration measured at the c.g. is included in the feedback term based upon terms more than another expression of a full state variable feedback control law. From the point of Stability Augmentation Systems - Controller and acceleration feedback SAS. view of a flight control engineer, eq. (9.57) is the practical alternative, requiring only pitch rate and normal acceleration to be measured. Both variables are relatively straightforward to measure using rate gyros and accelerometers located at the aircraft's c.g. The block diagram of a SAS, using the control law of eq. (9.57), is represented in Figure 9.13, with its corresponding dynamic response shown in Figure 9.14. This response should be compared with that of the uncontrolled aircraft which is shown in Figure 9.3(b). The closed loop responses shown in Figure 9.8, 9.9 and 9.12 should also be inspected for comparison purposes. Acceleration feedback is generally considered to 'stiffen' the system, i.e. the short period frequency is invariably increased. This can be

appreciated easily by considering FOXTROT-3 controlled by the law of eq. (9.57), where: -0.01 10 I I 1 I 2 I I 3 ' I 4 I I 5 Time (s) Figure 9.14 Response of blended control to a(O) = lo. Longitudinal Control r 7 e - I.' f -- "p, j i ~ l i ~ condition ht 2 1 F ~ l i g hcondition ht 2 1 F ~ l i g hcondition to a(O) = lo. Longitudinal Control r 7 e - I.' f -- "p, j i ~ l i ~ condition ht 2 1 F ~ l i g hcondition ht 2 1 F ~ l i g hcondition to a(O) = lo. Longitudinal Control r 7 e - I.' f -- "p, j i ~ l i ~ condition ht 2 1 F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondition ht 2 I F ~ l i g hcondit hcondition ht 2 I F ~ l i g hcondit hcondit h Response of pitch rate SAS for four flight conditions. The corresponding eigenvalues of the controlled system are: phugoid: XI, X2 = - 0.0066 f j0.005 short period: h3, X4 = - 26.4 f j43.55 In American papers, a control law such as eq. (9.57) is often referred as 'blended feedback control'; such a control law is usually used to achieve, as nearly as possible, invariant flying qualities throughout the flight envelope of the aircraft. The closeness with which this ideal is approached depends upon the ratio of the feedback gains, K, and K, . Usually, at low dynamic pressures (i.e. 1 1 2 ~ ~; being not very large) the controlled aircraft is arranged to behave as if it were a pure pitch rate SAS; at high dynamic pressures, the system behaves more noticeably as a normal acceleration control system. Such blended feedback systems can 'mask' the natural ability of an aircraft to provide an illustration of this point, Figure 9.15 shows the transient response of the aircraft FOXTROT for all four flight conditions of the same four flight condition 3. The transient responses for the same four flight conditions of the same four flight conditined four flight condit in Figure 9.16. It can be seen how effectively the blended control law of eq. (9.57) has provided invariant response, and this would ensure that the aircraft traversed the region confined in the flight envelope. The performance of such systems is greatly affected by the sensor locations; it has importance of pitch rate command and stability augmentation systems cannot be overemphasized. It can be debated with considerable force that pilots rarely, if ever, require from an aircraft to respond in some acceptable way to a manoeuvre command. In the UK what is referred to as the RAE principle (since the idea was developed in the 1970s in the Flight Systems Division of the Royal Aircraft Establishment at Farnborough) shows that a manoeuvre demand, essentially a commanded acceleration, is simply a scaled version of a pitch rate command. A block diagram of the principle is shown in Figure 9.17, from which it is easily seen that: However: If the aircraft is stable & + 0 as t -+ m. Therefore: Figure 9.17 Manoeuvre demand. Longitudinal Control, it was remarked how pitch rate response was dominant at low dynamic pressures, whereas at high q, the chief response was in acceleration. To maintain the relationship of (9.60) over the range of flight conditions, it is necessary to arrange that: K u;' (9.64) In other words, the scaling factor, K, must be scheduled. 9.4.5 SAS for Relaxed Static Stability Aircraft By relaxing the static stability of an aircraft it becomes possible to effect a considerable improvement in the manoeuvring performance of an aircraft shifts aft (see Figure 9.18). Were a gust to cause the nose of the aircraft in Figure 9.18(a) to move upwards, the angle of attack of the wing would increase and that increase would result in an increase in the lift. Since this lift would be acting behind the aircraft's c.g., the resulting moment would cause the aircraft's c.g., the resulting forces and it would result in less drag, greater manoeuvrability, and, because the stability requirements have been eased resulting in smaller control surfaces with less weight, better fuel efficiency. In Section 3.3 of Chapter 3 it is shown that: where A?, is the location of the aircraft's c.g. (expressed usually as a percentage of the m.a.c.) and i, is the location of the aerodynamic centre (also expressed as a percentage of the m.a.c.). As long as Cm is negative, the aircraft is statically stable; as the c.g. moves further aft, Cm bgcomes positive and the static stability is lost. For the fighter aircraft BRAVO, gf Appendix B, the following parameters relate to the statically stable state: In Appendix B it is seen that for aircraft BRAVO flight conditions 2 and 3 are identical, except that the location of the c.g. for flight condition 2 is forward of the aerodynamic centre, whereas for flight condition 3 it is aft. BRAVO-2 is statically unstable. The corresponding eigenvalues are: Stability Augmentation Systems Lift (due to wing and fuselage) W=mg (a) C Lift (due to wing and fuselage) Tail download a f Tail upload Figure 9.18 Static stability. (a) Conventional. (b) Relaxed. BRAVO-2 phugoid: XI, X2 = -0.005 short period: X3, X4 = -0.99 + j0.068 + j1.47 BRAVO-3 phugoid: XI, X2 = -0.005 short period: X3, X4 = -0.09 + j0.068 + j1.47 BRAVO-3 phugoid: XI, X2 = -0.005 short period: X3, X4 = -0.09 + j0.068 + j1.47 BRAVO-3 phugoid: XI, X2 = -0.014, Xg = -0.025 short period: X3, X4 = -0.09 + j0.068 + j1.47 BRAVO-3 phugoid: XI, h2 = -0.014, Xg = -0.025 short period: X3, X4 = -0.09 + j0.068 + j1.47 BRAVO-3 phugoid: XI, h2 = -0.005 short period: X3, X4 = -0.09 + j0.068 + j1.47 BRAVO-3 phugoid: XI, h2 = -0.005 short period: X3, h2 = -0.005 sh oscillatory, and has become an unstable motion, comprising two real modes, one convergent and the other divergent. In this condition, the phenomenon of 'pitch-up' is likely to occur: any tendency of the angle of attack to increase goes on increasing rapidly until the aircraft stalls, with the pilot unable to control the corresponding pitch-up. A more complicated SAS is then required, and one type which is effective is that referred to as a pitch orientation control system. Another system, which is effective is that referred to as a pitch orientation control system. Another system, which is effective is that referred to as a pitch orientation control system. gyro Figure 9.19 Pitch orientation control system. envelope, is to 'wash out' both the proportional feedback and the integral term which operates on the washed-out pitch rate. Washing-out is a method of permitting to be transmitted only the changes which are occurring in some variable, and blocking any steady value. A block diagram of a pitch orientation control system is shown in Figure 9.19. Note that the inner loop is a conventional pitch rate SAS, but its commanded and the achieved pitch rates. Any of the methods of Chapter 7 can be used to obtain suitable values of K, and K l. Using: then, from Figure 9.19: sE = -20SE + 20SE tiEc= qC + 1.5q 4c = 10q + 10qcomm Let: SE !L (9.73) 3% qc 4 x6 then, if: (9.74) A [U q u A qcomm f then: CX 8 8~ qc]' (9.75) (9.76) 292 Stability Augmentation Systems - 0 0 0 - 0.0475 0 1.28 0 - 0.0114 0.0179 0 - 0.113 - 0.723 1 A2 = B2 = [0 0 - 9.81 0.07 - 2.26 0 0 1 0 0 0 30 0 0 0 10 - 13.01 0 0 0 20 20 0 0 0 - 9.81 0.07 - 9.81 0.07 - 2.26 0 0 1 0 0 0 30 0 0 0 10 - 13.01 0 0 0 20 20 0 0 0 - 9.81 0.07 - 2.26 0 0 1 0 0 0 30 0 0 0 10 - 13.01 0 0 0 20 20 0 0 0 - 9.81 0.07 - 9.81 0.07 - 2.26 0 0 1 0 0 0 30 0 0 0 10 - 13.01 0 0 0 20 20 0 0 - 9.81 0.07 - 9.81 0.07 - 2.26 0 0 1 0 0 0 30 0 0 0 10 - 13.01 0 0 0 20 20 0 0 - 9.81 0.07 - 9.81 0.07 - 2.26 0 0 1 0 0 0 30 0 0 0 10 - 13.01 0 0 0 20 20 0 0 - 9.81 0.07
- 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9.81 0.07 - 9 0 0 0 -0.0475 0 0 -12.225 0 0 0 20 20 0 0, - 0 0 0 101' For flight condition 3: - 0.0116 - A3 - 0.017 0 0.113 - 0.723 0.06 = - 1 1.4772 -1.11 0 0 1 0 0 0 30 0 0 10 0 - - B3 is identical to B2. The two responses, obtained for the values of gains quoted and for a commanded step pitch rate of 1"s-l, are shown in Figure 9.20. Note how the control provides good dynamic response for the two cases of static stability. 1.21 0 I I 1 I 2 I I 3 I I 4 I Time (s) Figure 9.20 Step response of pitch orientation system. I 5 Other LongitudirialAxis SASs 9.5 Since it is known from the short peroid approximation that: then if Mw and Mk are both augmented, Ssp and o,, can be increased. One of the easiest ways of augmenting M, and M, is to use as an elevator control signal a feedback signal based upon the angle of attack and its derivative. M, is negligible, use of eq. (9.83) as a control law will have little effect on the phugoid mode. The method is not used very much, despite its effectiveness in augmenting both the damping and the frequency of the short period mode, since the stabilization reference for the system is the relative wind, i.e. the control system would cause an aircraft to rotate after a disturbance into a new relative wind direction. Essentially, the system tends to hold constant both the load factor and the angle of attack of an aircraft. Furthermore, it is relatively difficult to satisfactorily sense a and &; usually measured at some other location, XA. Hence, By manipulation of the appropriate transfer functions it is easy to show that acceleration and angle of attack are directly related by a simple proportional relationship holds over only a limited range of frequency. If the acceleration at this proportionality factor: /a [Zw - (ZG ~ / MG ~) M ~ I U O (9.86) It must be understood that this proportional relationship holds over only a limited range of frequency. If the acceleration at an end angle of attack are directly related by a simple proportional relationship holds over only a limited range of frequency. If the acceleration at a non-construction at this proportional relationship holds over only a limited range of frequency. If the acceleration at a non-construction at this proportional relationship holds over only a limited range of frequency. If the acceleration at a non-construction at a non-construction at the acceleration at this proportional relationship holds over only a limited range of frequency. If the acceleration at a non-construction at the acceleration at this proportional relationship holds over only a limited range of frequency. If the acceleration at a non-construction at the acceleration at a non-construction at the acceleration at the acce instantaneous centre of rotation, i.e. it is a point, a centre of percussion, at which, as a result of some deflection of the elevator results in an initial vertical acceleration (owing to Z6 SE) which is just balanced E by the pitching acceleration term, xAg. However, this centre of rotation will shift as the aircraft's c.g. shifts in flight, so that a location close to XA, as defined by eq. (9.87), is the best possible practical solution. Stability Augmentation Systems 294 If, instead of angle of attack, normal acceleration is used in a feedback control law, it should be appreciated that the phugoid mode will also be affected Usually the undamped natural frequency of the phugoid mode is decreased. If the aircraft is operating at some flight condition at which one of the zeros of the transfer function relating normal acceleration deflection is negative then, if the aircraft is operating at some flight condition at which one of the zeros of the transfer function relating normal acceleration feedback signal is not washed out, instability (of the phugoid motion) may occur Angle of attack sensors are available, but their use is confined chiefly to military aircraft at present. However, at high speeds, angle of attack can be computed using the signals from a vertical accelerometer and from the air data unit, if the aircraft is equipped with one. The method is also used in aircraft with angle of attack sensors to give some redundancy to a signal of primary importance. The lift coefficient of the aircraft is given by: where q, m and S denote the usual quantities. But: If the parameters m, S, CLOand CL are stored in a computer, the angle of attack can be computed using the measure "ments of normal acceleration, obtained from the acceleration, and dynamic pressure, 4, from the air data unit. 9.6 SENSOR EFFECTS It is important to understand that the sensors to provide the required feedback signals. In every case it has been entirely correct. Since the most usual sensors for SASS are linear accelerometers or rate gyros, and, in the case of the pitch orientation control, an integrating rate gyro, it is essential to know what are the effects upon the AFCS performance if the assumptions do not hold. 9.6.1 Rate Gyro It is usually quite simple to locate and align pitch rate gyros, except when the aircraft structure deforms easily, in which case special care must be taken to avoid Sensor Effects 295 locating the rate gyro, is given by: q = 0 cos + + @ cos 0 sin + (9.91) If a vertical attitude gyro is used it measures the Euler axis rate (not the bodyfixed rate) and measures: 0 = q cos+ - r s i n + (9.92) At large bank angles, the rates measured in body-fixed axis and Euler axis systems are not equivalent and cannot be zero simultaneously. This fact has great importance for pitch attitude control (see Chapter 10). Of great significance, however, to the performance of SASS is what happens when the sensor saturates, i.e. its output signal is limited. This can result in special problems if the command signal does not simultaneously saturate, there results a sudden error command (see Figure 9.1) which, provided the control surface actuator is not saturated, increases the manoeuvre. This can result in pitch-up, an attendant loss of pitch rate damping and, of equal significance, the stick force per g is reduced. Such an effect is sensed by the pilot as an impairment of the aircraft's handling quality. Similarly, if the AFCS commands a control surface deflection greater than its authority, there is also an impairment of the flying qualities. In effect, both these limitation problem is very much worse for the pitch orientation control, since the integration in the forward loop continues to increase the command signal to the feedback-limited inner loop SAS. 9.6.2 Linear Accelerometers are usually mounted rigidly in an aircraft with the sensitive axis perpendicular to an axis usually chosen to be nearly horizontal when the aircraft is in cruise flight. It is necessary to bias the output signal from the accelerometer to allow for the acceleration component of l q due to gravity; otherwise the accelerometer will not be properly sensing changes from level flight, which is at 1q. The static output from the accelerometer is approximately: O0 is the steady value of the pitch angle of the accelerometer relative to the gravity vector. Therefore, in unaccelerated, non-level flight the feedback signal which results from the accelerometer is (1 - cos O cos +); a command signal is then needed to prevent the feedback signal from producing, via the controller, a 296 Stability Augmentation Systems control surface deflection which will return the aircraft to a level flight path, i.e. at 1g. When the value of the normal acceleration being sensed is less than 1g it must be arranged that the sign of the feedback signal from the accelerometer is such that the control surface deflection produce a nose-down manoeuvre is arranged. For unaccelerated descent (i.e. nose-down attitude) such a feedback arrangement tends to increase further the climb attitude. If any integration is present in the forward loop, or if there is automatic trim actuation available, the effect of the acceleration feedback can be hazardous since a divergence in both flight path and speed can obtain which, if uncorrected, can lead to the aircraft's stalling. Obviously, it is an unsatisfactory arrangement to provide command inputs, via the stick, say, in steady flight conditions; in general aviation aircraft and commercial airliners this need is circumvented by adding an electrical trim command input to the SAS (see Figure 9.1). In high performance military aircraft, when manoeuvring flight is the principal means of accomplishing the aircraft's mission, it is impractical trim signal to offset every change in the gravity component: stick commands are used, and, in constant g manoeuvres, such as 360" rolls, the accelerometer feedback results in the pilots having to provide considerable longitudinal motion of the accelerometer. Assuming a rigid aircraft, the full output (in units of g) from a biased accelerometer, expressed in terms of a stability axis system is given by: n, = 1 - COS 0 COS c+ azcg + (p r - 4)lx + (qr + p)l, + (q2 + p2)lZ (9.94) g I, I, and 1, are the distances between the sensor's location and the c.g. of the a i r ~ r a f t Generally, .~ the lateral offset, I, is usually small and can be neglected. The troublesome terms in eq. (9.94) which are significant are - 41, and p21x. If the accelerometer is located at the point where elevator deflection produces pitch rotation, i.e. at: then the high frequency zeros in the aircraft's transfer function relating normal acceleration to elevator deflection are effectively moved to infinity in the splane which makes simpler the task of maintaining stability of the closed loop system. The effects caused by I, can also be great, surprisingly so when rolling manoeuvrable aircraft, flown at a high angle of attack, is considered, with an accelerometer located forward of the
aircraft's c.g. along its Sensor Effects Accelerometer input axis -7 Figure 9.21 (a) Location geometry for accelerometer input axis -7 Figure 9.21 (b) A typical gain schedule. large. See Figure 9.21(a). From the figure it is evident that: I, = $(9.96) \times \cos 0$ In the sensor's axis the accelerometer is located above the accelerometer will be rectified because $\cos 2 0$ and p 2 are both even functions. Thus, whenever the accelerometer is located above the roll axis, OX, of the aircraft, the term of eq. (9.97) will cause a feedback signal which will result in an upward deflection of the elevator. At large values of angle of attack such a tendency results in prostall, a condition of wing rock. It is also inimical of recovery in oscillatory spins. 298 9.7 Stability Augmentation Systems SCHEDULING It can be seen from the data presented in Appendix B how the characteristics of an aircraft exhibits a response as nearly invariant as possible throughout its flight envelope. Consequently, the use of the types of SAS discussed in this chapter, with fixed gains and forms, is unlikely to satisfy this preference. If the gains, for example, are left fixed at values designed for one condition, then the closed loop response at the other flight conditions will be different from what is required (see Figure 9.15, for example). To overcome this deficiency, gain or sensor scheduling is frequently used. Gain scheduling means that the gain in a control law, say K,, is changed as height or speed, or (rarely) both, change. How the gain is 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the AFCS function, but a representative 'scheduled' will depend upon the aircraft, and the air 1. Thereafter, it reduces uniformly at 0.00125 m-I s-' until the forward speed reaches a value of 0.1, remaining constant at that value as the aircraft speed increases. At 10 000 m the same constant value of 0.1, remaining constant at that value as the aircraft speed of 300m s-l. The same slope is used so that the gain schedule starts to operate at 60m s-' for this aircraft height. It is quite common to schedule either the gain of the controller, or the sensor sensitivities, with dynamic rather than with just speed or height. It is quite common to schedule either the gain of the controller, or the sensor sensitivities, with dynamic rather than with just speed or height. Introduction In conventional aircraft, there are usually three, relatively independent modes of lateral motion: roll, spiral and dutch roll. These modes correspond to a well damped response in roll rate, p, to a long term tendency either to maintain the wings level or to 'roll off' in a divergent spiral, and to 'weather cocking' directional stability. However if the dihedral effect of an aircraft is high, the roll damping is low, i.e. Lk + 0 and, as a consequence, the corresponding roll and spiral modes may converge into that single, rollspiral, oscillatory mode referred to as the lateral phugoid (see Section 3.7). If the dutch roll mode of such an aircraft is also very lightly damped, then its piloting can become very difficult, particularly in the execution of co-ordinated turns, for which there must not exist sideslip motion. It is evident, therefore, that if an aircraft is deficiencies. Three types of SAS are commonly used for lateral motion: yaw damper, roll damper and spiral mode SAS. Lateral Control 299 Because lateral motion in conventional aircraft is controlled by the simultaneous use of two independent control surfaces - the ailerons3 and the rudder - lateral motion studies are more involved than those in consequently, less satisfactory than those obtained in studies of longitudinal motion. Nevertheless, such approximations provide useful insight into the physical problem. However, the three SASSmentioned involve the use of both control surfaces is dealt with in Chapter 10. 9.8.2 The Yaw Damper Few aircraft have a degree of inherent damping of the dutch roll motion, with some coupling into the rolling motion, there are used, the lack gives rise to oscillatory yawing motion, with some coupling into the rolling motion, there are used in Chapter 6. As a result, whenever their rudders are used, the lack gives rise to oscillatory yawing motion, with some coupling into the rolling motion, there are used in Chapter 6. As a result, whenever their rudders are used in Chapter 6. As a result, whenever their rudders are used in Chapter 6. As a result, whenever their rudders are used in Chapter 6. As a result, whenever their rudders are used in Chapter 6. As a result, whenever their rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, whenever the rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapter 6. As a result, when rudders are used in Chapte significance of which depends upon the relative size of the stability derivative, Li (see, for example, Figure 9.23(a)). The use of an SAS to artificially increase the damping, by augmenting N : , is universal. A block diagram of such a yaw damper, using proportional feedback, is shown in Figure 9.22. The aircraft dynamics correspond to CHARLIE-4 of Appendix B and were obtained from the two degrees of freedom approximation. Once again the actuator's dynamics have been represented here by a simple, first order, transfer function. Such an approximation is very much less satisfactory than in the case of the pitch rate SAS, for example, since the response of the rudder actuator is less rapid than those of the other control surfaces, and a more powerful, but consequently more sluggish, actuator is required to be used. Rarely should a mathematical model of the dynamics of a rudder actuator of order less than two be used. For verisimilitude, the actuator may be required to be represented by a fourth or fifth order transfer function and it may possibly have to include a number of significant, non-linear characteristics. To illustrate the principles of operation of the yaw damper, however, the gross simplification used in Figure 9.22 will be retained, for a little while. The actuator has been assumed to provide one degree of rudder deflection per 1 V input, and to have a time constant of 0.25 s. The effects upon the response of the yaw damper of a higher order representation of actuator dynamics are shown later in this section. The sensitivities of the yaw damper of a higher order representation of a higher order representation of a higher order deflection per 1 V input, and to have a time constant of 0.25 s. The effects upon the response of the yaw damper of a higher order representation of a higher order deflection per 1 V input, and to have a time constant of 0.25 s. the rate gyro used in the feedback is 0.1 V degP1. The controller gain, Kc, has to be chosen to ensure that the closed loop response results in dutch roll motion, relating yaw rate, r, to rudder deflecction, SR, for CHARLIE flying qualities. 4 is given by: Stability Augmentation Systems Rudder actuator I Controller Rate gyro I Figure 9.22 Yaw damper block diagram. It can be seen that the damping ratio is less than 0.1 which is too small a value to result in acceptable dutch roll motion. The objective of using the yaw damper is to ensure that the damping ratio of
the resulting controller motion is much larger, say about 0.4 or 0.5, with possibly an increase in the corresponding natural frequency. The dynamic response of the uncontrolled aircraft CHARLIE-4to an initial disturbance in the yaw rate of los-' was obtained from a simulation of the complete equations of lateral motion and is shown in Figure 9.23(a). The response of the same uncontrolled aircraft to an initial disturbance in roll rate of 1"s-' is shown in Figure 9.23(b). Note the oscillatory reT6nse which is predominant in all the motion variables shown in Figure 9.23(a). The absence of such oscillatory motion in the same variables shown in part (b) arises solely because the mode which was initially disturbed was the roll mode. Since L: for CHARLIE-4is negligible, there has been no significant coupling of the dutch roll into the rolling subsidence motion. This observation supports the use of approximations in deriving transfer functions for r (s)/SR(s) and (later) for p (range of values of controller gains, Kc, are shown in Figure 9.24. Note that, although the most rapid response corresponds to Kc = ISVIV, it is the one with the lowest damping (although the value is acceptable, being 0.4). Increasing Kc to 196.875 results in the yaw damper's response being unstable. A Kc value of 10 provides a well damped and reasonably rapid response. The effect of the dynamics of the actuator on the performance of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of the yaw damper can be assessed by considering Figure 9.25, in which is shown the response of noted that the yaw damper of Figure 9.22 does not completely remove the effect of the initial disturbance in yaw rate; there are nonzero steady state values. In addition, such a system tends to oppose any change in yaw rate; there are nonzero steady state values. heading, for e ~ a m p l eTo rate, being used as feedback signal to the controller, is first passed through a wash-out network for the purpose of differentiating the signal from the yaw rate gyroscope (see Figure 9.26). Such a filter is easily synthesized by means of active electronic components, such as operational amplifiers. A block diagram Stability Augmentation Systems Time (s) Figure 9.27. Values of Kc and Two are easy to obtain from any of the conventional control system design methods (see Chapter 7). To illustrate a number of features of the results which can be obtained, feedback. Letting: X' = [p p r \$ SR ewol U = rcomm then the yaw damper, with wash-out network, can be represented as: (9.103) S=Ax+Bu where: -0.056 1.05 - 0.465 0.6 A = _0 - 1 0.39 0.032 - 0.115 0.042 0.0022 0 0 0.153 0.475 - 0 0 0 0 1 0 0 0 0 0 - 4 4 6.0 - 0.32 - 1.15 0 - 4.75 - 1.0 Step responses for this yaw damper to a commanded yaw rate of los-' for a range of values of controller gain are shown in Figure 9.28. Note that the washed-out Figure 9.28 Response of yaw dampers care must be taken with the choice of value of the time constant of the wash-out network, for, if it is too short, the yaw damper, having less time to act, is less effective. If it is too long, then stability problems arise. In aircraft of the general aviation (GA) type, arrangements are usually made to allow the pilot to switch out the yaw damper so that it does not operate. In this way the pilot can carry out landing manoeuvres without the rudder pedals being moved automatically and continuously as a result of the action of the yaw damper. Such pedal motion is particularly distracting to a pilot during a flight phase as busy as the approach, and it would occur in GA aircraft since it is customary, as a weight reducing measure, to install any AFCS with the actuator of each control surface in series with the control runs from the primary flying controls. In combat aircraft and large transport aircraft the surface actuators are usually installed in parallel with the control cables or rods. yaw damper in flight to be provided: it operates continuously throughout the flight. 9.8.3 Effect of Tilt Angle of the Rate Gyro Upon the Performance of the sensor has been used so far, the effects of sensor characteristics being deferred until the end of the chapter, the effect of gyro tilt will be considered here since it is an effect which may be used deliberately by a designer to enhance the dynamic performance of a yaw damper. Lateral Control Sensitive axis of rate gyro \ Figure 9.29 Geometry for tilted yaw rate gyro. Normally, a rate gyro is a designer to enhance the dynamic performance of a yaw damper. Lateral Control Sensitive axis of rate gyro. c.g. However, this will hold at only a single flight condition; at others, the gyro will be aligned in the fashion represented in Figure 9.29. The output signal from such a rate gyro, usually a voltage, may be denoted as : v, = O.l{r cos(a + aR) + p sin(a + aR)} (9.106) In a yaw damper this signal is used for feedback: a block diagram is shown in Figure 9.30. By increasing the tilt angle (aR) more aftwards (i.e. aR is increasingly negative) the dutch roll damping may be further increased. The technique is p(s) = O.1(s-2.83) (s2 0.19s + 1.04) &(s) + Aircraft dynamics sin(a+aR) Wash-out network e, (s) S s+11T, Controller e~(s) Kc Yaw rate gyro cos(a+aR) +- L Figure 9.30 Block diagram of yaw damper with tilted gyro. Stability Augmentation Systems Figure 9.31 Response of yaw damper with tilted gyro. often used in high performance aircraft. The effect can be seen in the transient response shown in Figure 9.31 Response of yaw damper with tilted gyro. gyro tilt angle is reduced to zero the response of the system of Figure 9.20 is identical to that shown for Kc = 10 in Figure 9.28. 9.8.4 Roll Rate Damper This type of AFCS is usually fitted when the roll performance of an aircraft is considered to be inadequate, by which it is meant that the time to attain a desired value of roll rate is too long. The roll rate damper augments the stability derivative, LL, thereby reducing the response time of the aircraft. This SAS is seldom used as a command controller on its own, but rather as an essential inner loop of another lateral AFCS. The customary assumptions about the dynamics of aileron actuator and the associated rate gyroscope are involved: both are assumed to act instantaneously, the aileron actuator having a gain, K,,,, and the rate gyro a sensitivity of K,. A block diagram of a typical roll rate, p, and the aileron deflection, ti, and derived from the single degree of freedom approximation. From the block diagram it can easily be shown that: Lateral Control)n actuator Aircraft dynamics P(S) I Controller Rate gyro I Figure 9.32 Roll rate damper block diagram. where: If the designer can arrange that KaciL&, KCKp > 1 / T R then: Such a reduction in the time constant of the system results in an improvement of the dynamic response of the aircraft's rolling motion, which often also has a beneficial effect upon the dutch roll motion. The practice of using the single degree of freedom approximation to represent the aircraft dynamics is almost universal; how justified it is depends upon the nature of the aircraft being studied. Figure 9.33 shows the roll rate responses of CHARLIE-4 to an initial disturbance in roll rate of 1"sC1; one response relates to the single degree of freedom approximation, the other to the motion. It is apparent that not much is lost in using the simpler form to represent the aircraft. The corresponding roll rate response obtained from the roll rate damper, with a value of controller gain of 30.0, is shown in Figure 9.32. The settling time for the basic aircraft is 10 s; for the roll damper, the response is an order faster. However, it must be appreciated that the roll rate damper does not affect the initial rolling acceleration which is available, although it does reduced, thereby causing the aircraft can produce. Hence, the bank angle reached in some specification of flying Stability Augmentation Systems 0,1 1approximation (a) '-.-Time (s) Figure 9.33 Roll rate response for CHARLIE-4. qualities. In such a case, more aileron control power is needed, i.e. the product LE, must be exercised in locating the rate gyro on the aircraft. Usually it is
mounted with its sensitive axis aligned with the centreline of the centr aircraft. However, since an aircraft rolls about its velocity vector, there is a misalignment between this roll axis, which is directly related to the aircraft's angle of attack. The voltage output signal from a rate gyro being used to sense roll rate is given by: vp = ps cos (Y - r, sin (Y (9.113) where the subscript 's' is used to denote that the variable has been measured in the stability axis system. Generally, it is true for conventional aircraft that: p, cos (Y S=-r, sin a (9.114) Hence: vP = p, cos (Y (9.115) and the effect upon the operation of the roll rate damper of such a misalignment is that the feedback gain is modulated by the instantaneous value of the aircraft's angle of attack 9.8.5 Spiral Mode Stabilization The method to be described is particularly effective in stabilizing a spiral mode. The three degrees of freedom approximation relating yaw rate, r, to aileron Lateral Control -0.2 1 0.0 I 0.5 I 1.0 I 1.5 I 2.0 I 2.5 Time (s) Figure 9.34 Roll damper and uncontrolled roll rate response. deflection, a*, is usually used, i.e.: For a number of aircraft types, however, the stabilization system is represented in the block diagram of Figure 9.35 in which the aileron actuator is assumed to be adequately represented by a simple gain of loV-' so that the actuator block is subsumed in the controller. Furthermore, the yaw rate gyro is assumed to have a sensitivity of KR VI0/s. It can then be deduced from Figure 9.35 that: Stability Augmentation Systems. Hence, the natural frequency of the closed loop system is reduced, thereby increasing the damping, which is the desired result. Note that if, for some particular aircraft, N' and Y; are not negligible, every coefficient of the S.A denominator polynomial of eq. q9.118) would be altered. If Nk is negative (i.e. an adverse yaw effect), the gain (K,,,KR) cannot be made ar6itrarily large without causing the dutch roll motion to be unstable. If, however, a proverse yaw effect is evident, i.e. NkA is positive, the damping of the dutch roll motion is augmented by the spiral mode stabilization. Often spiral mode stabilization is obtained by means of a kind of 'piggy-back7operation involving the yaw damper: the feedback signal from the yaw rate gyro is also used to drive the ailerons. That technique is referred to as aileronlrudder interconnection (ARI) or control systems which are closed loop control systems used on aircraft to remedy those deficiencies in flying quality which are due to basic aerodynamic or geometric inadequacies in the aircraft. Feedback control is used to augment some particular stability derivatives, thereby improving the parameters which directly govern the flying qualities. Both lateral and longitudinal motion systems have been considered and the most common types of SAS, such as pitch, roll and yaw dampers, are treated. A number of methods of designing such SASShave been discussed and the effects on the closed loop performance of actuator and sensor dynamics have also been dealt with. SAS are important since they invariably form the innermost loop of an integrated AFCS. Exercises 371 9.10 EXERCISES 9.1 The linearized equations of perturbed lateral motion for a Tristar (L-1011) passenger aircraft in a cruising flight condition are given by: fi = -0.13P - r + 0.04 + +0.02SR + = p Using appropriate approximations, design a yaw damper to increase the damping of the dutch roll mode from its uncontrolled value of 0.14 to a new value of 0.67. Calculate the natural frequency of the yaw damper. 9.2 The short period dynamics 'of a fighter aircraft are represented in the s-plane diagrams of Figure 9.36. Design an SAS (ignoring actuator dynamics) to obtain a closed loop damping ratio of 0.6. 9.3 A VTOL aircraft of the AV8B type has the following linearized equations of motion: + X8,SE + XsTST + XsNSN + (Uo Zq)q+ZS,SE+ZsTST+ZsNSN+Mqq+Ms,SE+Ms,ST+MS,SN u = X,U - g 0 w = Z,W 4 = M,w where u, w, q and 0 represent the changes in forward velocity, Figure 9.36 Pole zero map for Exercise 9.2. 3 72 Stability Augmentation Systems pitch rate and pitch attitude respectively. The control inputs are represented by the elevator deflection, SE, the change in thrust, ST, and the deflection of the reaction nozzles, SN. At hover, the corresponding stability derivatives are as follows: Xs,=-0.56 Zs,=-0.1 Ms,=O.O It is required that in hover the vertical velocity be controlled such that it has a characteristic equation of the form w 5w = 0. (a) Design a feedback control system to achieve this requirement. (b) Sketch a block diagram of the resulting control scheme. (c) If the aircraft is equipped with sensors for pitch rate and angle of attack, discuss the consequences for the synthesis of the control law determined in part (a). + 9.4 A strike aircraft has the following linearized equations of lateral motion: p = -0.lp - r + O. 1 + + 0.048, p is the sideslip angle, in radians p is the roll rate (rad s-l) r is the roll rate (rad s-l) r is the roll and yaw rates of the basic aircraft are to be directly controlled to improve the handling qualities. The desired handling qualities are assumed to be those obtained from the dynamics of some ideal aircraft which has the model equation: where Assume that the dynamics associated with the control surface actuators, are negligible. (a) Obtain a feedback control law which will provide the required handling qualities. (b) Show that this law results in perfect matching. (c) By means of a block diagram show how the control law of part (a) could be implemented. 9.5 The state vector of an oblique-winged research aircraft is defined as The aircraft has been provided with five controls such that ur = [6EL 6ER 8AL & AR 6 ~ 1 where aELdenotes left stabilizer eigenvalues corresponding to the phugoid, the short period and the convergent modes of longitudinal motion, and also those corresponding to the heading, spiral convergence, roll subsidence and dutch roll modes of the 3 14 Stability Augmentation Systems aircraft's lateral motion. (b) Find an optimal feedback control law for the weighting matrices Q = diag[1 10 0.1 0.1 10 10 1 1 1 11 (c) 9.6 Calculate the corresponding closed loop eigenvalues. For the oblique winged aircraft of Exercise 9.5 it is desired to have the closed loop dynamics match those characterized by the vector differential equation k, = L x, where L is defined as: L = diag[-2.0 -0.5 -1.0 -4.0 -10.0 -3.5 -3.5 -5.0 -5.01 (a) Find a feedback control law to achieve these model dynamics. (b) Calculate the eigenvalues of the resulting closed loop system. (c) How do these values compare with the eigenvalues of the model aircraft? 9.7 A hypothetical aircraft is considered to have the following matrices when it is flying at a height of 6 000 m and a Mach number of 0.8. The state prototype fighter aircraft has been built with a vertical fin of reduced size such that the directional stability derivative, Nb, becomes negative and the derivatives N i , N&, and Lb are reduced in amplitude. The change of sign of Nb makes the aircraft directionally unstable. (a) The prototype aircraft has been flight tested at sea level and at a Mach number of 0.4 and the following stability derivatives were determined: $Y_{r} = -0.2$ N& = -2.0 N h = -2.0 N evaluate the transfer s), thence design a yaw damper to improve the dutch function, r (~) / 6 ~ (and roll mode. (Note that the stable roots of the corresponding stability quartic are both real). (d) Will the feedback control law which will result in the controlled aircraft being stable. (f) Discuss in operational terms the benefits which relaxed static stability (see Chapter 12) is likely to bring. (c) 9.9 (a) A flight control system has been designed for the aircraft ALPHA-2 to reduce the acceleration experienced as a result of encountering turbulence. It is necessary to provide as a feedback signal some measure of the angle of attack, but no suitable sensor is available. An accelerometer is used, but it has to be located 1.55 m ahead of the aircraft's c.g. Show that the measured value of the initial normal acceleration in response to a step deflection of the elevator? 316 Stability Augmentation Systems 9.11 NOTES 1. All-electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric airplanes, proposed by the Americans for development by the year 2000, will remove such actuators and use electric actuators are proposed by the Americans for the proposed by the propos with the distances lxp and lzp, quoted in Appendix B, which represent the distances between the pilot and the c.g. of the aircraft. In a number of high speed aircraft, spoilers are used instead of the ailerons; the principles to be discussed are not materially affected, whichever is used. In some of the earlier, single-axis, autopilots which consisted solely of yaw rate feedback to the rudder, it was because they did oppose the almost steady turn associated with an unstable spiral mode that they were
successful. 2. 3. 4. 9.12 REFERENCE 1987. Eighty years of flight control: triumphs and pitfalls of the systems approach. J. Guid. Cont. 4(4): 353-62. McRUER D.T. and D.C. GRAHAM. Attitude Control Systems 10.1 - INTRODUCTION Attitude control systems find extensive employment on modern aircraft. They form the essential functions of any AFCS, in that they allow an aircraft to be placed, and maintained, in any required, specified orientation in space, either in direct response to command, or in response to command signals obtained from an aircraft's guidance, or weapons systems. It is through their agency that unattended operation of an aircraft is possible. In AFCS work, attitude hold, the commonest function, is often referred to, especially in the USA, as a control wheel steering (CWS) mode. inner loops of attitude control systems; the attitude control systems then form the inner loops for the path control systems, which are discussed in Chapter 11. It is often the case that attitude control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use simultaneously several of the aircraft's control systems meed to use variables other than those being controlled directly. Attitude control systems are, consequently, more complex in their operation than stability augmentation systems. A block diagram of a typical system is shown in Figure 10.1; the assumptions adopted in Chapter 9 about the representations of the dynamics of both the elevator's actuator and the sensor of pitch attitude are still maintained here. Therefore, the feedback control law being considered in this section can be generally expressed in the form: As the feedback gain, KcK,, is increased it is found that the aircraft's short period frequency w,, also increases, although its damping ratio, c,,, increases until the mode becomes over-damped and, consequently non-oscillatory. An early view (c. 1940) that the best results are Attitude Control Systems I Actuator dynamics Aircraft dynamics Controller Attitude gyro I Figure 10.1 Block diagram of pitch attitude control system. obtained when the value of the feedback gain is chosen such that the phugoid mode is critically damped, thereby making the phugoid mode is critically damped. attitude causes the damping of the phugoid mode to increase at the expense of the short period mode. Whenever the feedback signals, being used in an AFCS for longitudinal motion, depend solely upon motion variables which do not result in the augmenting of the stability derivatives Xu, Z,, Ma, or M,, then the total damping of the system is unchanged by the application of feedback. Consequently, the total damping of an open loop system can then be redistributed only among the resulting closed loop modes as a result of linear feedback control. If the phugoid damping is increased, for example, it can only be at the expense of the short period damping. If Kc is so chosen that the phugoid mode is heavily damped then, in the pitch attitude response of the controlled aircraft, the phugoid motion will be almost completely absent. The response of a pitch attitude gyro, KO, being taken as 1 Vldeg. It has sometimes been claimed that whenever the pitch attitude of an aircraft is tightly controlled the phugoid mode are usually real and negative. When the phugoid mode is so heavily damped, any changes, which occur in other motion variables (such as speed and height) as a result of the pitch command signal, are small and the responses associated with such variables are well damped, with long period. It is for such reasons that the use of pitch attitude feedback to the elevator has been, and will go on being, one of the most successful feedback control techniques used in AFCSs. However, such a system is said to be type 0 - see Figure 10.1 - and there must then exist, in response to any step command or disturbance, a steady state error - see Figure 10.2. Moreover, the loss of short period damping to augment the damping of the phugoid mode has resulted in a rather unsatisfactory dynamic response because the stability margins have been degraded. The steady state error can be removed by including an integral term in the control law; the inclusion of this additional term, however, may further reduce the damping of the pitch rate (thereby implementing an SAS function) such that: Pitch Attitude Control Systems -0.4 0 I 1 I 2 3 I 4 I I I I I 5 6 7 8 9 Time (s) 1 1 1 0 Figure 10.2. Response of system using such a control law is shown in Figure 10.3. The use of such a three-term controller is not universal, however, and in many systems the degree of steady state error which exists with Elevator actuator . "(') + + Aircraft dynamics qp) 1 KI - \ Rate - gyro Kd 4 K4 K2 - Atitude l+-, F".. Integrating A" Attitude Control Systems Time (s) (a) Time (s) (b) the chosen values of controller gains, Kc and Kd, is acceptable. As a result, many pitch attitude control systems have a 20 Time (s) Figure 10.4 (a) Step response of system of Figure 10.1. (b) Step response of system of Figure 10.3. (c) Response of pitch attitude. (d) Step response of pitch attitude control with C* pre-filter. 322 Attitude Control Systems Figure 10.4(c) shows the transient response of the same system for the same aircraft and flight condition using control law (10.4b). Note that the steady state error has been removed. These responses should be compared with that shown in Figure 10.2. The improvements, which the use of an integral term and pitch attitude control system using a control law such as (10.3) can produce a better command signal, ,,,8, from a pre-filter which has as transfer function: where the time constants have been chosen to suit the aircraft being dealt with. The resulting step response of the pitch attitude control system which uses this pre-filter will correspond to the C * criterion discussed in Section 6.5 of Chapter 6. With the use of this pre-filter it is often possible to change the values of the step response of the system whose response without pre-filter was given in Figure 10.4(a). For the system corresponding to Figure 10.4(d) the feedback gains were changed to Kq = 5.0 and KO = 5.0. Note the improved response of Figure 10.4(d). aircraft is flying in the presence of atmospheric turbulence, the pitch attitude control system tends to hold the pitch angle at a constant value. This fixity of attitude opposes the natural tendency of an aircraft to nose into the wind, thereby reducing with the gust. The net result of these two effects is that the accelerations experienced in gusty conditions are higher than they would be otherwise, with a consequent increase of the load being imposed upon the structure of the aircraft. When a pitch attitude control system is operating a problem can arise if the aircraft is banked at some large angle. problem depends upon whether a rate gyro or a vertical gyro has been used as the means of providing the feedback signal representing pitch attitude rate in the control law. The rate gyro or a vertical gyro signal is related to the body axis system, i.e.: 8 cos + + 4~cos o sin + 8 = q cos 4 - r sin + q = (10.6) (10.7) At large bank angles these signals q and 0 cannot both be zero simultaneously. Nor are they equivalent signals. For the wings-level flight situation, either gyro can be used with no discernible difference in performance; but in turning flight the system performance will be quite different, depending upon which gyro has Roll Angle Control Systems 323 been used. If the vertical gyro is used, the operation of the pitch attitude system must be restricted to a limited range of bank angles. 10.3 ROLL ANGLE CONTROL SYSTEMS 10.3.1 Introduction Roll angle is generally controlled simply and effectively by the ailerons at low-tomedium speeds on all types of aircraft; on military aircraft, at high speed, spoilers are used. Such spoilers, on the wing of an aircraft is usually produced by a proverse yaw moment as well as producing considerable drag. Roll control for swing-wing aircraft is usually produced by means of control surfaces, moving differentially, and located at the tail. Swing-wings generally contain spoilers to augment the roll control power of the tail surfaces. These spoilers are activated whenever the wings are forward of some value of sweep angle, typically 40-45". Except at high speed, a differential tail is not very effective at producing rolling moments, since the differential deflection which can be applied is necessarily restricted to allow the same surfaces to be
used (symmetrically) for longitudinal control. Associated with the rolling moments produced by this method is a large, adverse yawing moment. sible for the spoilers and the differential tail to produce rolling moments which oppose, and yawing moments. The complete transfer function relating bank angle to aileron deflection is given by: 1 pically, is can be very large; see lable 10.1 Therefore, the spiral mode can correspond to either a slow convergent or a divergent motion. In an early (and excellent) textbook Langeweische (1944) on flying, stated that 'any aircraft which was spirally stable was unpleasant to fly in rough air, for it was wallowy and unsteady and wore you out'. However, for unattended operation, neutral and divergent stability are undesirable since any disturbance can cause an aperiodic, divergent motion of the aircraft, which pilots have referred to as the 'graveyard spiral'. One of the most important functions of any AFCS operating on lateral motion must be, therefore, to attain to a high degree of spiral stability, but it must also improve the other lateral flying qualities so that a pilot is not 'worn out' Attitude Control Systems Table 10.1 Spiral mode time constants TS Aircraft -85.28 23.42 35.34 68.03 ALPHA CHARLIE DELTA FOXTROT 2849.00 111.11 97.09 103.20 534.76 97.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.09 126.58 57.0 achieving good spiral stability is to provide the aircraft with good lateral static stability. To achieve the degree of dynamic stability desired in roll requires the use of a roll attitude control system. Such a control system is a feedback control system is a feedback control system. roll commands from the pilot or a quidance system. For most aircraft, the following assumptions hold: (1) TR T,, and (2) the quadratic term in the denominator. When these assumptions are true, or are nearly so, the aircraft's roll dynamics may be represented by a single degree of freedom approximation: * where: K+ = L i A TR = - (LA)-' p = d+ldt 10.3.2 A Typical System A block diagram representation of a typical, roll attitude control system, in which the actuator response is assumed to be instantaneous, is shown in Figure 10.5. It can easily be shown that: Hence: Roll Angle Control Systems Controller Aircraft dynamics Attitude gyro 1 Figure 10.5 Bank angle control system. For a specific damping ratio of this roll attitude control system, the. value of controller gain needed is given by: It is interesting to consider what must be done to this value of controller gain if it is hoped to maintain the damping ratio of the closed loop system at a constant value throughout the flight envelope of the aircraft. In Section 2.10 of Chapter 2 it is noted that the stability derivatives LA and LbA could be expressed as: Consequently: z A Therefore, the gain must be steadily increased with increase in forward speed. Example 10.1 For CHARLIE-2 it can be shown that: Suppose 5 = 0.6 is required. Therefore: Attitude Control Systems where: w2.4 0.21 Kc But w, = 0.74 rad s-l, hence: Kc = 2.6VIV Figure 10.6 shows the step response of this example system. Time (s) Figure 10.6 Step response of bank angle control system. Time (s) Figure 10.6 Step response of this example system. component of dutch roll oscillation which may lead to serious difficulties for a pilot flying that aircraft. Whenever this situation is likely to arise, the control law is changed from ?iA = Kc& to: where: The additional rate term in eq. (10.18) introduces damping and corresponds to a phase advance term. The corresponding block diagram is represented in Figure 10.7. With a control law such as eg. (10.18), the closed loop system has a transfer function given by: Roll Angle Control Systems Controller Aircraft dynamics ll.-++Fbl Attitude gyro Figure 10.7 Bank angle control system with phase advance. By proper selection of values of Kc and Tc it is possible to achieve the transfer function of eg. (10.21): where: Example 10.2 Consider CHARLIE-2once more. Let: Hence: 0.21Kc = 1/TcT6 Attitude Control Systems Since there are two possible values of T, which can be used, namely Tc = 3.107s or Tc = 0.2742s. If the former value is used, the result is: +(s) 4.41 c o m m (1 + s 1.737) The corresponding value for T+, becomes 1.737, i.e when Tc = 3.107s then: +(s) = c o m m 4.41 (1 + ~0.153) When Tc = 0.2742s. If the former value is used, the result is: +(s) 4.41 c o m m (1 + s 1.737) The response of the system corresponding to eq. (A) is ten times faster than the response obtained from a system corresponding to eq. (B). Hence, system (A) would be the preferred system because the quality of rolling motion from the aircraft would be the preferred system (A) is ten times faster than the response obtained from a system corresponding to eq. (B). Loop Using phase advance compensation is often unsuccessful in practice, in situations where the feedback, or the command signals, are subject to noise interference. An alternative scheme, which permits a designer to use considerable freedom in arriving at the required dynamic performance of the roll angle system, is to employ as an inner loop the roll damper SAS discussed in Section 9.5. The roll damping of the aircraft can be considerably augmented by such an inner loop, to values even greater than that needed by the roll angle system, in order to sacrifice some in the outer loop, to values even greater than that needed by the roll angle system. representing a typical system is shown in Figure 10.8; the actuator dynamics are represented as a simple first order lag. It should be noted that using this technique requires that there be available another motion sensor, namely, a rate gyroscope must be washedout in an appropriate filter. From Figure 10.8 it can be established that the closed loop transfer function of the roll angle controller Kc, - 7 - "(s)- Lki (s-L'p) - P(S) 1 ' S 4(~) - Rate gyro 0.1 t Roll damper Attitude gyro 1.0 Figure 10.8 Bank angle control system with roll rate inner loop damper. By using the roll damper as an inner loop, the frequency of the roll angle systems is associated with locating the sensors to avoid the unwanted effects of structural flexibility. The effect of the aircraft's angle of attack should also be considered. (See the discussion on the roll rate gyro in Section 9.5 of Chapter 9). Example 10.3 For CHARLIE-2the single degree of freedom approximation for rolling motion as a result of aileron deflection can be approximated by the transfer function: If the system used as a roll angle control system is that represented by Figure 10.8, then the corresponding closed loop transfer function is: Kcl and Kc2 can be obtaine~dfrom any of the methods outlined in Chapter 7. System A. If Kc2 is chosen to be, say, 10.0 and Kcl is selected to be 31.55, then the characteristic polynomial of the roll angle system becomes: which is identical ti:) the polynomial which obtained for Example 10.2. Attitude Control Systems 330 However, since phase-advance is not being used, there is no numerator term and the factor (1 s3.107) is not cancelled. As a result, the response of this system, although heavily damped, is sluggish. + System B. A better choice of Kcl is 95.156 (KC2remains fixed at 10.0), for this results in the system being critically damped, i.e.: The step responses for systems A and B are shown in Figure 10.6; the superiority of B is evident from inspection. 10.3.5 Use of a Yaw Term in the Roll Control Law If the control law being used in a roll angle control system is modified to become: 8 ~ =~ Kcl+~+ Kc2y ~ , (10.24) The mode associated with the 'yawing' motion of the aircraft can then become a subsidence mode, with its damping being increased substantially, as Kc2 is increased. The dutch roll damping is decreased, however. From experiment and flight tests, it has been found (McRuer and Johnston, 1975) that the best practical arrangement results when: The step responses of a roll angle control system, used with CHARLIE-2, and using the control law eq. (10.24) for three values of the ratio, Kc{Kc2, are shown in Figure 10.9. It is evident from Figure 10.9. It is evident from Figure 10.9. It is evident from Figure 10.9. These results should be compared
with those shown in Figure 10.9. It is evident from Figure 10.9. These results should be compared with those shown in Figure 10.9. It is evident from Figure 10.9. It is evident aircraft, the pilot usually controls the roll angle indirectly through a CSAS, a commanded role damper, since such CSASs are necessary to assist the aircraft to provide the rapid roll performance which is essential for modern aerial combat, or for evasive manoeuvres during low level strike missions. To achieve the performance required inevitably means the use of high loop gains. Such high values of gain cause a number of problems, although it is worth noting that the gains of such CSASs are often fixed throughout the flight envelope. Among the problems are the following: 1. The command signal from the pilot must usually be 'damped'. If the input signal to the CSAS corresponding to a small deflection of the pilot's stick is too large then pilot-induced oscillations may result. This is particularly Roll Angle Control Systems Time (s) Figure 10.9 Step response of bank angle control systems added. likely when the aircraft is being used on a precision tracking task. This problem is general for any high gain CSAS. The obvious remedy of reducing the value of input signal corresponding to the stick deflection often results in the system's performance being inadequate. 2. When the speed of the aircraft is sluggish. To achieve the roll response required in this condition means that a pilot has to apply large deflections to the control stick. These large values can result in limiting of the feedback signal. Both limiting of the feedback signal. Both limiting conditions with the high loop gain, can result in limiting conditions with the high loop gain. can result in degraded roll performance if the roll control system is not well designed. 3. A system with a too high value of loop gain precludes control of bank angle by use of the rudder, which is a technique often used by pilots in making S-turns during landing, or during manoeuvres in aerial combat. This problem can be overcome by carefully scheduling the control gains with the correct flight parameter. 4. On swept-wing aircraft, as the stall condition is approached, it is essential to reduce the value of the loop gain by a substantial amount to avoid very large deflections of the control surfaces. 10.3.7 Roll Ratchet Caused by Excessive Roll Damping In-flight experiments with modern fighter aircraft have indicated that excessive values of rolling accelerations are experienced by pilots when trying to reach some Attitude Control Systems 332 desired values of rolling accelerations the pilot inust apply more slowly, through the primary flying control, the input to the roll control system. But, frequently, a pilot's reaction is instinctive and sudden, with the result that the closed loop system, formed by the pilot and the aircraft dynamics, oscillatory motion is typically of high frequency (1.8-3.0 Hz) and when it occurs is referred to as 'roll ratchet'. The phenomenon arises with aircraft in which the roll damping is excessive. Suppose the closed loop transfer function of a roll damper system is given by: $+(s) = c \circ m m K s (1 + ST)$ If the damping is large T +0 and eq. (10.26) can be approximated to: $= -K + (s) c \circ r n I s$ When a pilot closes the command loop around a roll damper SAS the system may be represented as shown by Figure 10.10. The form of mathematical model used to represent the pilot's reaction time) of about 0.13s. Therefore: Model of pilot + Kpe-n - Roll damper ~cornrn(~) K - ~ - 4(s) * S Figure 10.10. Pilot-in-the-loop roll ratchet. When the loop gain (K,K) has a value of, say, 12, and the time delay function is approximated by e-" = $(2 - \alpha s) / (2 TS)$, then: + +(s) 92.512(2 - 0.13s) s2 3s + 185 + cornm(s) + Therefore, the system will oscillate with very little damping (5 -- 0.01) at a frequency of 13.6rad s-' in response to a unit step function. The result of applying a unit step function to a digital simulation of eq. (10.28) is shown in Figure 10.11(a). The roll ratchet oscillation is clearly evident, at a frequency of 13 rad s-1. Figure 10.11(b) shows two step responses for the Roll Angle Control Systems 1.0 445.1 0.8 - - ---=- 4comm(s) 0.6 k l2 s+k s+12 - 4 0.4 - 0.2 - 0.0 0 I 2 I 4 I 6 I 8 1 10 Time (s) (a) Roll ratchet frequency o = 12.7 rad s-' -1.0 0 (b) 2 4 6 8 10 Time (s) Figure 10.11 (a) Bank angle control system: pilot reaction instantaneous. (b) Bank angle response with pilot reaction instantaneous. (b) Bank angle response with pilot reaction instantaneous. (c) Bank angle control system: pilot reaction instantaneous. (c) Bank angle response with pilot reaction instantaneous. (c) Bank angle control system: pilot reaction instantaneous. (c) Bank angle response with pilot reaction instantaneous. (c) Bank angle response with pilot reaction instantaneous. (c) Bank angle control system: pilot reaction instantaneous. (c) Bank angle c be seen that roll ratchet is only evident in case B, where T has increased, i.e. the roll damping has been reduced, the roll ratchet vanishes. Readers should refer to Chalk (1983) for further discussion of these topics. 10.3.8 Unwanted Pitching Motion In Section 2.6 of Chapter 2 it is shown that, in a steady turn, there occurs a steady pitch rate, the value of which is: 334 Attitude Control Systems g tan qss = - uo + sin + = r sin + It is necessary to use as feedback a signal proportional to this steady state pitch rate signal being used in the pitch attitude control system will not perform properly in banked turns. This matter is discussed in Section 10.2. To obtain this signal, qss, requires that the output signal from the yaw rate gyroscope be multiplied with that from a resolver driven by a bank angle servomechanism (or the product can be determined in an on-board digital computer). 10.4 WING LEVELLER In small, general aviation aircraft there is a need, sometimes, for a regulating system which will hold the wings level in the presence of atmospheric disturbances. Although any roll angle control system performs this function, in such a class of aircraft the use of a roll attitude gyro may be avoided by means of setting the command signal, to zero and using a tilted rate gyro in a wing leveller system such as that represented in Figure 10.12. This system has proved to be very effective. The principle of the tilted gyro is the same as that explained in Section 9.8. ,,,+, - Aircraft dynamics Controller ~A(s) Kc Tilted rate gyro P(4 US), - 4s) 7 Us) cos (a+aR) + 0.1 sin (a+aR) -= Figure 10.12 Roll rate system with tilted gyro. P(S) Coordinated Turn Systems 10.5 CO-ORDINATED TURN SYSTEMS 10.5.1 Introduction A co-ordinated turn is one in which both the lateral acceleration, a,, and the sideslip velocity, v, are zero. In such a turn the lift vector is perpendiated turns, and the sideslip velocity, v, are zero. In such a turn the lift vector is perpendiated turns is one in which both the lateral acceleration. In such a turn the lift vector is perpendiated turns reduce adverse sideslip and, therefore, roll hesitation. In such turns, there is minimum coupling of rolling and yawing motions. Provided that the side force due to aileron, Y: A, and the side force due to the yaw rate, Y, are both negligible, then zero sideslip velocity (v = P/Uo = O), and zero lateral acceleration (a, = 0) are all equivalent conditions. Sometimes, particularly in early textbook? on flying techniques, a co-ordinated turn was assumed to be one in which the lateral acceleration experienced in the cockpit was zero - a condition displayed to pilots by the turn-and-bank indicator, with its black ball centred between the vertical lines. However, this condition is not one which finds much use in AFCS studies since the acceleration at the cockpit is a function of the distance from the aircraft's c.g. Generally, the acceleration at the pilot's station features in AFCS work only in relation to ride control systems, which are dealt with in Chapter 12. 10.5.2 Conditions Needed for a Co-ordinated Turn For a body axis system the side force equation is: Y = m (~W - P + UR) Following the development detailed in Section 2.4 of Chapter 2, it can be seen that the rate of change of sideslip angle can be expressed as in eq. (2.75), i.e.: If R0 = 0, WdUo = a0 and, if a co-ordinated turn is achieved, i.e. if: p = 0 then: If the aircraft has been trimmed so that olo is zero, then: 336 Attitude Control Systems Therefore, in a co-ordinated turn, the rate of turn develops in proportion to the bank angle, 4. Of course, neither Yv nor YzA is generally zero, nor may they be neglected. Consequently, if p is to be zero, so that eq. (10.35) obtains, a steady deflection of the ailerons is required to maintain the co-ordinated turn. The value of aileron deflection required is given by: There are a number of factors which may delay the establishment of a coordinated turn. They include the following: 1. An aileron deflection usually induces a yawing moment. 2. The build-up of yaw rate, as a result of any change in bank angle, is delayed by aerodynamic lag. 3. The action of the yaw damper, which is commonly fitted to aircraft, tends to reduce any transient yaw

rate. As an illustration of how these factors affect the turn, consider an aircraft, such as B, in which: CHARLIE in Appendix NkA > 0 (10.37) Whenever a positive roll rate is required i.e. BA < 0, a negative (adverse) yawing moment is negative, the sideslip is positive. 10.5.3 Sideslipping as a Result of Sensor Signals in Lateral AFCSs If the rate gyro used to measure the yaw rate in a yaw damper is of the strapdown variety (i.e. it is fixed to the aircraft and is not mounted on gimbals) its signal is a measure of the body rate, rather than of a windhody rate (i.e. one which has been measured in relation to the stability axes). However, the equations used in the yaw damper design have been derived using stability axes, so that there is a discrepancy when a strap-down gyro is used. The output signal produced by such a gyro is given by: rbOdy= I, cos (YO + ps sin a. (10.38) In a rolling manoeuvre, with a positive angle of attack, the component due to roll rate in the signal from the strap-down gyro will increase. But if this signal is used as the feedback signal in a yaw damper, that feedback signal in a yaw damper, that feedback signal is used as the feedback signal in a yaw damper. motion. Co-ordinated Turn Systems 10.5.4 Horizontal Acceleration During a Turn The situation is represented in Figure 10.13: fc denotes the centripetal force, VT the tangential velocity, m the mass of the aircraft, and R the radius of the turn. VT Figure 10.13: fc denotes the centripetal force, VT the tangential velocity, m the mass of the aircraft turn geometry. (10.42) But: o = (g/VT) tan + .'. aycg = g tan (10.43) + (10.44) The total acceleration is the vector sum of a, and the acceleration due to gravity. The maximum value of acceleration is the vector sum of a, and the acceleration due to gravity. acceleration is limited to some maximum value which corresponds to VT. For a given speed, Uo, and a constant rate of turn, o, the bank angle too large for the linearization of sin and cos to hold, the results obtained above are ~ o r r e c t . ~ The number of turns which are completed in a manoeuvre may be calculated from: + + Attitude Control Systems 10.5.5 A Steady Sideslip Manoeuvre This flight condition of non-symmetric, rectilinear translation is often used in light aircraft to correct for the presence of a cross-wind on the landing approach. At large values of sideslip angle, the drag on the aircraft increases; as a result, the aircraft's liftldrag ratio decreases. In this flight condition, rates of change are zero, i.e.: g cos @, + Yvp + uo + YgASA + yiRsR= 0 (10.48) i.e. Au = c . If, say, a value of bank angle is chosen, arbitrarily, the resulting sideslip angle P and the control surface deflections SA and SR required for the manoeuvre can easily be found, provided that the matrix A is non-singular. The control deflections required tend to be very large, since powerful controls are needed to sideslip an aircraft at large angles. If A is singular, it implies that the bank angle term on the r.h.s. should be transferred to the 1.h.s. of eq. (10.51) and the P term should be transferred from the 1.h.s. to r.h.s. The new matrix A which results is then nonsingular. The control deflections required to produce the specified sideslip angle can then be deduced from the discussion on co-ordinated turns that sideslip angle is the motion variable whose control is central to the achievement of a co-ordinated '! Sideslip; the vane sensors which are used in some low speed aircraft are affected by problems concerning the local aerodynamic flow around the vane. They are also physically vulnerable. Some types of stagnation point sensor are useful for sensing flow direction, but have not yet found general application for AFCSs. Thus, the obvious means of controlling sideslip angle, by using a feedback control law based on sideslip sensing, is rarely used on high performance aircraft. However, its design and use will be covered first, to indicate the effectiveness of such systems, before presenting some other methods which are commonly used. These include: lateral acceleration feedback, computed yaw rate feedback, and control cross feeds. Further discussion of this topic can be found in McRuer and Johnston (1975). 10.6.2 Sideslip Feedback Figure 10.14 shows a typical system in which the sideslip angle, P, is sensed and used as a feedback signal to drive the rudder so that the system includes a yaw rate gyro with a sensitivity, K R, of 0.1 V deg-l, a value of controller gain, Kc, of 10, and a wash1 out time constant of 1.0 s. The state equation for the yaw damper was taken as rCom,; from Figure 10.14 it can be seen that when the sideslip suppression system is added, the command input to the yaw damper is now: Wash-out network 1 a- Pilot's Rudder actuator rudder + command @ s ~ (s) _ 4 + - 4 Controller and rate gyro - ~R(s) Aircraft s+4 dynamics Controller and rate gyr sideslip suppression system, may be written as: where: k [P P r 4, 8~ ewol (as before - see eq. (9.72)) Because of the perturbed airflow surrounding the vane of a sideslip sensor, the output signal is prone to contamination by noise. ~Consequently, to avoid feedback of local flow disturbances, it is customary to use vane sensors of low sensitivity. To illustrate the effectiveness of the system, a value of sensitivity, Kp, for the sideslip sensor in Figure 10.14 of 0.05 V deg-I has been chosen. Figure 10.15 shows the system responses to an initial sideslip disturbance of l o , (Kc2 = 100.0, Kc = 10.0, yaw damper only: the other responses should be compared to this one to observe the relative effectiveness of the sideslip suppression system. The effect of the sideslip suppression system. The effect of the sideslip suppression system can also be appreciated from an examination of the system eigenvalues. Table 10.2 shows the eigenvalues. damper only, and the combined system with sideslip suppression. The corresponding values of controller gains Kcl and KC2 are indicated. From Table 10.2 it can be seen how the sideslip transient more effectively. It is worth appreciating, finally, that a (s) (a) (b) 1.0 (4 Time (s) Sideslip suppression system: KG= 100, Kc,= 10, Time (s) Figure 10.15 Response to P(O) = lo. (a) Uncontrolled aircraft. (b) Yaw damper system - wash-out time constant = 1 s. (c) Sideslip suppression systems 342 Table 10.2 Eigenvalues of sideslip suppression systems 342 Table 10.2 Eigenvalues of sideslip suppression system. Yaw damper and wash-out network Sideslip suppression - sideslip feedback as a result of the rudder deflection, the acceleration which is sensed (assuming linear relationships) is: Thus, the sideslip suppression system can now have a block diagram like that shown in Figure 10.16. The system requires, however, that the sensitivity of the accelerometer be high, since Yv is usually small, e.g. for CHARLIE-4, Yv = - 0.056. In addition, the acceleration threshold of the acceleration threshold of the acceleration threshold value means that the system is subject to spurious inputs from structural effects, and, possibly in very high performance aircraft, from the effects of Coriolis acceleration. The state equation of eq. (10.53) also applies for this system, and for the example of CHARLIE-4, the only change which occurs is to the element asl of A. It now becomes: Rudder actuator 'R(s) * st4 Wash-out network S - s+1/T,, - Aircraft dynamics B(s) * Y" -- Controller and rategyro Kc, KR - - Sideslip controller Accelerometer K, Ka" Figure 10.16 Block diagram of lateral acceleration control system. 343 Sideslip Suppression Systems This type of sideslip Suppression Systems This type of sideslip Suppression Using Computed Yaw Rate It is shown in Section 10.5 that, in a co-ordinated turn, the rate of turn develops in proportion to the bank angle, i.e. g sin 4 r =- uo By using this signal as a feedback signal in the system (the block diagram of which is given in Figure 10.17) unwanted sideslip angle only if eq. (10.35) does not hold. In other words, if the sideslip angle has a value other than zero, the feedback operates. The error signal in
Figure 10.17 is given by: ess = Kc2KEs r - - sin 4 o: { I The system is effective, if the resolver is accurate; with modern, airborne digital computers computing equations such as eq. (10.35) can be carried outle to very good accuracy. With old-fashioned, electromechanical resolvers, the accuracy was often difficult to achieve and, as a result, the rudder could be held over at either extreme of its range of deflection during a turn. This tendency was objectionable to many pilots and, although its use was confined to high performance, military Pilot's command to command to command to end over at either extreme of its range of deflection during a turn. rudder 6 Rudder actuator (4 4 st4 Wash-out network Controller and rate gyro 3 Ideslip controller 4x1 Attitude gyro 1 Figure 10.17 Block diagram of computed yaw rate system Kc, = 0. (b) Computed yaw rate system Kc, = 0. (b) Computed yaw rate system Kc, = 0. (c) Kc, = 10.0. Sideslip Suppression Systems 345 aircraft, the system never enjoyed much popularity. The responses of such a system, for CHARLIE-4, using the same parameters for the inner loop as were used in the yaw damper discussed in Chapter 9, are shown in Figure 10.18. The sensitivity of the resolver was chosen to be 1V deggl. The value of the gain of the controller is indicated at the appropriate response curve in Figure 10.18. 10.6.5 Control Crossfeeds Introduction It must be remembered that turn co-ordination is most often required when either stopping or completing a lateral manoeuvre during the final approach. Such manoeuvres are usually controlled by use of the ailerons; however, the use of ailerons can result in a significant yawing moment if the stability derivative, Nk, is relatively large. This yawing moment can make a substantial control systems to incorporate a control crossfeed to remove that source of sideslip. There are two types of crossfeed (sometimes referred to as 'interconnects'): aileron-to-rudder interconnect (ARI) and bank angle-to-rudder interconnects): aileron-to-rudder interconnects (ARI) and bank angle-to-rudder interconnects): represented in Figure 10.19. The most suitable value of the crossfeed gain, Kc-, has been found from flight studies (see McRuer and Johnston, 1975) to be: However, the student should understand that if the ARI is to be a permanent connnection throughout the flight envelope, and not active just at the terminal phases of flight, being switched in, say when the flaps are deployed, or when the landing gear is lowered, gain scheduling of Kcf will be needed (it should vary inversely with the forward speed of the aircraft). Additionally, if the structural modes of the aircraft are significant, then some form of frequency compensation must be used. In aircraft with a variable configuration, such as a swing wing, the sign of the crossfeed signal may also need to be changed as a function of the system. The presence of the wash-out network in the crossfeed path is required to permit the aircraft to produce steady sideslipping manoeuvres, unopposed by the system. required in cross-wind landings. One concern of flight control system designers using ARI, to suppress sideslip to improve turn coordination, must be the situation, the pilot will 346 Attitude Control Systems need to command a constant aileron deflection to counter the resulting yawing motion caused by the engine failure. The ARI system may aggravate this control problem. From Figure 10.19 the following relationships can be established: 1 - ecfT2 10FCaA+ 10Kc2Kc&co,m - KclKc\$ - 10Kc2Kc&co,m - 10 + 0.042+ + 0.00226, Wash-out network I Rate evro Wash-out network Cross-feed gain Rudder actuator 5 s+4 10 - - 1 s+10 Aircraft dynamics ' (CHARLIE-4) 7 Aileron actuator Rate evro Wash-out network I Rate evro Wash-out network I Rate evro Attitude gyro n Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: From Figure 10.19 Block diagram of ARI system. = ' fi P r = 4) Sideslip Suppression Systems 347 Let: Sid Kcl = 9.5156VN7 Kc = 10.0 V N, Kcf = 0.035 V deg-l, KR = 10 V deg-l, KR = 10 V deg-l, R = 10and its flight condition, Kcf = 0.035 is the best choice. Large values of T2 (slow wash-out) result in poor sideslip responses of this system are shown in Figure 10.20. It is evident how effectively the sideslip has been suppressed, and it can be seen that the bank angle response has not been seriously affected by the crossfeed (cf. Figure 10.6). Roll Angle to Rudder Crossfeed This type of control crossfeed is used most with large transport aircraft, the time constant needed for the wash-out network in the yaw damper is frequently unsuitable to achieve the required value of damping ratio for the closed loop system. The situation arises because the Attitude Control Systems - O .2 ~3 4~ 5 ~ 6 I7 8I 9 I 10 I 11 I 12 13 I 14I I 12 13 I 14I I 1 2 13 I 14I I 1 Time (s) Figure 10.20 Step response of Figure 10.20 Step response of Figure 10.20 Step response of Lateral acceleration cannot be used, otherwise there would be considerable coupling of the rigid body and structural motion. Furthermore, the relatively slow response of the rudder fitted to such large aircraft precludes the use of high loop gain to suppress unwanted sideslip motion. Furthermore, the relatively slow response of the rudder fitted to such large aircraft precludes the use of high loop gain to suppress unwanted sideslip motion. crossfeed signal, ecf, is introduced into the summing junction of the yaw damper as if it were a command signal for some value of yaw rate which corresponds to zero sideslip angle. It has already been shown in this chapter that in a co-ordinated turn the yaw rate is given by: g sin r=- uo + However, for small bank angles (which is likely to be the case for large transport aircraft) the command signal for yaw rate can be taken as: In using such a crossfeed to the rudder channel some phase advance compensation is introduced into the system which causes an increase in the damping of the dutch roll motion. The state equation which corresponds to the system of Figure 10.21 can be represented once more by eqs (9.74)-(9.76) save that the fifth row of the matrix A, now becomes I I 349 Sideslip Suppression Systems Pilot's rudder command Rudder actuator (') P(s) Aircraft dynamics s+4 4(s) 4s) Wash-out network ewo(s) Controller and rate gyro + s IIT,, Crossfeed gain n Figure 10.21 Block diagram of roll angle to rudder interconnect centre system For CHARLIE-4, it can be shown that: Hence: The response of this system to an initial value of sideslip angle of lo is shown in Figure 10.22 Where the response of yaw damper, ARI and @IsR crossfeed. 350 Attitude Control Systems same initial condition are also shown for comparison. Bank angle to rudder crossfeed is evidently less effective at suppressing sideslip, it is usually necessary, therefore, to use a roll angle control system of the kind discussed in Section 10.3 to augment the stability of the spiral mode. When this is done, there is a marked improvement in the sideslip suppression capacity of this type of crossfeed. Reliability is of the greatest importance in AFCS work, there are situations in which the loss of a feedback path will result in no more than a downwards change of level of an aircraft's flying qualities. In other situations, the dynamic stability of the aircraft and its occupants is imperilled. In AFCSs employing crossfeeds, if failure occurs in any feedback path, the flying qualities of the aircraft are usually so drastically impaired that it becomes necessary to disconnect at once the other feedbacks so that the aircraft is no longer under automatic control. This is the case for the bank angle to rudder. This is a difficult engineering problem and its partial solution is to be found in the technique of using redundancy in the feedback paths. Readers should refer to McRuer and Johnston (1975) for further discussion of control crossfeeds. 10.7 DIRECTIONAL STABILITY DURING GROUND ROLL The geometry of the situation is represented in Figure 10.23 in which y denotes the lateral displacement from the desired track (the runway centre-line), h is the azimuth angle of the aircraft experiences during its ground roll is: I; = vj, (10.80) With this acceleration being used as a negative feedback signal, and with the feedback gain represented by K, , then: Y = NBP + NDu- K, Y v); The sideforce equation is given by: (10.81)~ Directional Stability, Nu is the contribution of the undercarriage to the track stability, Yp is the aerodynamic sideforce stability derivative and Yu is the combined sideforce stability derivative from the tyres of the undercarriage. where: Taking Laplace transforms, eq. (10.83) and (10.84) can be re-expressed as: where [0] represents a null matrix. From this equation, the characteristic equation, the characteristic equation can easily be shown to be: s3 + [Yz - y;]s2 + [N b - N & - K~Y V (Y z - Y z)] s = s{s2 + [Yz - Y \$]} s+ [Nb - Nb &Y V(Y\$ - Yz)] (10.87) =0 The zero root means that if the aircraft is disturbed from its track there is no
inherent restoring moment unless the pilot applies rudder correction or nose wheel steering or asymmetric thrust. 352 Attitude Control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N\$ and N \$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N \$ >0, it is control Systems Examining the quadratic term, and noting that Y \$ < 0, N z < 0, Y z > 0 and N \$ >0, it is contr then evident that the sideforce contributions of the undercarriage contribute to the damping of the motion during ground roll. However, suppose V represents the ground speed and Vw represents The presence of the headwind now results in the real root of the characteristic cubic being finite, rather than zero, with the possibility of some stabilizes the ground tracking mode whereas N; destabilizes it. A discussion of the dynamics of aircraft rotation and lift-off can be found in Pinsker (1967). 10.8 CONCLUSIONS Automatic control systems for maintaining the attitude to a new commanded value, are introduced. To emphasize the principles of negative feedback control systems for maintaining the attitude to a new commanded value, are introduced. control system used on aircraft, the pitch attitude control system is dealt with extensively. The use of a pre-filter in conjunction with these types of AFCS to obtain the required handling qualities in the controlled aircraft is briefly dealt with, before a roll angle control system is dealt with extensively. networks, or a roll rate damper as an inner loop to achieve the required dynamic response is dealt with and gain scheduling as a means of maintaining the same closed loop performance over as much of the flight envelope as possible is also treated. The unwanted results of tight roll control, such as roll ratchet or pitching motion due to rolling, are treated briefly before the means of achieving automatically controlled coordinated turns by a variety of methods is explained. The chapter concluded with the important subject of controlling direction stability during ground roll. 10.9 EXERCISES 10.1 A transport aircraft, flying at a Mach number of 0.8 and a height of 10 000 m has to aileron deflection, SA, Gl(s) as as its transfer function, relating bank angle, +, Exercises 353 as defined below. When the aircraft flies at half the height and at a Mach number of 0.4 its transfer function becomes G2(s). The block diagram of the bank angle control system used on the aircraft shown in Figure 10.24. Controller I Aircraft dynamics m Attitudes at half the height and at a Mach number of 0.4 its transfer function becomes G2(s). gyro l Figure 10.24 Block diagram of a bank angle, for flight condition 1. (Hint: make the commanded bank angle, reasonable simplifying assumptions.) (b) What is the effect upon the dynamic response of the bank angle control system if the aircraft flies at flight condition 2? Assume the controller gain, K+, remains unchanged. (c) If the value of K+ is 2.5, and if the value of K+ is 2.5, and if the value of the commanded bank angle is 5.0°, sketch the closed loop response for both flight conditions. +,,- 10.2 If the experimental VTOL aircraft of Exercise 2.7 is flying at 15 m s-l, and has the same stability derivatives that were listed in that question, calculate the lateral acceleration at its c.g. for a flat, co-ordinated turn in which the yaw rate is 0.33 rad s-l. (The aircraft may be assumed to have zero sideslip suppression system, represented by the block diagram in Figure 10.25 a sideslip signal is used as feedback to drive the rudder so that sideslip is eliminated. The wash-out filter in the inner loop can be regarded as a blocking filter for constant manoeuvre commands, i.e. yaw rate feedback operates only during changes of the flight state. The dynamics associated with the rudder servo are negligible. The equations of motion of the aircraft are: The sideslip suppression system is to be designed so that its closed loop response resembles closely that obtained from an idealized model system governed by the Attitude Control Systems Rudder servo + 4 ' Pmmm *I 100 I L A Aircraft dynamics I T * KP - Sideslip sensor Figure 10.25 Block diagram of a sideslip control system for Exercise 10.3. characteristic equation + 5.OP, = 0. (a) By means of any suitable analytical method determine suitable values of the sensitivities of the sensitivities of the sideslip motion being suppressed in any manoeuvre. ,p 10.4 For the aircraft detailed in Exercise 3.5 design a simple bank angle control system, with full wing-tip fuel tanks, which will ensure that the aircraft can roll through a bank angle of 30" in 1.225 s in response to a step command. 10.5 (a) In a co-ordinated turn the lift vector is perpendicular to the aircraft axis OY and the lateral acceleration at the c.g., aycg, and the sideslip velocity, P, are both zero. The situation is represented in Figure 10.26. Derive, for small angles, a transfer function relating yaw rate, r, to bank angle, 4. (b) For an aircraft on approach, with a flight path angle of 2", the appropriate lateral stability derivatives are: (Height = SL; Speed = 37.73 m s-l): Figure 10.26 Geometry of a co-ordinated turn for Exercises N; = 1.0 Lb, N; = -0.2 Nb, = -1.0 = 0.1 N: = - 0.24 (c) 10.6 If the aircraft has been commanded to produce a steady sideslip at a constant bank angle of 10" determine the steady deflections of the ailerons and rudder required to be maintained by the pilot. The sideslip is to occur to starboard. What value of sideslip angle is produced? What numerical difficulty would occur in your calculation if the bank angle was to be zero during the sideslip manoeuvre? The high speed reconnaisance aircraft described in Exercise 3.9 is to be controlled such that its transient roll rate response is close to that shown in Figure 10.27. (Compare this response with that determined for the uncontrolled aircraft described in Exercise 3.9.) Determine a suitable feedback control law. Time (s) Figure 10.27 Model roll rate response for Exercise 9.1 the rudder is being executed under 'manual' control. If the steady sideslip angle is 5.73", calculate the lateral acceleration at the instantaneous centre of rotation of the aircraft. (Note: the equations of Exercise 9.1 were derived using SI units.) (b) Suppose the aircraft is flying over the North Pole at the cruising speed which corresponds to the equations of motion given in Exercise 9.1. Calculate the value of the Coriolis acceleration to which the aircraft will be subjected. Comment upon whether there would be any significant effect upon the performance of the aircraft's yaw damper if the aircraft were to carry out over the North Pole a co-ordinated turn of the kind defined in part (a). 10.8 A transport aircraft with twin piston engines has the following equations of lateral motion: (a) Attitude Control Systems + 0.048, + FT p = - 2.0P - 6.5p + 2.5r + 4.0SA + 0.088, + LT i = p - 0.2p - 0.6r + 0.05SA - SR + (NT + AD) p = - 8.6p - 0.25 + and SR denote the sideslip angle, roll rate, yaw rate, aileron and p, p, r, rudder deflections respectively. The terms FT, LT and NT represent, respectively, the changes in sideforce, rolling and yawing moments which occur when an engine fails. AD is the yawing moment due to drag caused principally by the feathered propeller. When both engines are operating satisfactorily the four terms are zero. When the starboard engine fails determine the approximate maximum angle of sideslip which develops aircraft, with a roll control system, is flown by a novice pilot. The transfer function of the roll system is given by: +(s)/+,,,(s) = 4.5/(1 + 0.01s) Whenever the novice pilot commands a bank angle, roll ratcheting is observed in the aircraft's motion. From several assessments of his tracking performance it has been established that his performance can be reasonably represented by the following transfer function: $Gn_s = 2.67 e-0.32S$ (a) When the same aircraft is flown by an experienced test pilot whose mathematical representation is Gtp(s) = 3.2-0.12s will the phenomenon of roll ratchet occur when the aircraft was flown by the novice pilot? 10.10 A sideslip suppression system is to be designed for the aircraft FOXTROT-4 using computed yaw rate as the feedback signal. The sensitivity of the resolver is 1 V deg-' and the product of the values of the gain of the controller and the sensitivity of the resolver is 1.0. A time constant of 1.0 s is
employed in the wash-out network used in the feeback path of the yaw damper. If the dynamics of the rudder actuator can be represented by the first order transfer function: 8 R (\sim) / 8 K (= \sim) 41(\sim + 4) (a) Find a suitable value for the gain, Kc,, of the controller of this sideslip suppression system; (b) Determine the response of the system to a commanded change in yaw rate. References 0 1 357 A wing leveller is required for the aircraft GOLF. Design a suitable system such that it can produce almost'. 10.10 NOTES 1. 2. 3. 4. For the purpose of comparison with results presented earlier, the actuator dynamics here have been assumed to be instantaneous, and represented by a transfer function of unity. The same results are obtained using the non-linear equations of motion. If X A = - Ygn/NiR, then a, = aycg + x A i Primed stability derivatives are not involved since it is assumed that during ground roll any rolling motion is negligible. 10.11 REFERENCES 1983. Excessive roll damping can cause roll ratchet. J. Guid. and Cont. 6(3): 218-9. LANGEWEISCHE, W. 1944. Stick and Rudder. New York: McGraw-Hill, p. 133. MCRUER, D.J. and D.E. JOHNSTON. 1975. Flight control systems properties and problems. Vol. I. NASA CR-2500. PINSKER, W.G. 1967. The dynamics of aircraft rotation and lift-off. A R C R & M 3560. STENGEL, R.F. 1983. A unifying framework for longitudinal flying qualities criteria. J. Guid., Cont and Dyn. 6(2): 84-90. CHALK, C.R. Flight Path Control Systems 11.I INTRODUCTION There are a number of flight missions which require that an aircraft be made to follow with great precision some specially defined path For fixed-wing aircraft there are four positioning tasks which must be performed with extreme precision. These tasks are: air-to-ground weapons delivery, air-to-air combat, in-flight refuelling and all-weather landing.' Whenever a conventional aircraft is to be controlled, a pilot can command rates of rotation in any or all of three axes: pitch, roll and yaw. On such aircraft his direct control of translation is restricted to the control of airspeed either by means of changing the thrust being delivered by the engines, or by the use of any speed brakes or drag modulators. Conventional aircraft have no special control surfaces to permit the control of translation in either the normal or lateral directions Consequently, the reduction of an inadvertent lateral displacement from some desired track, for example, has to be achieved indirectly by means of a controlled change of aircraft heading. As another example, has to be achieved indirectly by means of a controlled change of aircraft heading. attitude. As a consequence of such limitations, a number of the attitude control systems. In Chapter 12, active control technology (ACT) is discussed and its use with control configured vehicles (CCVs) is dealt with. Because such CCVs are provided with many more control surfaces than are usually to be found on a conventional aircraft positioning control systems are dealt with more appositely in that chapter 1, it is stated that the control of the attitude angles of an aircraft is the special function of flight control, whereas the control of its path through space is more properly a guidance function. But path variables, such as heading and pressure altitude, need to be measured in the aircraft; there is some logic, then, in considering their control in a treatment of flight control. Automatic tracking and terrain-following will be shown to involve merely linear approximations to those kinematic transformations in the guidance loops which place an aircraft and its destination (or target) on comparable terms. Since these approximations are linear as well as sufficient, such systems can be regarded as Height Control Systems 359 members of the class of flight path control systems, and are so treated in this book. 11.2 HEIGHT CONTROL SYSTEMS 11.2.1 Introduction When a system is used to control the height at which an aircraft is flying, it acts as a feedback regulator to maintain the aircraft's height at a reference (or set) value, even in the presence of disturbances. The pilot can either fly the aircraft by manual control or use the pitch attitude control system to control the climb (or descent) of the aircraft until it has reached the required height. When that height control system to control the climb (or descent) of the aircraft until it has reached the required height. usage, however, which merit distinct treatment: automatic landing and terrain following. In each of those special cases, the height control system is required to control system is often referred to as a 'height hold' system. Supersonic transport (SST) aircraft are known to have phugoid modes of very long period and it has been observed that pilots of such as 160m. Upset recovery is also known to be prolonged and as much as 5000 m may be needed to recover the aircraft's attitude and height after an upset. For such SST aircraft, a height hold system is shown in Figure 11.1. The height of the aircraft can be seen to be controlled by means of elevator deflection; that deflection is produced by an actuator, the dynamics of the altimeter have also been represented as a first approximation by a first order, with the same value of time constant. For its successful operation, the system requires a longitudinal accelerometer to provide a feedback signal proportional to u. The sensitivity of the altimeter, denoted by Kh, can be taken to be unity without loss of generality. Obviously, being a closed Poop feedback control system, the height hold system may be stable, unstable, oscillatory or over-damped depending upon the aircraft dynamics and the values of the controller gains, K, and Kc. With K, being selected at - 200.0, it is found that a somewhat oscillatory, but very slow, response results with an evident error Flight Path Control Systems Feedback Accelerometer Controller Aircraft dynamics 1+O. Ip Feedback Figure 11.1 Height hold system 1. in the steady state value of the height (compared to the reference height) when the controller gain is chosen to be 0.08 mV m-l. It is obvious from an inspection of the response shown in Figure 11.2 how large are the variations in flight path angle and for how long they persist. Doubling the value of Kc leads to obvious dynamic instability - see Figure 11.3 With the value of the controller gain reduced once more to 0.08mV m-l, but with the value of the gain of the accelerometer increased to - 300.0, the dynamic reponse can be seen from Figure 11.4 to be much better damped, and that very much smaller values of flight path 7 Steady state error -1.0 L -2.0 0.0 0.4 0.8 1.2 1.6 Time (xIOZs) Figure 11.2 Response of height hold system I. 2.0 Height Control Systems Time (X lo2 s) Figure 11.3 Response of height hold system I - increased damping. Flight hold system I - increased damping. Flight hold system I - increased damping. system 11. The block diagram of an alternative height hold system is shown in Figure 11.5. Notice that it represents a pitch attitude control system, with a pitch rate SAS as its inner loop. An outer loop, involving the use of an altimeter to provide a feedback signal proportional to height, is used to achieve the height hold function. It can be seen from Figure 11.6 how much improved is the dynamic reponse and how the steady state error has been very nearly eradicated. The amplitude of the necessary changes in flight path angle has also been reduced. Time (~ 1 0 s) ' Figure 11.6 Response of height hold system II. Height Control Systems -0.5L -0.1 1 0.0 I 0.4 I 0.8 I 1.2 I 1.6 I 2.0 Time (X 10' s) Figure 11.7 Commanded step response of height hold system II. Time (X 10' s) Figure 11.8 Commanded step response and the sizeable steady state error still remain. A different control structure is needed to avoid this steady state error There is some scope, however, for choosing different values of Kq, Kc and KO differ from those relating to Figure 11.6), in changing from the first set height of 4 000 ft to be the reference height of 5 000 ft, there is a peak height of 6 000ft at about 15 s. This large peak, which occurs in an oscillatory but heavily damped, response, comes about as a result of the existence of a significant zero in the transfer function relating the change in height to the elevator deflection which caused it. Other choices for the values of the controller gain lead to improved dynamic reponse The dynamic response for a commanded change of height of - 10 ft is shown in Figure 11.8. It can be seen from that figure that the dynamic response is non-oscillatory, smooth, and rapid. There is a pronounced difference in the effectiveness of the two systems. phugoid mode, although the period of the phugoid oscillation is itself reduced. For really quite moderate values for Kh, instability results. Consequently, the second type of system relates to the 'backside' parameter, a, namely: where D represents the aircraft's drag force, and T is its thrust. The parameter a is one of the zeros of the transfer function relating height to elevator deflection. In certain aircraft, a performance reversal can arise (on the backside of the power curve) in which BTIBu S=- aDlau; a is then negative. controller to assure stability of the height hold systems. In that case a more complex form of control law than the simple proportional feedback control being used in these two systems 11.3 SPEED CONTROL SYSTEMS Although speed is not truly a path variable, its exact control is essential for many tasks related to the controlled. A block diagram representing a typical airspeed control system is shown in Figure 11.9. Speed is controlled by changing the thrust, &th, of the engines; such a change in thrust is obtained by altering the quantity of the fuel flowing to the engines; such a change in thrust is obtained by altering the thrust. thrust setting and the flight condition. For the purposes of illustration, TE will
be assigned a value of 0.5 s. Although the thrustlthrottle angle relationship is not linear, in practice, it will be assumed to be so here. The system depends upon a feedback signal based on sensed longitudinal acceleration. However, the dynamics of the accelerometer are such that its bandwidth is much greater than that of the aircraft system so that its response in this application can be assumed to be instantaneous. Since the airspeed sensor is usually a barometric device, it has been represented by a first order transfer function, with a time constant of T,. The controller is a proportional plus integral type; the integral term has been added to remove, if required, any steady state error in the response of the airspeed system to constant airspeed, u, should persist, Hence uref if taken to be zero. The dynamic response of the system of Figure 11.9 to an initial airspeed error of -t 10 m s-I in the equilibrium (approach) airspeed sensor was taken to be 0.1 s, and the controller gain Kc was chosen to be 2.0. The sensitivity of the accelerometer K, was 2.0 V m-' sL2. The integral term was omitted. Note the small error at values of time greater than 12s. In Figure 11.10 the longitudinal acceleration, u, is also shown. The key factor in the response of this speed control system is the authority allowed over the engines' thrust. However, if 10 per cent authority is allowed, say, then it is possible to evaluate KE by knowing that for steady flight: T = W(D1L) (11.4) For the approach flight condition, the weight and liftldrag ratio of CHARLIE are known to be: W = 2450000N LID = 8.9 T",,, = 800 kN Hence the available excess thrust on approach is 525 000 N. Only 10 per cent of that excess thrust on approach is 525 000 N. Flight Path Control Systems Controller Throttle actuator Jet Aircraft dynamics i=Ax+Bu Accelerometer Ku I fisensed Airspeed control system is very greatly affected by the actuator dynamics. In Figure 11.11 are shown the speed responses which result for the same conditions and values of parameters that, in case A, the time constant of the actuator has been Time (s) Figure 1 1.10, except that, in case A, the time constant of the actuator has been Time (s) Figure 1 1.10 are shown in Figure 11.10, except that, in case A, the time constant of the actuator has been Time (s) Figure 1 1.10 are shown in Figure 11.10, except that, in case A, the time constant of the actuator has been Time (s) Figure 1 1.10 are shown in Figure 11.10 T,,,=0.25 s Case B T,,,=0.5 s ---- Time (s) Figure 11.11 Response to initial u (0) - effects of actuator time constant. doubled (Tact = 0.25 s) and, in case B, the actuator's response is beginning to be oscillatory. Further increases in the time constant of the actuator will lead to instability of the speed control system. Similarly, the dynamics of the airspeed sensor are crucial. Figure 11.12 shows the dynamic responses to the same flight condition and control parameters (the value of the time constant of the actuator being restored to 0.125 s). Case A represents the response when the value of time constant of the airspeed sensor was increased to 0.4 s and case B when its value was increased further, by a factor of 10. With the value of the system to a reference speed command, which is a linear change of airspeed from 75.0 m s-' to 70.0 m s-' shows the responses to the same initial speed error of + 10m s-' but, in case B, with Kc1 = 10.0, and Ki, = 2.0, and, in case B, with Kc1 = 10.0, and Ki, = 2.0, and Ki, = 1.0. Case B is the case used to obtain the ramp response shhwn in Figure 11.13. In Figure 11.13. In Figure 11.14 the incipient oscillatory response with increased values of Kc, can be seen in the acceleration (u) responses. Flight Path Control Systems ,.' UB Time (s) A.)'~ Figure 11.12 Response to u(0) - effects of sensor time constant. Time (s) Figure 11.13 Ramp response to u(0) - effects of sensor time constant. Time (s) Figure 11.13 Ramp response to u(0) - effects of sensor time constant. (1973) and Blakelock (1965). MACH HOLD SYSTEM 11.4 Modern jet aircraft are often fitted with such a control system; its purpose is to hold the set Mach number in the presence of disturbances, provided that the change in height is not very great. Variations in Wach number in the presence of disturbances is to hold the set Mach number in the presence of disturbances. (11.8) A block diagram of a typical system is shown in Figure 11.15. Note that speed is being controlled in this system by using elevator deflection. Since the elevator is being used, and the aircraft will be flying at large subsonic, or even supersonic, Mach numbers, the basic short period dynamics usually have to be augmented. A pitch rate SAS has been used as an inner loop in the system represented by Figure 11.15. For BRAVO-4, of Appendix B, the aircraft has a Mach number of 0.8. To illustrate how effective the system is, Figure 11.15. For BRAVO-4, of Appendix B, the aircraft has a Mach number of 0.8. To illustrate how effective the system is, Figure 11.15. For BRAVO-4, of Appendix B, the aircraft has a Mach number of 0.8. To illustrate how effective the system is, Figure 11.15. For BRAVO-4, of Appendix B, the aircraft has a Mach number of 0.8. To illustrate how effective the system is, Figure 11.15. For BRAVO-4, of Appendix B, the aircraft has a Mach number of 0.8. To illustrate how effective the system is, Figure 11.15. For BRAVO-4, of Appendix B, the aircraft has a Mach number of 0.8. To illustrate how effective the system is a mach number of 0.8. To illust u,, defined by: Flight Path Control Systems p = A- d dt number ,/ Elevator actuator Controller Aircraft dynamics @ + 10) - Disturbance I m*, u Rate gyro - Accelerometer and airspeed sensor Figure 11.15 U Mach hold system. (i.e. u, changes from 0 to - 20 m s-I in 20 s). It is evident from Figure 11.16 how effectively the speed and Mach number have been held nearly constant. This splendid regulatory performance is not achieved, however, without adjustment of other motion variables of the aircraft climbs by approximately 1800 m to a new 8.000051 - 2.0 - 8.000030 - 1.2 - M - I ?j8.000011 - w c? L Ei D 2 X w 3 5 7.999990 - -0.4 -7.999970 - -1.2 - 7.999950 - -2.0 s I 0.0 4.0 I I 8.0 12.0 Time (s) I 16.0 Figure 11.16 Response of Mach hold to horizontal shear I 20.0 Directional Control System Time (s) Figure 11.17 Response of Mach hold to horizontal shear I 20.0 Directional Control System Time (s) I 16.0 Figure 11.16 Response of Mach hold to horizontal shear I 20.0 Directional Control System Time (s) I 16.0 Figure 11.17 Response of Mach hold to horizontal shear I 20.0 Directional Control System Time (s) I 16.0 Figure 11.16 Response of Mach hold to horizontal shear I 20.0 Directional Control System Time (s) I 16.0 Figure 11.16 Response of Mach hold shear height of 11000 m. This dramatic climb occurs because the aircraft being studied is a very high performance fighter. 11-5 DIRECTION CONTROL SYSTEM The purpose of such a system is to allow an aircraft to be steered automatically along some set direction. A block diagram representation of a typical system is shown in Figure 11.18. The heading of the aircraft makes under automatic control will be coordinated. Hence, any sideslip angle, p, is zero. It is shown in Section 10.5 of Chapter 10 that for small bank angles: This equation is represented in Figure 11.18 by the blocks which have been labelled 'aircraft kinematics'. The aircraft kinematics'. path. The control law for this direction control system is simply: where the value of the controller gain, KT, can be determined by any of the appropriate-design methods discussed in Chapter 7. The system shown relates to CHARLIE-2 and the bank angle control system being used is that derived as system B 372 Flight Path Control Systems Aircraft kinematics Aileron servo 10 10 & A + Aircraft dynamics (~O+P) 4 PA uo Rate gyro and amplifier -9.5156 + -. Note: pA here denotes aircraft's roll rate; p denotes dldt Figure 11.18 Direction control system. in Example 10.3. An appropriate value for Kyr was selected to be: The unit step response of the system is shown in Figure 11.19 in which the corresponding bank angle, and aileron deflection, tiA are also shown. The long settling time required to achieve the new heading should be noted. Although it has been assumed that the turn was co-ordinated, there is some residual sideslip angle, f3, with a peak deviation of 0.38"; see Figure 11.20. The response of the system can be made more rapid by using an improved value of Kw. Figure 11.21 shows the step responses of the system to a 1" direction change command, for different value. Using this value, the system was subjected to a sideslip crosswind with a profile similar to that shown in Figure 11.22. The
heading reference was 0°, and the response to this crosswind disturbance is shown in Figure 11.23. Note that the peak heading deviation was merely 0.0085 degrees, which caused a bank angle change of 0.013". The effectiveness of this direction control system can also be seen by considering how well it performs to suppress the effects of a sideslip shear. Figure 11.24 shows the response of the system when subjected to a sideslip shear with the profile represented by BCW in Figure 11.24, a change in sideslip of 3.5" in 3.5 s. The peak deviation in heading was 0.012", and the set heading was regained in about 12 s after the onset of the shear. There is an associated peak bank angle of 0.2". This direction control system forms the basis of the automatic azimuth tracking systems. +, Directional Control System Time (s) Figure 11.19 Step response of direction control system. Time (s) Figure 11.20 Yaw and sideslip response to step change in direction. Flight Path Control Systems Case A K, 2.0 Time (s) Figure 11.22 Sideslip cross-wind profile. Directional Control System Time (s) Figure 11.23 Response of direction control system to cross-wind. -4.0~ 0.0 I 2.0 I 4.0 6.0 I 8.0 1 10.0 Time (s) Figure 11.24 Response of direction control system to sideslip shear. Flight Path Control Systems 11.6 HEADING CONTROL SYSTEM The heading angle, A, of an aircraft is defined by: In the preceding section, the direction control system operated by means of coordinated turns, thereby ensuring that the sideslip angle, p, was effectively zero. To do that, however, required the turning manoeuvre to be effected by means of the ailerons. If rudder use is involved, then it would seem that the yaw angle, could be controlled by means of a yaw damper system, and with sufficient sideslip suppression could provide the basis of a heading control system. However, there are fundamental control problems involved with this approach and it is not much used. Nevertheless, for the purpose of instruction, a heading control system, with a block diagram like that shown in Figure 11.25, can be considered. For CHARLIE4, and using the yaw damper of Section 9.8.2 of Chapter 9, it can be shown that if the state vector is defined as: *, and the control vector is defined as: the corresponding coefficient and driving matrices, A and B, for KA = 1.0 and Two = 3.0, are given by: For the heading is shown in Figure 11.26 for two values of yaw damper gain, K, namely 1.0 and 0.5. The yaw rate response is identical for both cases, but the heading angle, A, has a different steady state value in each case. To remove such steady state errors normally requires the use of an integral term in the controller. 377 Heading Control System I Wash-out network Rate gyro and controller I I Gyrocompass KA f Figure 11.25 Heading control system. From Figure 11.25 it can be seen that the control law for the heading control system is given by: = - K*Kcl(P If we let: J + *)- KAKC1KC2 (P + p = XI A is Figure 11.26 Response of heading control system to Vr(0). Flight Path Control Systems then: x = 1 (11.20) pdf And if we let: $+ = xg A x^9$ then: $\sim c_{0,1} = -(11.22b) K \sim Kc \sim x l KhKclKc2x^3 - K Kcl(P If we let: J + *) - KAKC1KC2 (P + p = XI A is Figure 11.26 Response of heading control system to Vr(0). Flight Path Control Systems then: <math>x = 1$ (11.20) pdf And if we let: $+ = xg A x^9$ then: $\sim c_{0,1} = -(11.22b) K \sim Kc \sim x l KhKclKc2x^3 - K Kcl(P If we let: J + *) - KAKC1KC2 (P + p = XI A is Figure 11.26 Response of heading control system to Vr(0). Flight Path Control Systems then: <math>x = 1$ (11.20) pdf And if we let: $+ xg A x^9$ then: $\sim c_{0,1} = -(11.22b) K \sim Kc \sim x l KhKclKc2x^3 - K Kc^2 + (11.22b) K \sim Kc \sim x l KhKclKc2x^3 - K Kc^2 + (11.22b) K \sim Kc^2 + ($ Using a value of Kc of 0.875, 1 with Kc = 0.01 results in a stable, but lightly damped and oscillatory, closed loop 2 system with the following eigenvalues: Heading error. The response to initial heading error. The response of this closed loop 2 system with the following eigenvalues: Heading error. The response to initial heading error. noting that the choice of Kc is most important. 2 It can easily be shown that for stability the value of Kc, must not be greater than 0.0222. Much of the type being considered, relates to the presence of the wash-out network in the feedback path of the yaw damper. If it is removed, then the yaw system shown in Figure 11.28 can be represented by the following state equation: where: Aircraft dynamics Rate gyro - KR 4 Figure 11.28 Block diagram of heading control systems For CHARLIE-1 - 0.189 - 0.146 0 0.005 - 1 1.33 - 0.98 0 0 0.06 0.17 - 0.217 0 0 - 0.15 0 1 0 0 0 0 0 0 0 0 0 0 0 - 60 0 0 - 4 0.17 A 0 - 0.33 - = -B = [OOOOO4]'(11.30) C = [IOOOIOO](11.31) D = [0](11.32) Using LQP, or pole placement, or any other appropriate method outlined in Chapter 8, provides a feedback control law. One such law is: u = e = -2.8448 + 11.80 ~+ 53.16r + 23.5 + 47.161)(11.33) With attitude, and rate gyros, and a radio compass, it is possible to measure p, r,and A. Measuring sideslip angle is not particularly simple or successful. However, using the radio compass to measure heading, A, means that the control law of eq. (11.33) can be re-expressed as: +, + e = -2.8841 + 11.80 ~+ 53.16r + 23.5+ + 50 II (11.34) The step response of the system is shown in Figure 11.29. Case A represents the Case A: (11.34); case B shows how little affected is the response if eq. (11.33) is synthesized, but omitting the first term completely, thereby avoiding the need to measure sideslip, or to carry a radio compass. 11.7 VOR-COUPLED AUTOMATIC TRACKING SYSTEM To achieve automatic tracking of an aircraft's lateral path requires the use of a navigation system to provide the AFCS with the appropriate steering commands. Radio navigation systems are very commonly used and the VOR system is one of the most popular and effective of these systems. It is used in conjunction with DME (distance measuring equipment) transmissions so that both, working together, provide a rholtheta navigation system. VOR provides the bearing (0) information. The VOR system operates in the frequency range of 108-135 MHz; DME operates at UHF, in the range 960-1 215 MHz, in the range 960-1 215 MHz. The principle of providing bearing information by means of VOR is relatively simple (see Kayton and Freid, 1969). The ground transmitter has an antenna system which is so arranged that the transmission pattern is a cardioid rotating at 30 rev s-l. When this signal is received in the airborne receiver, the resulting output signal is a 30 Hz sine wave. There is also transmitted from the ground station an omni-directional signal is received on the airborne receiver, the output signal is demodulated 30 Hz tone. There is a phase difference between these two 30 Hz output signals which depends upon the bearing of the aircraft in relation to the transmitter. The beam width of the VOR transmission is relatively coarse, being about f 10". However, it should be remembered that even when a navigation system has a large bearing error it does not mean that an aircraft cannot home onto the source of the bearing information. Accuracy of bearing of 1" is achieved with operational airborne VOR systems. Therefore, VOR guidance can be regarded as accurate for the reception range which, because the transmission is VHF, is line-of-sight, i.e. about one hundred miles. But as an aircraft nears a particular transmitter the system inherently becomes more sensitive. It can easily be understood, from the beam's centre-line the greater is the error angle, T, as range reduces. The output voltage from an airborne VOR receiver is proportional to the measured error angle r. Such an increase in the sensitivity of the receiving system, as the aircraft flies nearer the transmitter, will have a destabilizing effect on the closed loop VOR-coupled system. the distance to the transmitter reduces. The basic geometry of the navigation system is represented in Figure 11.30(b). The output signal from the VOR receiver is proportional to I?. It is this Flight Path Control Systems 382 /~eam centre-line I d !1 0 Range North VOR transmission --- Figure 11.30(b). displacement. (b) Geometry of VOR system. signal which is used as a command signal for the direction control system to drive the aircraft back on to the centre-line of the VOR beam, thereby reducing r to zero. From Figure 11.30(b) it can be deduced that: The error angle, T, is assumed to be not greater than 15", i.e. small. It can also be seen that: If Laplace transforms are taken then: sd(s) where = $(Ud57.3)(+(\sim)- ref(^{)}) + and IJJ, \sim are$ in degrees. A block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram of VOR geometry. Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37)
is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) is shown as VOR-coupledAutomatic Tracking system Note: R R(t) Figure 11.31 Block diagram representation of eqs (11.35) and (11.37) generality. It is supposed that if the VOR-coupled system causes the aircraft to change direction to restore its path along the centre-line of the VOR beam, it will do so in a manner that results in any turn being a co-ordinated one. Consequently, the direction control system of Section 11.5 can be used to provide the required heading, It is the function of the flight controller and the VOR receiver to provide the comparison of the receiver's output signal with Tref (which is zero, by definition) the controller (sometimes referred to as the coupling unit) provides the required command signal, However, to ensure that the complete system performs correctly, even in the presence of a severe cross-wind, there must be an integral term in the control law. Consequently, the controller must have a transfer function at least of the form: +. Of course, Gc may also have a rate term, or a phase advance compensation term, depending upon the nature of the aircraft's dynamics. Note that if the initial capture of the VOR beam is at some large angular deviation, say f 20°, then if the proportional gain, Kc, in the controller is large the bank angle. In the example which follows, only a simple proportional plus integral controller is used, but the reader should remember that these limiting circuits are necessary for practical applications. A block diagram of this system will become unstable since R contributes to the open loop gain. In some systems, the loop gain, is scheduled with range measured from the DME system. No such scheduling is assumed in this system, although it has been arranged for R to reduce linearly with time from Ro to Rmi, (actually Rmi, + 200 m). The response 384 Flight Path Control Systems +,,,,,,,,,,, PC 4 Aileron servo - -12 - lipl* hA KC2 P Aircraft dynamics B --4 + Rate gyro ur 1 P i ,> - uo VOR RX Figure 11.32 Block diagram of VOR-coupled system. of the VOR coupled system to an initial bearing error, for a variety of controller gains, is shown in Figure 11.33. Note that an overshoot occurs when the gain, Kc, is increased from 20 to 30 (with the gain of the integral term being zero). With some integral action present, i.e. Kc = 0.025 s-l, and with Kc = 25, the resulting response has reduced the overshoof! and has 'locked on7 to the VOR bearing in about 80 s. The same response has reduced the overshoof! and has 'locked on7 to the VOR bearing in about 80 s. The same response has reduced the overshoof! and has 'locked on7 to the VOR bearing in about 80 s. The same response has reduced the overshoof! and has 'locked on7 to the VOR bearing in about 80 s. The same response has reduced the overshoof! Rmi, is reached at which point the system becomes unstable. 11.8 ILS LOCALIZER-COUPLED CONTROL SYSTEM ILS equipment is located only at airports in which the runway length is greater than 1 800 m. ILS is often referred to as the instrument landing system, but should more correctly be called the instrument low approach system. It is an important distinction since the system is insufficiently accurate (owing to the nature of the propagation characteristics corresponding to the transmission frequencies) to permit its use by an aircraft right down to t o u ~ h d o w neven , ~ though the system does form an essential element of all aircraft automatic landing systems. The ILS involves a number of independent low-power radio transmissions: Localizer-coupled Control System Time (X lo2 s) Figure 11.33 Response of VOR-coupled approach response. Flight Path Control Systems Table 11.1 ILS transmitter (VHF) Carrier frequency 108-122 monose. Flight Path Control Systems Table 11.33 Response of VOR-coupled approach response. MHz (USA) Radiation: Polarization Horizontal Power 100 W Modulation: Frequencies 90 Hz and 150 Hz Depth (on course) 20% for each frequency (tone) 1020 Hz Depth 5% Voice communication Depth 5% Transmitter (UHF) Carrier frequency Radiation: Polarization Power Modulation: Frequencies Depth (on path) 329.3-335 MHz (USA) Horizontal 2W 400 Hz (outer marker) All marker frequencies Radiation: Polarization Power Modulation: Frequencies Depth 75 MHz Horizontal 2W 400 Hz (outer marker) 1300 Hz (middle marker) 3 000 Hz (inner marker) 95% 1. The localizer which provides information to an aircraft about whether it is flying to the left or the right of the centre-line of the runway towards which it is heading. 2. The glide path (or slope, in American usage) which provides an aircraft with information about whether it is flying above or below a preferred descent path (nominally 2.5") for the aircraft intends to land. 3. Marker beacons which indicate to an aircraft its precise location at fixed points from the runway threshold. Localizer-coupled Control System 387 The characteristics of the radio transmitters involved are summarized in Table 11.1. A representation of the transmission characteristics of the ILS localizer and glide path systems is shown as Figure 11.35. When an airport runway are represented in Figure 11.35. When an airport runway is fitted with an ILS system which is certified to provide category I11 landing information, the third marker is used. It is located at a distance of 305 m (1 200 ft) from the runway threshold: for a touchdown point some 366 m (1 200 ft) from the runway threshold this location of the inner marker means that an aircraft correctly positioned on the glide path will be at a height of 100ft above the ground. This height is the decision height for a category I11 landing (see Section 11.10). Provided an aircraft is equipped with the necessary airborne receivers and aerials for the localizer, glide path and marker transmissions, it has available signals which indicate its location lefttright of runway centre-line and marker transmissions. or whether it is abovelbelow the glide slope, depending upon whether the demodulated 90 Hz signal or vice versa (see Figure 11.35). A different method of using the ILS has been under consideration for many years: it is known as the two-segment approach system. In it an aircraft is required to descend at a rate of about 1400 ft min-' along a glide path of 6" before intercepting, at a height of 800ft, some 5 000 m from touchdown, the normal glide path of 2.5". There has been considerable pilot opposition to the scheme, not least because a failure to effect the transition from steep to normal segment could result in ground impact as much as 2 400 m short of the runway. Nevertheless, the principles involved in the proposed system are the same as those just discussed, apart from the glide path angles and the transition point. Using such output signals, an ILS localizer-coupled control system can be arranged which will steer an aircraft automatically towards a runway, minimizing any deviations from the centre-line of the runway. The block diagram of the system is essentially the same as that given in Figure 11.32 Runway Localizer modulation 90 Hz Frequency 150 Hz Glide, path Localizer beamwidth (depends on topography at airport) Figure 11.35 ILS localizer and glide slope transmissions. Flight Path Control Systems Sited to provide 551t5 ft runway threshold crossing height VHF localizer transmitter antenna array Typically 1 000 - 2 000 m from runway threshold Runway UHF glide slope transmitter antenna Middle marker 75 MHz Morse code: dot - dash repeated Outer marker 75 MHz Morse code: 2 dashes per second Figure 11.36 Location of ILS ground transmitters and antennas. except that r represents the angular deviation from the localizer centre-line, and a localizer receiver is used, not the vor care must be exercised with the controller gains since the beamwidth of the system is less (- 3"). There are also present in the transfer function representing the localizer receiver, the dynamics associated with the low-pass filters needed to remove the 90 and 150 Hz modulation tones from the output signals. The range involved in this system is much less than that which obtains with VOR coupling, being not greater than about fifteen miles, usually less. However, like the VOR hold system, the localizer-coupled control system cannot operate below a certain minimum value of range, otherwise the open loop gain will increase beyond the critical value and the closed loop system to an initial angular displacement of 1" to the right, at a range of 15 000 m, for CHARLIE-1, is shown in Figure 11.37(a). The corresponding values of gains are shown in Table 11.2. The minimum value of range for stability is approximately 200 m; the simulation was stopped when the range reached 1800m. This simulation was only illustrative since the airspeed was maintained at a constant value of Uo = 60.0 m s-l. The response of the same system to a crosswind corresponding to a side gust of f 1" in 10 s, and with the airspeed being reduced steadily from 60 to 40 m s-' throughout the approach, is shown in Figure 11.37(b). Note how effectively the system restores the aircraft to the localizer centre-line and maintains it there: the peak displacement in heading is only 8 x loy4 degrees. The dotted line represents the trajectory corresponding to KCI2 = 0.25 s-l. In Figure 11.37(c) the trajectory is shown for an initial range of 40 000 m and a constant speed of 158 m s-l; the purpose of including this trajectory is to show the behaviour of the system when minimum range is approached and reached: the system becomes unstable 11.9 ILS GLIDE-PATH-COUPLED CONTROL SYSTEM This system uses the output signal from the aircorft. The loop is If S Glide-path-coupled Control System Time (x lo2 s) (a) Time (x 10' s) (b) Figure 11.37 (a) ILS-coupled trajectory. (b) Response to side gust. Flight Path Control Systems Time (x 102 s) (c) Unstable response. Table 11.2 Gains for ILS
localizer-coupled system U o = 60ms-I Ro = 15 x lo3m closed via the aircraft into a displacement from the preferred descent path (the glide path) into the airport. The situation is represented in Figure 11.38(a). The glide path angle is denoted by y, and its nominal value is - 2.5". If an aircraft is flying into an airport, but it is displaced below the glide path by a distance, d, that distance is negative. The geometry is shown in Figure 11.38(b). If the value of the aircraft's own flight path angle is - 2.5", the displacement is 0. Any angular deviation from the centre-line of the glide path transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmitter. Since the value of y, is so small, it is customary to regard the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement, d, and the slant range from the transmission is measured by the airborne glide path receiver: that deviation depends upon both the displacement depend course: In this section, x and R are taken as identical. Therefore, the angular deviation, r, is defined as: Glide path Horizontal TX (a) Glide path r=Angular deviation, or glide path error Ground Figure 11.38 (a) The glide path geometry. (b) Aircraft below glide path - geometry. (c) Angular deviation from glide path - geometry. (d) Slant range definition. Flight Path Control Systems Figure 11.39 Block diagram of glide path is Uo sin r; this guantity represents the rate of change of the displacement, i.e.: d = U, sin r (Ud57.3)r (11.41) where - However, for the situation shown in Figure 11.38 the aircraft's flight path angle is less than 2.9, therefore r is positive, the situation shown in Figure 11.38(c) represents the case when the aircraft is approaching the glide path from below: I d = (Ud57.3) (y + 2.5")dt = (Ud57.3) r d t The block diagra representing eq. (11.42) is shown in Figure 11.39. The aircraft flight path angle is most effectively controlled by using a pitch attitude control system, with a pitch rate SAS as an inner loop, to effectively t, , , ,@ Attitude controller Elevator actuator 2.5" a Glide-path-coupled control system. 57.3 393 ILS Glide-path-coupled control system glide path. The block diagram of a typical glide path control system is shown in Figure 11.40. The gain of the glide path receiver, KR, can be considered, without loss of generality, to be 1V deg-l. The control l a 6 used is then: ~ c o m m = - GC(p)r (11.44) The transfer function, Gc, of the glide-path-coupled controller represents essentially a proportional plus integral term controller. The phase advance term has been added to provide extra stabilization, if required. So far it has been presumed that the airspeed, Uo, is constant throughout the coupled system, is essential to ensure that the aircraft's flight path angle, y, in the steady state, has the same sign as the commanded pitch angle. The speed control system also ensures that the airspeed of the aircraft is reduced from Uol at the start of the approach to a lower value, UO2, at its finish, the change in speed corresponding to the appropriate speed schedule, U,,(t). A typical

speed schedule, for CHARLIE-1, is given in Figure 11.41. At the start of the coupled glide path descent, the airspeed, Uol, is 85.0 m s-1; thirty seconds later, it is 65.0 m sf1. During that time the aircraft will have travelled a slant distance of: Time (s) Figure 11.41 Airspeed schedule for CHARLIE-1. The horizontal distance covered is actually 2 248 m (assuming aircraft descends along the glide path). The height at the start of this manoeuvre is 320 ft. A typical set of parameters, corresponding dynamic response to an initial displacement, d, of 100 ft above the glide path, is shown in Figure 11.42. It is evident how effective the system is in restoring the aircraft to the glide path and maintaining it there subsequently. Flight Path Control Systems Table 11.3, ~it can be shown that the closed loop dynamics can be represented in that form, representing a generalized AFCS, which was explained in Section 7.2 of Chapter 7. Aircraft dynamics where: X' & [U w q 8 SE] ui ! [SE) - $0.021 \ 0.122 \ - 0.22 \ 0.22 \ - 0.021 \ 0.122 \ - 0.021 \ - 0.0$ coupled 'ControlSystem Output equation is: Yc = Ccxc + DCY (11.56) From Figure 11.40 and Table 11.3 it can be deduced that for the glide path coupled control system the control law is: Yc = Ccxc + DCY (11.56) From Figure 11.40 and Table 11.3 it can be deduced that for the glide path coupled control system the control law is: Furthermore, it can be seen that: If we let: I wdt 4 x, 396 Flight Path Control Systems then If the third term on the r.h.s. of eq. (11.57) is denoted by - KAKcg(p) then: Let: Then: +z = r 0.42 + 1.042 + 0.12 = g 0.042 Let: Hence: Hence: A, = 0 0.42 Let: Hence: Hence: A, = 0 0.42 Let: Hence: Hence: Hence: A, = 0 0.42 Let: Hence: Hence: A, = 0 0.42 Let: Hence: H 2.027 h hs, $\$_{-}$ - 4.21 j6.76 Al0 = - 24.944 + Range = 200 rn = 0.0 A3, 4 = 0.026 5 j0.024 A5 = - 4.004 h6 = - 8.91 k7, 8 = - 1.57 k j4.105 = 2.58 - 1 j4.77 A1, 2 + The eigenvalues of the closed loop system just described, which correspond to values of range, R, of 4000m and 200m, are shown in Table 11.4. It can be deduced from these values of the closed loop system just described, which correspond to values of range, R, of 4000m and 200m, are shown in Table 11.4. It can be deduced from these values of the closed loop system just described, which correspond to values of range, R, of 4000m and 200m, are shown in Table 11.4. It can be deduced from these values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the closed loop system just described which correspond to values of the cl loop roots that at a range of 4 000 m the glide-path-coupled system is unstable. In fact, there is a critical value of range below which the system is unstable. In fact, there is a critical value of range below which the system is unstable. The treatment above supposes that the glide slope receiver is located at the c.g. of the aircraft, and measures the aircraft's angular deviation from the glide path at the c.g. However, if the aircraft receiver is installed at, say, the nose of the aircraft, the dynamics of the system are affected, as follows. From Figure 11.43, the height measured at the receiver is: (assuming 9 is small). Therefore the flight path angle at the receiver is: (assuming 9 is small). located in aircraft nose. 399 Automatic Landing System Thus, the effect of locating the glide path receiver in the aircraft's nose is to introduce a phase advance term into the closed loop dynamics. 11.10 AUTOMATIC LANDING SYSTEM Although the contribution to the development of airborne automatic landing systems has been international, the basis of most of the operational systems in service is the system developed in the UK by the Blind Landing Experimental Unit (now disbanded) of the Royal Aerospace Establishment. It makes use of the ILS, and the entire automatic landing segment is made up of a number of phases which are shown in Figure 11.44. At the start of the final approach phase (point 1 in Figure 11.44), the aircraft being considered is assumed to be guided on the glide path by a glide-path-coupled control system described in Section 11.8. What has been described above is a category I1 automatic landing. What distinguishes landings into the various categories are the conditions of visibility. These categories are summarized in Figure 11.45; it can be seen that each category is defined as a combination of the decision height (DH), i.e. the minimum Point 1 Outer marker radio beacon 2 Middle marker radio beacon 3 Start of flare phase 4 Start of KOD manoeuvre 5 Point of touchdown G 10 0 8 m hard shoulder -, L 8 000 m Figure 11.44 / 3 000 m runway length BLEU aircraft automatic landing. Flight Path Control Systems 60 - - E ...* 8 * ... P 00 h .y. > 30I I e"IE - I = I I I I Category IIIa 200 I I I I I A00 600 800 1000 1200 RVR (m) Figure 11.45 Definition of landing categories. I permitted ceiling for vertical visibility for the landing to proceed, and the runway visual range (RVR). Categories I and I1 allow only coupled glide path approaches down to the DWRVR combinations defined in Figure 11.45. At the DH, the pilot either to attempt once more to land at the airport, or to divert to an alternative. Category IIIa allows the aircraft to make an automatic landing by providing an automatic landing by providing an automatic flare and roll-out, with the pilot assuming control only at some distance along the runway after touchdown. Category IIIc performance. The DHs which obtain for categories IIIa and IIIb vary with airline and aircraft type. A summary of some airlines and aircraft is given, merely for illustration, with no intention of being definitive, in Table 11.5. The automatic flare control system is arranged to provide a flare trajectory represents the path of the aircraft's wheels as the landing is carried out. During this flare manoeuvre, the flight path angle of the aircraft has to be changed from - 2.5" to the positive value which is recommended for touchdown; in other words, during the flare manoeuvre the control system must control the height of the aircraft's c.g. and its rate of change such that the resulting trajectory corresponds as nearly as possible to the idealized exponential path shown in Figure 11.46(a), while at the same time causing the aircraft to rotate in a fashion similar to the representation of Figure 11.46(b). The equation which governs the idealized, exponential flare trajectory shown in Figure 11.46(a) is Automatic Landing System Table 11.5 Landing categories for different airlines Airline Aircraft types Minimum values RVR (m) British Airways Lufthansa Air France Swiss ~ i KLM DELTA TWA r ~ DH (ft) Tristar Concorde B-757 A300 A300 Concordea DC-10 B-747 Tristar Tristar " A t Paris, Charles d e Gaulle, only. A t Zurich only. The distance from ho to the point of touchdown depends on the value of ho, the flare entry height, and the approach speed of the aircraft Uo. Usually the point of touchdown, which is aimed for,
is 300 m from the runway threshold which is the nominal location of the glide path transmitter (see Figure 11.35). Assuming that the airspeed does not change significantly throughout the flare trajectory (a not unreasonable assumption), then: Glide oath centre-line Exponential flare trajectory p' . CI., ~~ nntr.i 1LL'C CIIIIJ height Point of touchdown x, I Ground Glide path transmitter (a) Glide path centre-line Point of touchdown Runway threshold Ground Figure 11.46 (a) Flare trajectory. (b) Rotation of aircraft during flare. Flight Path Control Systems ho = Uo sin y = Uo sin(-2.5") (assuming landing speed for CHARLIE-1 of 57.3 m s-l). From eq. (11.84) it can easily be shown that: If the time to complete the exponential flare is taken as 57 then: (x + 300) = u057 = 286.57 (11.87) From eq. (11.86); ho = - h o / ~ (11.88) hence: -2.5 = - h d ~ ... h o = 2.57 From Figure 11.46(a), ho = x tan 2.5" = 0.0435x, therefore: 7 = (0.043512.5)~ Hence, substituting eq. (11.89) yields: x + 300 = - h o / ~ (11.89) yields: x + 300 = - h $300 = (286.5 \times 0.043512.5) \sim :$: T = x=75.3m (11.91) ho = 3.25m = 10.65 ft (11.92) 1.3 s (11.93) Hence, the ideal flare manoeuvre is assumed to take 6.5 s to completion. The law which governs the flare trajectory is given by: (11.94a) h = -0.77h A block diagram of an automatic flare control system is shown in Figure 11.47. Automatic flare controller em, - Pitch attitude control system kinematics I - + T ~ ~ Radio altimeter Figure 11.47 Block diagram of automatic flare control. Automatic flare control. Automatic flare control system is used: changing 0 results in a change in flight path angle, and consequently, a change in height. Because the heights involved are very low, an accurate measurement of height is necessary for this control system: a low range altimeter is used. The control law used can be simply: but,. to ensure accuracy, it is usual to add an integral term, and the need to remove, by filtering, any noise from the height signal obtained from the radio altimeter, tends to destabilize the closed loop system. Consequently, it is customary to include a phase advance network with the feedback terms to improve the stability, i.e.: -4.0 - -12.0 - 16.0 0.0 I 4.0 I I 8.0 12.0 Time (s) Figure 11.48 Flare trajectory response. I 20.0 404 Flight Path Control Systems where p = dldt and TI 9 T2. The results of a digital simulation of such an automatic flare control system for a particular flare entry condition is shown in Figure 11.48. Because the model flare trajectory is exponential it takes infinite time to reach zero height. In the UK, href is taken as - 1.5 ft, thereby ensuring that the wheels will touch the runway at a time much nearer 57 s. Obviously the methods of modern control theory can as easily provide a feedback control laws. 11.11 A TERRAIN-FOLLOWING CONTROL SYSTEM II.II.IIntroduction The terrain following situation is represented in the sketch shown as Figure 11.49. A change of range, the horizontal distance, is denoted by Ax; Ah denotes a change of the height of the aircraft to be considered, which is a representative (or generic) strike aircraft, there are two controls, namely a commanded change in normal acceleration measured at the aircraft's centre of gravity, a, which is denoted ul, and a cg' change in the rate of opening or closing of the throttle which is denoted ul, and a cg' change in the rate of opening or closing of the throttle which is denoted ul, and a cg' change in the rate of opening or closing of the throttle which is denoted by u2. It is customary in terrain-referenced navigation to use, as a technique for determining the aircraft's centre of gravity, a comparison of the return signals of the radar Pushover ++----Ax Figure 11.49 Terrain foliowing geometry. A Terrain Following Control System 405 altimeter, (an accurate, short range, vertical radar system), with those obtained from a three-dimensional terrain model which has been digitized and stored on an on-board computer; this complex method is used to avoid, as much as possible, active sensing of the terrain, thereby reducing the chances of being detected. However, there may be unmapped obstacles, so that terrain ahead using a laser rangefinder, or a forward-looking infra-red (FLIR) system, is often taken. For the purposes of explaining the terrain-following control system it will be assumed that range can be measured by FLIR or laser rangefinder and that changes in height are measured by FLIR or laser rangefinder. 11.11.2 Equations of Motion The aircraft's equations of motion, assuming a stability axis system and small perturbations, can be expressed as: Note that the elevator is not being used as a control. To relate the aircraft's motion at point A to the obstacle, and thereby to design an effective control system to control the aircraft automatically to avoid the obstacle, and thereby to design an effective control system to control system to control system to control system to control the aircraft automatically to avoid the obstacle, and thereby to design an effective control system to cont being studied, the flight path angle at point A is assumed to be zero. Hence: :. h = UoO - w = - a, cg Therefore: Flight Path Control Systems Figure 11.50 Block diagram of terrain following system. Any change in thrust depends upon u2, and, as a first approximation, the aircraft engines are represented by a simple, linear model, namely: The system is represented in the block diagram shown as Figure 11.50; KE represents the gain of the engine and TE its time constant. From Figure 11.50 it may also be deduced that: ~ = ~ ~ - h (11.108) By defining state variables in the way shown in Table 11.6 it is possible to obtain as a mathematical representation of the aircraft dynamics involved in the terrainfollowing situation a state equation, namely: Table 11.6 State variables definition for terrain-following system State variable Denotation XI Vertical acceleration Rate of change of height Height Thrust commanded Change in thrust Change in thrust X3 X4 Xs X6 h h STH, ~ T H u 407 A Terrain 0 0 1 (11.112) 0 -0 0 0 0 0 1 - The following parameters and coefficients obtain for the strike aircraft at Mach 0.85 at a height of 40000ft. Table 11.7 Output variable XI Vertical acceleration Height Thrust rate Change in thrust Change in airspect X3 X4 X5 X6 - X5 Flight Path Control Systems Hence: The aircraft dynamics are represented in Figure 11.50. 11.11.3 The Control System By framing the terrain-following problem as a LQP, with an appropriate performance index to be minimized, the feedback control is found as a solution to the LQP. The performance index is chosen to be: where: z denotes the vector defining the desired flight path. This vector is defined in terms of vertical acceleration, height, required thrust rate, thrust and set speed. These desired inputs have to be available as instantaneous functions of time. Therefore: el denotes the error in the vertical acceleration, e2 the deviation in the aircraft's height from the desired path, e3 the error in the thrust rate, e4 the error in the thrust being developed, and e5 the deviation in the aircraft's speed from the reference value. A method of solving such an LQP was discussed in Section 8.4 of Chapter 8, from which it can be learned that the required feedback control is given by: A Terrain Following Control System 409 where: = (A - B G - ~ B ' K) '-~ C'QZ with: K is the solution of the corresponding ARE. It should be appreciated that the feedforward function, h(t), involves z as a forcing function, h(t), involves z as a forcing function, h(t), involves z as a forcing function and is obtained by eqs. (11.120) in 'reverse' time (i.e. by starting at t = a and returning to t = 0). To illustrate one particular solution, for the strike aircraft defined by eqs. (11.109), (11.112), (11.114) and (11.115), the parameters given in Table 11.8 Elements for weighting matrices in LOP The reference vector, z, is chosen to be: i.e. the aircraft encounters a sudden step change in the terrain of 200 ft. Using matrices A, B, C, Q and G so far given, the resulting control law is given by: The appropriate variables, hl, h2, h3, h4, h5, h6, which make up the vector, h, are shown in Figure 11.51. These represent the solutions to eq. (11.120) for the forcing vector z
given in eq. (11.122). The resulting terrain following response for an initial aircraft height of 300ft is shown in Figure 11.52; the variables, yi, correspond to the definitions given in Table 11.6. Note how height (y2) changes from 300 to 95 ft and then to a peak height of 350 ft before settling to the required height of 350 ft before settling to the required height (y2) changes from 300 to 95 ft and then to a peak height of 200 ft, after 1 minute of flight time. In executing that manoeuvre the 4 10 Flight Path Control of the required height of 350 ft before settling to the required height of 350 ft before settling to the required height of 200 ft, after 1 minute of flight time. Systems Curve A = hl Curve B = h2 Curve C = h3 Time (X lo2 s) (a) h5and h6 o...H /A- Time (X lo2 s) (b) Figure 11.51 Solution of eq. (11.120) for given z. A Terrain Following Control System Curve A = y, Curve B = y2 0.01 0.0 I 0.2 I 0.4 I 0.6 I 0.8 Time (X lo2 s) Time (X lo2 s) (b) Figure 11.52 Response of terrain following system. I 1.0 4 12 Flight Path Control Systems aircraft has to undergo a - 2g to + 2g (approximately) change in about 10 s. The aircraft is required to go supersonic at t = 30 s: this is most improbable. To avoid such a speed excursion it is necessary to solve the problem again with a much larger value of g5, to penalize such deviations in airspeed. 11.12 CONCLUSIONS Although the chapter is devoted to path control systems, it is opened with detailed studies of automatic control systems, it is opened with detailed studies of automatic control systems. transmitter. Direction and heading control are treated next, so that they can be used as elements in the automatic tracking systems which depend upon the radio transmissions and appropriate airborne receivers for VOR, ILS localizer and glide path to obtain the appropriate guidance commands for these tracking systems. From a detailed consideration of ILS localizer and ILS glide-path-coupled systems it was the next step to consider automatic landing, including a flare phase, and then finally a terrain following system which allows an aircraft to be guided automatically over ground obstacles. 11.13 EXERCISES 11.1 A business jet aircraft uses a speed hold system to assist the pilot with ILS coupled approaches. The block diagram of the system is shown in Figure 11.53. The following parameters and stability derivatives relate to the aircraft: $U_0 = 72m$ spl W = 98065N Maximum thrust = 54655N Xu = -0.0166 LID (on approach) 2, = -0.175 = 8.0 Controller Throttle and actuator ~osition Engines Change in thrust Aircraft dvnamics Pitot system Figure 11.53 Block diagram of a speed control system for Exercises 413 (a) If the system has 20 per cent authority, calculate the gain of the engines, K E . (b) If the time constants associated with the engine, the throttle actuator and pitot system are all negligible show that it takes 167s to achieve a new commanded speed when K1 = 1.0. (c) If the effective time constant of the engines is 1.0 s, K2 = 7.5 and K3 = 5.0 what is the maximum value that K1 can take before the closed loop system becomes unstable? 11.2 (a) For the landing approach represented in Figure 11.54 an aircraft is coupled to the glide path via its receiver and its AFCS. For any given departure from the glide path measured by the perpendicular distance, d, show that the angular error of the aircraft from the nominal glide slope increases with the integral of the flight path angle. Assume the airspeed is constant. transmitter Figure 11.54 Glide path geometry for Exercise 11.2. (b) The flare manoeuvre, performed at touchdown, results in the rate of descent of an aircraft being decreased in an exponential manner. If an aircraft has a contstant forward speed of 80 m s-' and the distance to the touchdown point from the runway threshold, at which the glide path transmitter is located, is 500 m, calculate the following for a glide path angle of 2.5": (i) The approximate time to execute the flare manoeuvre. (ii) The height of the aircraft at the start of the flare manoeuvre. (iii) The ground distance travelled during the manoeuvre. (c) Draw a block diagram of a complete automatic landing system which uses the glide path transmission to measure the departure of the aircraft from the approach trajectory. Show clearly all the significant variables of the system and indicate the function of each block. Detailed transfer functions need not be shown. 11.3 The bank angle control system whose block diagram is shown in Figure 11.18. With a value of controller gain, KT, of 2.0 the response was very slow, the settling time being about 28.0s, although the response was well damped with no oscillatory response, which is unacceptable. (a) Find a value for the gain, KT, of the controller of the direction control Flight Path Control System such that the direction response to a step command exhibits no overshoot and settles within 7.0 s. (b) What is the maximum value which KT can take before the direction control system to the sideslip cross-wind profile shown in Figure 11.22. 11.4 A cargo aircraft, with four turboprop engines, has been proposed for use as a gunship for counter-insurgency (COIN) operations. A control system is to be used so that the errors in positioning the aircraft in relation to its target are minimized, even in the presence of head and cross-winds. Three aerodynamic control surfaces - elevator, ailerons and rudder - are to be used. The state variables of the aircraft have been defined as follows: change in sideslip angle, f4 change in gitch rate, g change in gitch attitude, 0 target attitude error, ET, target elevation error, ET ~ target azimuth error, ET ~ target azimuth error, ET ~ target azimuth error, A, and the driving matrix, B, are: - 0.013 21.35 - 8.17- 15.11 - 0.95 corresponding feedback gain matrix. (b) Determine the eigenvalues of the closed loop system. Compare these with the values corresponding to the uncontrolled aircraft. (c) Does the closed loop system reduce the target errors in response to a crosswind? 11.5 The following stability derivatives relate to a VTOL aircraft in hovering motion: - 2.78 x 1 0 - ~ his used to control the If a feedback control law, ST = h,,,, aircraft, show that its change of height is characterized by a critically damped transient mode. At what time after a step input of h, does the rate of change of height reach its maximum value? Exercises 477 The block diagram of a speed control system used with the aircraft dynamics - Accelerometer Figure 11.55 Block diagram of a speed control system for Exercise 11.6. Making use of appropriate numerical approximations determine the maximum permissible value of accelerometer sensitivity for stability of the closed loop speed control system. 11.7 The glide slope. The receiver is mounted in-line with the aircraft 's c.g., but directly below the pilot. (a) Determine the transfer function y 1 (s) / 0 (s) which is appropriate to this location of the receiver. (b) Show how Figure 11.40 must be modified to take account of the receiver. (c) Is the location of the receiver. (b) Show how Figure 11.40 must be modified to take account of the receiver. (c) Is the location of the receiver. (c) represented in Figure 11.5 with K, = 1.5 and an accelerometer time constant of 0.5 s, find a suitable for localizer coupling, for the aircraft approach. It can be assumed that over the approach phase the aircraft speed is constant. At what range from the runway threshold will your system performs in the presence of a constant cross-wind from the right (i.e. from starboard) of 20 knots. ALPHA'S 11.10 The VOR-coupled control system, represented by the block diagram of Figure 11.32, uses the directional control system of Figure 11.32, when the value of the value of K*, evaluated in answer to Exercise 11.3(a), with the system of Figure 11.32, when the value of the value of the value of the value of K*, evaluated in answer to Exercise 11.3(a), with the system of Figure 11.32, when the value of the of the gain associated with the integral term is 0.025. Compare the response obtained with that shown in Figure 11.33. Which is better? Suppose Kc, is increased to 0.25. What is the likely effect upon the performance of the VORcoupled control system? Is the integral term is 0.025. Compare the response obtained with that shown in Figure 11.33. Which is better? merely a euphemism used in aviation circles to describe the bad weather which involves the use of transmission frequencies which avoid those errors which limit the ILS. However, the principles involved in providing the guidance signals are sufficiently similar for the account being given here based on the ILS to suffice. Although it is not shown in the diagram, the speed control system (of the type represented in Figure 11.9) is assumed to operate and it has the following parameter values: KE = 30 000, KCu = 3.0, KuI = 0.4. 2. 3. 11 .I5 REFERENCES BLAKELOCK, J. 1965. Automatic Control of Aircraft and Missiles. New York: Wiley. KAYTON, M. and W.R. FREID. 1969. Avionics Navigation Systems. New York: Wiley. McRUER, D.T., I.L. ASHKENAS and D.C. GRAHAM. 1973. Aircraft Dynamics and Automatic Flight Control. Princeton University Press. 1979. Airplane Flight Dynamics. Kansas: Roskam Publishing. ROSKAM, J. Active Control Systems 12.1 INTRODUCTION Although there is now available a large amount of published material relating to active control technology (ACT), which is being added to continuously, there are few satisfactory definitions of what ACT is. It has been called an extension of conventional feedback control systems which provides a multi-input, multi-output feedback control systems definition is complete. In this textbook, it is proposed to use a definition which is based upon those earlier versions, namely: Active control technology is the use of a multivariable AFCS to improve the manoeuvrability, the dynamic flight characteristics and, often, the structural dynamic flight
characteristics and often an appropriate of a multivariable approprise of number of control surfaces and auxiliary force or moment generators in such a fashion that either the loads which the aircraft produces a degree of manoeuvrability beyond the capability of a conventional aircraft. The purpose of ACT is to provide AFCS with the additional means to increase the performance and operational flexibility of an aircraft. Modern aircraft, and for the missions they are to perform, are such that the resulting configurations are considerably changed from the familiar designs of earlier times. In meeting the new requirements, the designs duriting a high level of stress, and low load factors. These features have resulted in aircraft which are of the required structural displacement and accelerations of large amplitude as a result of the structural deflections and the rigid body motion of the aircraft. The structural deflections and the rigid body motion of the structural deflections and the rigid body motion of the aircraft. can arise either as a result of some manoeuvre Active Control Systems 420 command from the pilot, or from a guidance or weapons system, or from a guidance or weapons system, or from encountering atmospheric disturbances. The structural vibration which results can impair the life of the airframe because of the repeated high levels of stress and the peak loads to which the aircraft is subjected. With such new aircraft, a new class of flight control problems has emerged, including: the need to minimize the loads experienced by the aircraft to precisely control the location of an aircraft's c.g. over its entire flight envelope; the suppression of flutter; the reduction of the amplitude of the disturbed motion of an aircraft which is caused when it encounters turbulence. To solve such problems by using modifications of the conventional control surfaces, usually by increasing them, even in the few cases where there are feasible solutions, would impose severe economic penalties upon the aircraft's operation, mostly in terms of reduced performance in terms of range or speed. Consequently, ACT was proposed to meet the evolving demands for more effective and efficient aircraft. 12.2 ACT CONTROL FUNCTIONS It is generally agreed that the most beneficial effects of using ACT will be secured by using any, or all, of these six ACT functions: relaxed static stability (RSS), manoeuvre load control (MLC), ride control (MLC), ride control (MLC), ride control (RC), flutter mode control (RC) By relaxing the requirement for static stability it is possible to achieve better dynamic controls and to reduce the trim drag and thereby enhance the aircraft's dynamic stability. It is necessary when doing this to restore the aircraft's dynamic stability and its handling qualities by using an ACT system. When the need for static stability is relaxed, the empennage required on the aircraft is smaller: an empennage is sized to provide the aircraft are possible. An SAS for an aircraft with RSS - aircraft BRAVO of Appendix B - is discussed in detail in Section 9.4 of Chapter 9. By using RSS on an aircraft it becomes feasible to provide 'carefree manoeuvring'; sometimes the RSS function is an element of an aircraft during a manoeuvre. By the symmetrical deflection of control surfaces, mounted at proper stations on the trailing edge of the wing, in response to load factor commands, it is possible to reduce the increments in the stress by arranging for an inboard shift of the wing. This shift also reduce the increments in the stress by arranging for an inboard shift of the wing. wing. MLC is sometimes referred to as active lift distribution control (ALDC), or a structural mode control (SMC) system, which it is called on the USAF bomber, the B-1. Occasionally such systems are provided simply to ensure that any loads which arise from the execution of some particular manoeuvre do not exceed some specific limit. The purpose of the RC system is to improve ride comfort for the crew or passengers by the reduction of objectionable levels of accelerations which are caused by the rigid body and/or structural motion of the aircraft. For an aircraft carrying passengers the requirement may be for a reduction in accelerations to be achieved over the whole length of the passenger cabin. For an interdiction aircraft, carrying out high speed strike missions at low level, the principal need is to prevent the pilot's ability to track his target from being impaired by the accelerations at the cockpit which are a result of flying in turbulence. Obviously the mission requirements considerably influence the purpose and the design of the system used to improve an aircraft's ride characteristics. 12.2.4 GLA GLA is a technique which controls the contribution of the rigid body and the bending modes to the complete dynamic response of an aircraft to a gust encounter. Its purpose is to reduce the transient peak loads which arise from such encounters. Since their purpose is similar in nature to that of RC, the two functions are often achieved by a single system. Moreover, a successful GLA system will contribute to the reduction in structural loading so that MLC and GLA are quite likely to be used in conjunction with each other. 12.2.5 FMC By properly controlled deflection of certain auxiliary control surfaces it is possible to damp the flutter modes of an aircraft without having to increase in flutter speed. The principal benefit of FMC in fighter and strike aircraft is a resulting increase in the permissible wing-mounted stores which can be carried within the same speed envelope. The benefit for bomber and transport aircraft to a possible reduction in the weight of the wing. Of all the ACT functions, FMC is the most sensitive to configuration, particularly to planform and thickness of the wing. To reduce the rate of fatigue damage, FR systems minimize the amplitude and/or the number of transient bending cycles to which the structure may be subjected during flight in turbulence. This ACT function has not yet been implemented physically on any aircraft, and, at present, its objective is achieved indirectly as a result of the combined action of the other five ACT functions. It remains, however, as potentially one of the most economically advantageous ACT functions depend upon several aircraft parameters. However, the only function which will provide its benefits independent of the speed range of the aircraft because of the low wing-loading which such aircraft because of the low wing-loading which such aircraft because of the speed range of the speed ran commercial operation, supersonic transport aircraft must operate over a very wide range of dynamic pressure which greatly affects the handling qualities. Some form of stability augmentation is required to provide the dynamic stability augmentation is required to provide the dynamic stability augmentation. in addition to improving the handling qualities. SST, and modern bombers such as the B-1, have a long slender fuselage, the forward position of which acts as a cantilevered beam mounted forward of the stiff structure. This configuration results in the lateral and vertical accelerations in the forebody having natural frequencies of about 1Hz which is a vibration frequency causing considerable discomfort to passengers and crew alike. Hence, on such aircraft, an RC system is needed. On the B-1, the SMC system also acts as a ride control system at low altitudes; the function is referred to in this aircraft as a low altitude ride control system. altitude, over long stage lengths. Consequently, new transport aircraft have their configurations designed particularly for energy conservation. As a Gust Alleviation 423 result, the aircraft's handling qualities in high altitude turbulence will be poor and, therefore, a GLA system will be required. The principal area in which the many benefits of ACT will be seen to greatest advantage is that of aerial combat. Some present-day fighters required to provide six degrees of freedom, even though the two extra degrees of freedom in translation provide. The extra degrees of freedom which ACT can provide that improvement in manoeuvrability which can lead to superior combat tactics. 4 Yawing moment Direct sideforce (drag modulation) Rolling moment Direct lift force pitEhing moment Figure 12.1 Six degrees of freedom of an aircraft flies is constantly in turbulent motion. Consequently, the aircraft's aerodynamic forces and moments fluctuate about their equilibrium (trimmed) values. These changes cause the aircraft to heave up or down, to pitch its nose up or down, to roll about the axis OX, or to yaw from side to side about the aircraft's heading. These motions result in accelerations it is necessary to cancel the gust effects by other forces. The general principle of gust alleviation is that specially located sensors provide motion signals to a controller which causes appropriate deflections of suitable control surfaces to generate additional aerodynamic forces and moments to cancel the accelerations caused by the gust. achieving such alleviation have been proposed - almost since the beginning of manned flight, Lillienthal, was killed in 1896 when his glider was so upset. A patent was granted in 1914 in the USA to a Mr A. Sprater for a 424 Active Control Systems 'stabilizing device for flying machines'. The device was claimed 'to counteract the disturbance and to prevent it from having an injurious effect on the stability of the machine'. In 1915 the very first NACA report by Hunsaker and Wilson (1915) contains a reference to the problem. Continual reference to the problem was made by early British workers in aircraft stability and control from 1914 up to the Second World War. The foundation papers which established the basis for suitable mathematical representations of turbulence were published by Von Karman (1937). There was a proposal for a gust alleviation system in 1938 by a
Frenchman which was eventually flight tested in the USA in 1954. In 1949, the Bristol Brabazon aircraft (then the largest in the world) was fitted from the design stage with a GLNMLC system was provided, the wing structure of the prototype aircraft was 20 per cent weaker than the design figure required to meet the specified discrete gust levels (with a GLA system). The Brabazon system used symmetrical deflections of the ailerons in response to signals from a gust vane mounted on the aircraft's nose. The system was not proven in flight before the project was scrapped in 1953. A series of flight tests with other aircraft types were carried out in the USA in the period 1950-1956 and experiments were carried out from 1955 to 1960 by the RAE in England using an AVRO Lancaster. All these attempts, except the Brabazon, were concerned solely with alleviating the effects of gusts on the rigid body motion. In every case, however, the results achieved were unsatisfactory. In the case of the RAE experiments with the Lancaster, a considerable loss of stability was observed which arose as a result of the larger pitching moment which was created by the symmetrical aileron deflection. This moment led to a decrease in the effectiveness of the alleviation system at large gust gradient distances. The American systems, like the Brabazon system, depended upon a gust vane to detect the aircraft's entry into the gust field by sensing either changes of pressure or a change of direction of the relative wind. They were unsatisfactory chiefly because it was not appreciated that any gust has components normal to the plane of symmetry of an aircraft and because secondary effects, such as changes in flight condition, downwash effects on the tailplane, the time delay between the wing's encountering the gust and then the tail, were not considered. Thus, gust vane systems tried, in effect, to provide control correction in advance of the actual gust and were really feed forward systems. system could not be designed then to provide the necessary speed of response, nor made insensitive enough to the secondary effects mentioned earlier. In many of these tests, the operation of the GLA systems is that, when a gust has been sensed, the system cannot take action until it is too late to achieve much effect. These defects were noted and avoided by Attwood et al. (1961) who proposed in their patent applications and should use auxiliary control surfaces to produce the countering forces and moments required to minimize the unwanted accelerations. Some further developments continued from that work including, notably, the prototype UK fighter-bomber, the TSR-2, which depended upon augmented static directional stability to reduce its sensitivity to side gusts in its high speed, low altitude role; and also the prototype American bomber, the XB-70. It was an event in 1964, however, which accelerated the present interest in gust alleviation. A B-52E bomber of the Strategic Air Command of the USAF encountered severe turbulence, with an estimated peak velocity of 35 m s-l, on a low-level mission over territory in the western USA. Approximately 6 s after penetrating the gust field its yaw damper was saturated and the response of the then 'unaugmented' rigid body dynamics was such that about 80 per cent of the fin broke off. This event led in 1965 to an extensive flight development programme, known as the load alleviation and mode suppression (LAMS) program, being carried out by the USAF and its contractors. The results of the programme was presented in the RCS which was developed to provide improved ride quality (Stockdale and Poyneer, 1973). 12.4.2 Gust Alleviation Control The amplitude of the response caused by the structural vibration excited by turbulence may be reduced or any energy which is absorbed by the bending modes is rapidly dissipated. Both methods should be employed simultaneously for optimal effectiveness. To reduce the energy being transferred requires a countering moment (or force) from the deflection of some control surface. The method requires an accurate knowledge of the aircraft's stability derivatives change with flight condition, with mass and the mass distribution of the aircraft with changes in dynamic pressure, etc. Consequently, the aircraft dynamics are known too imperfectly to admit of perfect cancellation of any gust forces or moments. Once the energy has been absorbed, its dissipation can be controlled by augmenting the damping of the elastic modes. It is difficult, however, to achieve a sufficient increase in structural damping by such a method if there structural modes are close in frequency, for then they are usually closely coupled, and there is then a periodic exchange of energy between the modes which corresponds to the behaviour of very lightly damped structures. To actively suppress the bending of a structure it is necessary to be able to sense either the structural displacements or the associated rates of change. It is possible to sense these quantities to provide motion signals for feedback in the GLA control system. However, as it had been discovered in the early GLA tests, the control system. consequently, auxiliary control surfaces are required. 12.4.3 Ride Quality Almost every modern aircraft has an SAS which is used to control its rigid body motion, and for which the locations of the sensors are carefully chosen to pick up the minimum of spurious signals from any structural motion. Such SASs do not control or deliberately alter the structural vibration of the aircraft. Yet it should be remembered that such SASs do provide a large amount of reduction of the unwanted motion produced by an aircraft in response to any gust disturbance. However, from operational records and simulations, it is known that those symmetrical structural modes with the lowest natural frequencies contribute substantially to the levels of acceleration which are present at various points of the fuselage, such as the cockpit. For example, it has been found that at the crew stations on the B-52E, without any SAS, 60 per cent of the total normal acceleration measured at those locations could be attributed to the first three longitudinal bending modes. of the remaining 40 per cent, three-quarters was due to rigid body motion and the other quarter was caused by the structural modes of highter frequency. If the accelerations are unacceptable, resulting in discomfort for passengers or crew or impairment of the pilot's ability to fly, then an RC system is needed to reduce the accelerations being experienced at particular locations. One of the best methods of designing such a system is to solve the LQP, discussed in Chapter 6. 12.5 LOAD ALLEVIATION SYSTEM FOR A BOMBER AIRCRAFT 12.5.1 The Aircraft Dynamics The differential equations which represent the B-52E heavy bomber can be expressed as: where x represents the state vector relating to longitudinal motion. In deriving the longitudinal equations it was assumed that the rigid body motion of the aircraft was adequately represented by the short period approximation. Included in both sets of equations were the dynamics associated with five structural bending modes: in the longitudinal set there were modes 1, 5, 7, 8 and 12, and in the lateral set they were 1, 2; 3, 9 and 10. The 427 Load Alleviation System for a Bomber Aircraft control inputs which were employed for longitudinal motion were the deflections of the elevator and a horizontal canard; for lateral motion the three control inputs were the deflections of aileron, rudder and vertical canard. For longitudinal motion, the state vector x is defined as: a and q have their usual meanings of angle of attack and pitch rate, respectively. hirepresents the vertical displacement of the ith bending mode. The corresponding control vector, u, is defined as: For lateral motion, the corresponding vectors are defined as: xi = [V Pr + 4~ YI ?I YZ ?2 ~3 5 3 7 9 9 9 YIO (12.5) ?lo] The corresponding matrices A and B and Al and B1 are defined in eqs (12.7) to (12.10) (given in Figures 12.2 and 12.3). Since load alleviation is being considered, normal and lateral acceleration are motion variables of primary concern. If the measured normal acceleration at the pilot's station, location A, is taken as the output variable, y, it is easy to show, from the material presented in Section 2.7 of Chapter 2, that: (12.11) y 4 a,* = CAx + DAu where the state vector in (12.11) comprises solely the short period motion variables. - - 1.6 6.57 O -7.2 0 -1.35 A = 0 -2.1 0 0.31 0 -1466.1-1.75 - Matrices A and B. Active Control Systems 428 Figure 12.3 Matrices Al and B1. When bending effects are included in the aircraft dynamics, the acceleration becomes: where @A,i is the ith bending mode slope at body station A. Consequently, the matrices CA and DA in eq. (12.11) must be altered to account for the structural motion augmenting the state vector by the variables associated with the bending modes. For B-52E, location A is 4.4m from the tip of the nose of the aircraft (hence X A = 17.48 m)' and it can be shown (see Burris et al., 1969) that: It can also be shown from the work presented in Section 2.7 of Chapter that, at body station A, the lateral acceleration, with structural bending effects included, is given by: where rAJis the slope of the jth bending mode curve at body station A. load Alleviation Note: Eigenvalues have been denoted by X i; the symbol hi does not denote here the displacement of the ith bending mode. For the B-52E, at the pilot station: CIA= [- 0.072 - 3.56 34.6 0.98 0 14.94 0.29 - 181.77 - 4.1 100.5 3.8 - 7531.0 - 18.54 - 3321.35 - 3.181 DIA= [- 1.401 - 0.94 - 1.0431 (12.16) (12.17) The eigenvalues corresponding to the matrices A and Al are given in Table 12.1. 12.5.2 Alleviation Control System Designed as Optimal Control System Since both the normal and lateral accelerations are linear functions, it is possible to use in a gust alleviation system the feedback control
which results from minimizing the performance index, J, where: The performance index in eq. (12.18a) corresponds to longitudinal motion; if lateral motion was being considered, the appropriate performance index would be: ll m J = (x;Qlxl + u;G1ul)df (12.18b) Active Control Systems Figure 12.4 Feedback matrices. and Q (Q1) is a symmetric, non-negative definite matrix, weighting the elements of the state vector, x(xl), and G (GI) is a symmetric, positive definite matrix weighting the elements of the control vector, u(ul). (See Section 8.4 of Chapter 8.) When Q (Q;) was chosen to be a diagonal matrix such that the state variables corresponding to rigid body motion were weighted at unity, while those state variables associated with the flexible modes were weighted at 10.0, then when G (GI) was taken as I2 (I3), the resulting feedback control matrix, obtained by solving the associated LQP, for longitudinal motion, is given by eq. (12.20). Both are shown in Table 12.2.4. The eigenvalues corresponding to the closed loop alleviation control system are shown in Table 12.2.1. Note from a comparison of Tables 12.1 and 12.2 how, for longitudinal' motion, the damping ratio of the first bending mode (X3, X4) is unaltered in the closed loop gust alleviation optimal control system Load Alleviation System for a Bomber Aircraft Time (s) -0.2 Time (s) 0.2 Time (s) (J) Figure 12.5 Acceleration. reduced. For lateral motion, the damping of the rigid body motion is very much augmented. Note how the dutch roll mode, for example, has been changed from a very lightly damped, oscillatory mode to become two real modes (A3, A4). The same effect has occurred with the first bending mode (A5, A6) This substantial increase in the damping of the third and tenth lateral bending modes. Some acceleration responses as a result of an initial disturbance are shown in Figure 12.5 for the uncontrolled aircraft when fitted with the optimal gust alleviation control systems. It is evident that the accelerations occurring at the pilot's station have been reduced by the optimal control systems. With another choice of weighting matrix for the state vector, namely: for longitudinal motion, and: for lateral motion, it is found that the control law, which results from minimizing the performance index, depends almost entirely on the state variables which 2.0 - a(0)=lo 0 .- .* -8 2 0 -2.0 - 1 2 3 Time (s) 4 5 + 3 -0.5 Time (s) Figure 12.6 Optimal responses of gust alleviation system. (a) Normal acceleration. (b) Lateral acceleration. I Active Control Systems 432 contribute most to the output a, (or a,,), e.g. for longitudinal motion, using CA 4 defined in eq. (12.13), the resulting feedback matrix, K, was found to be: G was again taken as 12. The elements denoted by the symbol * were all less than 1 x lo-' and consequently can be neglected. The only gain associated with variables of the bending motion is the 0.41 associated with the vertical displacement of X I without fitting a sensor specifically to measure it. If no measure can be found the feedback gain matrix of eq. (12.23) cannot be synthesized directly and the optimal control law to achieve gust alleviation methods were outlined in Chapter 8. 12.5.3 Sensor Blending Suppose an attitude gyro is located somewhere on an aircraft to measure pitch attitude. If the aircraft is flexible, the output signal from the attitude gyro will contain components proportional to the displacement of each significant, the output signal from an attitude gyro located at point number 1 will be given It two more attitude gyros, are located on the same aircraft, but at points number 2 and 3, say, their output signals can be represented as: ~2 ~g Let: + k22X1 + k23X5 = kgla + kg2X1 + k23X5 = kgla + kgla + kg2X1 + kgla + of M are the blending gains. For example: By combining the signals obtained from these three, independently located, attitude gyros in the correct proportions it is possible to obtain the displacement of the first bending mode which may then be used with the feedback control corresponding to eq. (12.23). Note that the dynamics associated with each gyro have been considered to be negligible: this is a very important assumption for sensor blending. 12.5.5 Model-following control for gust alleviation that all the state variables, including the bending modes, of the gust alleviation system were to behave as first order modes, of rapid subsidence, then the technique of implicit model-following dealt with in Section 7.3 of Chapter 7, it can be shown that: K = [C B] ~(TC - C A) For longitudinal motion of the B-52E, suppose that the model matrix, T, was chosen to be: The resulting feedback matrix K can easily be found to be given by: Active Control Systems 434 It is most obvious from inspection that an arbitrary choice of model matrix has resulted in a feedback control law which depends very heavily on the rates and displacements of the bending modes: if sensor blending were required to obtain the feedback signals required, the design would be far too expensive, involving thirty attitude and rate gyros. A far better approach is to restrict the definition of the output matrix, C, to be c4= [I4:01 (12.38) so that only the rigid body motion and the first bending mode are controlled. Let: T 4 diag[-10.0-5.0] 1.0 - 4.01 (12.39) The corresponding feedback matrix, obtained by using implicit model-following theory, is now given by: The acceleration response to an initial disturbance in angle of attack for this model-following control is represented in Figure 12.7. The feedback control derived by this method is seen to be as effective for gust alleviation as that obtained from solving the LQP. Interested readers can find further discussion of load alleviation systems in Burris et al. (1969). "Time (s) -1 Figure 12.7 Model-following control system response to normal acceleration. C RCS for a Modern Fighter Aircraft 0.0094 0.031 Horizontal tail actuator p+20 7.5p Symmetricalaileron actuator & -,Yrn - * - 20+ FASrn p+20 K2 Aircraft dynamics 0.046 K4 0.0067 Figure 12.8 Ride control system. 12.6 A RIDE CONTROL SYSTEM FOR A MODERN FIGHTER AIRCRAFT A modern fighter aircraft of the type represented by aircraft ECHO of Appendix B has an RCS fitted to provide a better aircraft path through low level turbulence so that the pilot's weapons tracking ability is not impaired by the accelerations which arise at the cockpit. A block diagram is shown in Figure 12.8 from which the following features will be noted: 1. There are two control surfaces employed: the horizontal all-moving tail and the symmetrical ailerons. 2. The motion variables sensed are angle of attack and pitch rate. 3. The control law being used in the pitch rate SAS is a proportional plus integral control. 4. The loop which acts through the symmetrical ailerons is washed out. 5. The equations of short period, longitudinal motion for the ECHO-2 (at Macli 0.81 at 4 600 m) are: 436 Active Control Systems The feedback control gains Kl to K4 may be found by using any of the design methods outlined in Chapters 7 and 8, but the preferred method is to use the LQP technique of Chapter 8 to minimize the performance index which results from considering the ride discomfort (RD) index (of Section 6.6). It is shown there that: where the subscript HT signifies horizontal tail. Thus, if a, cg and the control deflections are minimized by the optimal control system, then it can be shown (see eq. (2.130)) that: y=Cx+Du For ECHO-2, C and D can be shown to be: From eq. (12.41) it can be shown that: Thus, minimizing: where: results in the control law: where: Aircraft Positioning Control Systems Time (s) Figure 12.9. The response for RCS. The response to low-level turbulence is shown in Figure 12.10. The r.m.s. value of the intensity of the vertical velocity gust was 0.3 m s-l. 12.7 AIRCRAFT POSITIONING CONTROL SYSTEMS 12.7.1 Direct Lift control (DLC) and Sideforce Generation There are four positioning tasks which reguire great precision in aircraft: air-toground weapons delivery; air-to-air combat; in-flight refuelling, and all-weather landing. By using direct lift control (DLC) and direct sideforce generation (DSFG) it is possible to furnish an aircraft with additional degrees of freedom. Using DLC considerably enhances an aircraft with its wing level. See Figure 12.11. By this stage of his reading, the reader will be familiar with the idea that, in controlling conventional aircraft, a pilot can command angular rates in the three axes of pitch, roll and yaw. Such angular rates are achieved by the existing surfaces. But the direct control of translation in such conventional aircraft is restricted to what can be achieved by using the throttles or any speed brakes. And the use of these controls inevitably also generates moments simultaneously. There are no dedicated control of translation in the normal and lateral direction but, by using DLC and DSFG, complete control of the six degrees of freedom Active Control Systems -3.20 1 0 I 4 (a) I 8 Time (s) I 12 I 16 Time (s) (b) 1.Zr -1.21 0 I 4 (4 Figure 12.10 I 8 I 12 I 16 Time (s) Response to turbulence of RCS. (a) Uncontrolled aircraft. (b) Gust. (c) Aircraft with RCS. can be generated by either aerodynamic or propulsive means. Aerodynamic or propulsive means. Aerodynamic of RCS. (a) Uncontrolled aircraft. even with conventional aircraft, some degree of DLC and DSFG is possible by using such auxiliary control surfaces as flaps, slats and drag petals. 12.7.2 Longitudinal motion can be written as: Aircraft Positioning Control Systems "" 4 6~ = deflection of direct lift surface 6, = deflection o moment surface incidence wing Figure 12.1 1 DLC aircraft. If constant angle of attack flight is required, it is necessary to remove the influence of pitch rate from both the a and q equations. Thus, if a control law is used in which: then: which is the desired result. How can the control be synthesized? From eqs (12.53)-(12.55) it can be shown in Figure 12.12. It is
sometimes preferred to emphasize the 'over the nose' visibility of the aircraft in flight, or, perhaps, to control the 'tail scrape' angle at the 'tail scrape' at the 'tail scrape' angle at the 'tail scrape' at the ' take-off. For such flight situations, constant pitch attitude is preferred, with a constant maintenance of the stability requirements on the flight path. Thus it is necessary to remove the influence of the stability requirements on the flight path. of the form of eq. (12.59). This is achieved by mechanizing the following equations: Use of this control law, eq. (12.61), results in: Its synthesis is represented in Figure 12.13. For aircraft ECHO the equations of motion for flight condition 2 were given in eq. (12.41), from which it can be inferred that: With the feedback control of Figure 12.12 being used, and with a pulsed Figure 12.13 Controller/surface interconnect for constant 0. Aircraft Positioning Control Systems Time (s) Figure 12.14 Response of angle of attack, a, and q to pitch rate command. change of input command of los-' for 0.25 s the response shown in Figure'12.14 was obtained. Similar responses can be obtained for constant pitch attitude flight. If it is necessary to arrange in the aircraft's pitch attitude be independent of any change in its lift (i.e. the motion of the pilot's moment command! Hence, if a feedback control law is to be usFd7 then from eqs (12.52) and (12.63) it can be seen that it is necessary that: Thus: Active Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Hence: "A synthesis of the control law is to be usFd7 then from eqs (12.52) and (12.63) it can be seen that it is necessary that: Thus: Active Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.17 Decoupling Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.17 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control Systems Figure 12.16 Step response for a and q. Aircraft Positioning Control eq. (12.66) is shown in Figure 12.15. The dynamic response of this system for ECHO-& to a unit step command being applied to both Sp and SpL simultaneously, is shown in Figure 12.15. The dynamic response of this system for ECHO-& to a unit step command being applied to both Sp and SpL simultaneously, is shown in Figure 12.17, can be written as: [I] Y" = [L. 0 -1 Lf,L :] Nb N f, N : -; I I YL + - i f LAdw LkVf NAdw Nkvf 2] YL sfg - (12.70) Ssfg If it is required to provide the aircraft with a control system for constant heading operation, with wings level and with sideslip angle constant, then the interconnect to the control system for constant heading operation. zero sideslip is wanted, with wings level and yawing motion in operation, the input from the pilot's controller is rcommand the equations become: Active Control configured vehicle. LAdwSdw+ LAVFvf+ LA sfss s f g = - L:~co,, Nhdw8dW+ NA\$Vf (12.72) + NAsfg 6sfg= -N:rcomm If, however, it is required that the lateralldirectional motion be decoupled, then three separate inputs and feedback paths are required: r + YgdwSdw + Ygvf Svf + YgsfgSsfg = YgsfgSsf + L:r + LAdw&iw + LAv& + LAsfs = Lidwsnm Nbp + Ni + Nkdwsdw + NkV& + Nk s f gs s f g = N&v,6ym LLP (12.73) where Ssf is commanded directsideforce, S,, commanded roll moment, and S,, commanded yaw moment. For a future projects aircraft, coded OMEGA, which has differentially acting wing tips, a ventral fin, and a vertical canard to generate the sideforce, the equations of motion which obtain at FL 60 and at Mach 3 are given by: Conclusions 445 Using the control law of eq. (12.73) for unit step commands in SSf, S,, and 6,, results in the dynamic response shown in Figure 12.19 when eq. (12.73) can be expressed as: A block diagram of the closed loop lateral positioning control system is given in Figure 12.19 that the lateral/directional motion is truly decoupled: Figure 12.20 shows the response of the aircraft to step deflections of the control surfaces, without input command scaling or motion variable feedback. 12.8 CONCLUSIONS In this chapter the important topic of active control technology is introduced by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and FR. This brief qualitative treatment is followed by discussing some of the features of six ACT functions: MLC, GLA, RSS, RC, FMC and RS, RSS, RC, FMC and an account of the development of gust alleviation from the earliest days of aviation up to its application on the B-1 bomber used by the USAF. A detailed consideration of a load alleviation is treated as a special problem in optimal linear control. One feature of controlling flexible aircraft is proper location of the sensors upon which the control law depends for its feedback signals: the sensor Command inputs -4, ----t - "aw "SI . Aircraft Controller 7 6rm 7 - - Figure 12.18 Lateral position system." Time (s) Figure 12.20 Step response of aircraft. Exercises 447 signals are invariably contaminated with components caused by the flexing of the structure which the control system is trying to reduce. A method of using blended signals from several identical sensors at different locations to obtain a 'bending mode free' feedback signal was presented, before showing how the alleviation problem could also be treated as a model-following problem. To illustrate the performance and structure of an RC system for a fighter aircraft, an optimal control problem was once more solved, but on this occasion the performance index was chosen to reflect a ride discomfort (RD) index. Finally, longitudinal and lateral aircraft positioning control systems which used auxiliary control systems is fitted to an advanced fighter aircraft to provide it with improved air-to-air combat performance. Such a pitch pointing mode is characterized by the pitch attitude being decoupled from the flight path angle, i.e. both motion variables, y and 0, can be controlled independently. For such an aircraft a suitable model is defined by the equations: i=Ax+Bu y=Cx where: x' = [0 g a SE SF] U' = [SE, SF,] Y' = [O yl a, g, 0 and y have their usual meanings; SE and SF represent the deflections of the elevator and flaperon respectively. SE, and SF, are the corresponding command inputs. The appropriate matrices, A and B, are: (a) Determine the corresponding output matrix, C. (b) Find a control scheme which will result in it being possible to change 0 without disturbing y, and vice versa. (c) Sketch a block diagram to show how your control scheme can be implemented. 12.2 A 'superaugmented' aircraft is one with active control and a considerable degree of stability augmentation. Without augmentation. Without augmentation of the flight control system. An example of such an aircraft with its c.g. at 50 per cent m.a.c. has the following matrices: The state vector is defined as: the control law which will minimize the r.m.s. value of the normal load factor to any commanded change in the rate of change of height. 12.3 The fighter aircraft BRAVO is
statically stable only at flight condition 2. Design a pitch rate SAS which will provide satisfactory performance at this flight condition 4 without any change in the parameters or the structure of the controller. What advantages does relaxing the static stability of this aircraft bring? 12.4 Design a lateral ride control system for the B-52E such that the r.m.s. value of the side acceleration at the pilot's stations is minimized in response to a side gust velocity of intensity 3 m s-'. Use the mathematical model defined by the matrices References 449 A, B1, CIA and Dl, given in eqs (12.9), (12.10), (12.1 placed at a point A on a B-52E measures the yaw rate and also components of the lateral bending displacement rates, jl and j3. The output voltages are found to be: vq vc + 0.12jl + 0.27j3 = 0.1lr + 0.46jl - 0.32j3 = 0.34r Derive an expression for jl and j3in terms of the output voltages from the rate gyros and the corresponding blending gains. 12.6 Design a model-following control system to achieve gust load alleviation for the B-52E. The output matrix is restricted to the rigid body motion and the first and third lateral bending modes. The model matrix is defined as: 12.7 For the aircraft OMEGA, defined by eq. (12.72), show that the lateral positioning control law given as eq. (12.73) is correct. 12.8 Find a control law to achieve longitudinal positioning of the aircraft ECHO if its static stability is neutral. Determine the response in pitch rate and angle of attack to a pulsed change of input command of losC1 for 0.25 s. Compare your responses with those shown in Figure 12.14. Has the relaxed static stability been beneficial? 12.10 NOTE 1. The aircraft is c.g. is taken as being located at 21.88 m from the nose tip. 12.11 REFERENCES ATIWOOD, J.L., R.H. CANNON, J.M. JOHNSON and G.M. ANDREW. 1961. Gust alleviation system. US Patent 2, 985, 409. 1969. Aircraft load alleviation and mode stabilization (LAMS) - B52 system analysis, synthesis and design. AFFDL-TR-68-161. WPAFB, Dayton, Ohio. HUNSAKER, J.C. and E.B. WILSON. 1915. Report on behavior of aeroplanes in gust turbulence. NACA TM-1 (MIT). October. OSTGAARD, M.A. and F.R. SW~RTZEL.1977. CCVs: active control technology creating new military aircraft design potential. Astrophys and Aero. 15: 42-57. ROUGHTON, D.J. 1978. Active control technology. Inst. M. C. Colloquium, London. March. STOCKDALE, C.R. and R.D. POYNEER. 1973. Control configured vehicle - ride control system. BURRIS, P.M. and M.A. BENDER. 450 Active Control Systems AFFDL-TR-73-83. WPAFB, Dayton, Ohio. turbulence. J. Aero. Sci. 4: 311-5. the statistical theory of turbulence. J. Aero. Sci. 4: 131-8. TAYLOR, G.I. 1937. Fundamentals of Helicopter Sare a type of aircraft known as rotorcraft, for they produce the lift needed to sustain flight by means

of a rotating wing, the rotor. Because rotors are powered directly, helicopters can fly at zero forward speed: they can hover. They can also fly backwards, of course. At present, there are two main rotors in tandem. These helicopter types are illustrated in Figure 13.1. In the single main rotor type, the rotor produces vertical thrust. By inclining this lift vector a helicopter can be accelerated in both the fore and aft, and the lateral directions. This main rotor is usually shaft-driven and, as a result, its torque has to be countered, usually by a small tail rotor mounted at the end of the tail boom Yaw control is achieved by varying the thrust developed by this tail rotor. - Main rotor & + > Rotation Rotation (b) Figure 13.1 Most common helicopter Flight Control Systems In the USA and UK the main rotor rotates counterclockwise (viewed from above); in France, they use clockwise rotation. This has some significance in relation to the use of the tail rotor. To approach some point at which to hover, the pilots to sit in the right-hand seat in the cockpit, the external view can be restricted in this flare manoeuvre, and, often, a sidewards flare is executed, which requires the pilot to apply more pressure to the left pedal in order to sideslip to the right, but this increase in the thrust of that rotor. Pilots flying French helicopters do not have so great a problem in carrying out this manoeuvre. The two rotors of the tandem helicopters are normally arranged to be at the top and the front and rear of the fuselage. These rotors rotate in opposite directions, thereby ensuring that the torque is self-balancing. hub of the rotor at the front. The resulting aerodynamic interference causes a loss of power, but the amount lost, being about 8-10 per cent, is almost the same as that lost in driving a single tail rotor. Every rotor has blades of high aspect ratio which are very flexible. blades to allow free motion of the blades in directions normal to, and in the plane of, the rotor disc. A schematic representation of an articulated rotor hub is shown in Figure 13.2. At the blade hinge, the bending moment is zero; no moment is zero eliminated hinges: these are referred to as hingeless, or rigid, rotors. Figure 13.2 Rotor hub of an articulated rotor. The out-of-plane motion about the vertical hinge causes the blade to deflect in the plane of the disc and such motion is referred to as lagging motion. In hingeless rotors, flapping and lagging motion are defined as the out-of-phase and the in-phase bending, respectively. To control a rotor means that the pitch bearing is usually outboard of both the flapping and lagging hinges, but on a hingeless rotor the bearing may be found either in- or outboard of the major bending moment at the blade root. With any type of rotor there will be an azimuthal variation of lift as the rotor rotates. Such variation affects the degree of flapping motion and, consequently, the direction of the average thrust vector of the rotor. A cyclic variation of lift can be effected, therefore, by changing a rotor blade is being rotated. This altering of blade pitch is termed the cyclic pitch control; when it causes a pitching moment to be applied to the helicopter it is called the longitudinal cyclic, usually denoted by SB. If the applied moment is about the roll axis, the control is called the lateral cyclic, denoted by SA. Yaw is controlled by changing, by the same amount, the pitch angle of all the blades of the tail rotor; such a collective deflection of the blades of the tail rotor; such a collective deflection of the blades of the tail rotor are changed by an identical amount at every point in azimuth, a change is caused in the total lift being provided by the rotor. This type of control is called collective pitch control, since it is the means by which the direction of the thrust vector can be controlled. The importance of the collective to helicopter flight cannot be overemphasized: it is a direct lift control which allows the helicopter's vertical motion to be considerable energy stored when the rotor rotates (as a result of its angular momentum) only small changes in the collective setting are needed to change vertical motion without any accompanying exchange of height for airspeed. Moreover, for small collective inputs, the ability of the helicopter's engine (or engines) to change speed is not of great concern. However, this simple means of controlling height makes difficult the control of a helicopter's horizontal speed: to slow down, it is necessary to pitch a helicopter nose-up. Thus, a pilot achieves deceleration by means of pitch attitude, while maintaining his helicopter's height with the collective, which requires of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective, which requires a distinct of the pilot greater control co-ordination. It is characteristic of helicopter's height with the collective distinct of the pilot greater control co-ordination distinct of the pilot greater control co-ordination distinct disti collective which, in turn, causes a change in the helicopter's yawing motion, thereby resulting in the development of significant sideslip. These coupled motions subsequently result (in the absence of immediate and effective pilot action) in the helicopter rolling and pitching. 454 Helicopter Flight Control Systems helicopter's approach on the glide slope, for it can lead to deviation from the desired flight path. With tandem rotors, matters are differential collective pitch between the rotors is used, it is possible only to produce pitching motion; yaw control is provided by tilting the rotors in opposite directions. If the c.g. of the helicopter is not located exactly midway between the rotors, then use of the lateral cycle will inevitably produce a yawing moment. If such a tandem helicopter is rolled towards starboard (to the right), yawing motion towards port (to the left) will be induced. This characteristic is opposite, unfortunately, to that needed to produce a co-ordinated turn. The helicopter gives rise to a number of very distinctive AFCS problems, including the following: it is unstable; its control its lift force as well as controlling the motion about its three axes; and its speed range is narrow, the speeds involved not being very high (the upper limit is about 240 knots, i.e. 120 m s-l). Only the problems involving stability and control of the helicopter are dealt with in this book, and then only briefly. However, for helicopters, more acutely than for fixed wing aircraft, the control and stability characteristics depend very heavily upon the vehicle's distinctive flight dynamics. The reader should consult Johnson (1980), Mil et al. (1966, 1967) and Nikolsky (1951), which are outstanding books giving excellent and comprehensive coverage. Bramwell (1976), Gessow and Myers (1952), Lefort and Menthe (1963), McCormick (1967) and Payne (1959) so it must be read carefully. 13.2 EQUATIONS OF MOTION 13.2.1 Introduction Any study of the dynamic response of a helicopter is complicated because each blade of the rotor has its own degrees of freedom, which are in addition to those of the fuselage. Yet, for small perburbations in the helicopter's motion, a knowledge of the motion of each blade is not required: only the rotor's motion, a knowledge of the motion of each blade is not required such analyses are invariably carried out in a bodyfixed axes system (see Figure 13.3) and it is assumed that all perturbations are small, the inertia terms can be linearized and the lateral and longitudinal motions may be considered as being essentially uncoupled. It should be Equations of Motion Figure 13.3 Helicopter axis system. remembered, however, that because of the rotation of the rotors, a helicopter does not have lateral symmetry (except for coaxial or side-by-side rotor configurations). There is, consequently, considerable coupling which can result from yawing motion. However the pedals in the cockpit are moved, a rolling acceleration is experienced because the tail rotor is generally above the roll axis. This can be easily seen from an examination of the equations governing rolling and yawing motion: $L = -(13.1) N = I_{,,*} - 1 - -4$; (13.2) For a helicopter, if T_,, represents the thrust produced by the tail rotor, h represents the height of the hub of the tail rotor above the helicopter's c.g. and 1 is the distance aft of the c.g. at which the tail rotor is located, then: It is simple to show that the ratio of rolling to yawing acceleration can be expressed as: Since I, < I, in general, then: 456 Helicopter Flight Control Systems Ix, IIxx can take a value in the range 0.1-0.25. For more on equations of motion, readers should refer to Johnson (1980), Nikolsky (1951), Mil et al. (1966, 1977), Gessow and Myers (1952), Lefort and Menthe (1963) and Bramwell (1976). 13.2.2 Longitudinal Motion In wind axes the linearized equations of motion are: mw = mveF - mgOF sin y + AZ = AM where hX and AZ
are increments in the aerodynamic forces arising from disturbed flight, AM the corresponding increment in pitching moment, y the angle of climb, and OF the pitch attitude of the fuselage. Because it is assumed that the perturbations in u, w and OF are small, the increments in the forces and the moment can be written as the first terms of a Taylor series expansion, i.e.: where SBis the cyclic pitch control term, and 6, the collective pitch control term. The coefficients aXldu, axlaw etc. (or in theoshorthand Xu, Xw, etc.) are the stability derivatives. Thus: (13.11) + mgOF cosy + XsBSB+ Zse Sea (13.12) 0 1 ~ = ~Mull 0 + ~ Mww + Mqq + M,w + MaBSB+ Ms Se0 mu = Xuu + Xww + X,q Oo (13.13) The term M, wis usually included to account for the effect of downwash upon any tailplane which may be fitted. Because lift is generated by the rotating blades whose tilt angles are considered as the control inputs, it proves to be helpful to employ a nondimensional form of those equations. Let the radius of the rotating blades whose tilt angles are considered as the control inputs, it proves to be helpful to employ a nondimensional form of those equations. Let the radius of the rotating blades be denoted by R. The tip speed of any blade is therefore given by LRR. The blade area is STR' where the solidity factor, s, of the rotor is given by: where b represents the chord of these blades (assuming, of course, that they are all identical). 457 Introduction Let: li = uIflR 19 = wlaR 9 = qlfl Let there also be defined as nondimensional time, 7: = tlt" 7 where: The reference area, Aref, is given by: Aref 4, r r ~ ' (13.20) Hence: t" = m ~ ~ b c=f ml l~p s~n ~ ~ fl (13.21) Note that: 4 # dOF/d7 (13.22) = flt"(d0~1d~) (13.23) but: 4 A relative density parameter, p*, is defined for longitudinal motion as: p* = flt" = mlpsArefR (13.24) Therefore: 9 = k*(dOF/d~) (13.25) The non-dimensional moment of inertia is defined as: iYY = - - R - (13.26) and the non-dimensional stability derivatives are defined as: xu = X, psArefflR (13.27) xw = X, psArefflR (13.28) x = X, z = - R - (13.29) (The significance of the prime is explained after eq. (13.45).) $z_{i} = ZUlp - ArefflR (13.30)$ $z_{i} = ZWlpsAreflR (13.27)$ Helicopter Flight Control Systems 458 z; = Z, IPSA, ~~~ R ~ m: = M, lp ~ A, ~ fl ~ ~ mk = MwlpsA, ffl ~ 2 mk = MqlpsAr, ffl ~ 3 XS, = x, H ~ ~ s A ~ ~ ~ ~ R ~ zsB = z, B ~ p ~ ~ r eff 1 2 ~ 2 mf, = M 8 J P ~ ~ r eff 1 2 ~ 3 xs = Xs lp s ~ r eff 1 2 ~ 2 "0 "0 = Z8 lp s ~ r, ffl z ~ 2 "0 "0 mf, = M8 lp s ~ , ~ lp s ~ r eff 1 2 ~ 2 mf, = M 8 J P ~ ~ r eff 1 2 ~ ~ 2 mf, = M 8 J P ~ ~ r eff 1 2 ~ ~ 2 mf, = M 8 J P ~ ~ r eff 1 2 ~ ~ 2 mf, = M 8 J P ~ ~ r eff 1 2 ~ ~ 2 mf, = M 8 J P ~ ~ r eff 1 2 ~ ~ 2 mf, = M 8 $\sim \sim \sim \sim$ "0 "0 Hence, if eqs (13.11) and (13.12) are divided by p s \sim , \sim f and l $\sim \sim$ eq. \sim , (13.13) by p s \sim r e f f 1 2 the \sim 3, following equations are obtained: dl4 d7 -= z,a + zwfi + (V + \$)%- mgeF sin y + zsBtiB + z8OoS0 (13.44) This non-dimensional form of the equations of motion (eqs (13.43)-(13.45)) is due to Bryant and Gates (1930); it is, however, a cumbersome notation. The prime has been used here to indicate that the form being developed is not the final one. For notational convenience it is proposed to write: xklp* = x, (13.47) Similarly, the circumflex will be dispensed with from hereon. Thus: dw dl - -- z, u + zww + (V + z,) OF - mgeF sin y d7 + z8B8B + zg"06"0 (13.49) Equations of Motion 13.2.3 Lateral Motion To control lateral motion the following inputs are used: the deflection angle of the lateral cyclic, SA1 and the collective pitch angle of the lateral cyclic, SA1 and the moments of inertia. $[VP r + \$FI x_{i,i}]$ and the following as control vectors: uion, 4 [SB so,] 'A = PA ST] Ulat 8 ~ (13.58) 8 ~ 460 Helicopter Flight Control Systems then the equations of motion can be represented in a. canonical form, namely: (13.65) %=Ax+Bu where: xu xw x, - mg cos zu z, (V + 2,) - mg sin y Along = mu mw mq in which mu = (mu + m+zu) mw = (m, + mz) mw = (mz) mw m+z,) + m+(V + 2,)) m, = (m, msB = (msB+ m+zsB) ms = Oo (mSg + m+zg) 0 80 and where: CYs, YsT - 18, Blat = - IsT nsA nsT 0 0 0 0 - 0 Static Stability and in which, for example: The other stability derivatives, I,, ir, fi, and fir, can be derived in similar fashion. 13.3 STATIC STABILITY 13.3.1 Introduction Static stability is of cardinal importance in the study of helicopter motion since the several equilibrium modes so much affect each other. For example, any disruption of directional equilibrium will lead to a change in the thrust delivered from the tail rotor, resulting in a corresponding change of the moment of this force (relative to the longitudinal axis, OX) which causes a disruption in the transverse equilibrium of the helicopter. But how does any disruption of directional equilibrium occur in the first place? Suppose the helicopter rotates about the transverse axis, OY, i.e. its longitudinal equilibrium is disrupted. The angle of attack of the main rotor will then change in thrust and, consequently, a change in thrust and consequently, a change in thrust and consequently a change in thrust and consequently are change in thrust and consequently. in the reactive moment of the main rotor. That change disrupts the directional equilibrium. The practical significance of this interplay between the disrupted equilibrium so that controlling (i.e. flying) a helicopter is more complicated and therefore more difficult than flying a fixed wing aircraft. That is why the simple question: 'Do helicopters possess static stability?' requires the examination of a number of factors before an answer can be attempted. Three factors are involved: (1) the static stability properties, if any, of the main rotor; (2) the static stability properties, if any, of the fuse factors are involved: (1) the static stability properties, if any, of the fuse factors before an answer can be attempted. tail rotor and any tailplane on any static stability properties. Further discussion of static stability can be found in Johnson (1980) and Mil et al. (1966). 13.3.2 Static Stability of the Main Rotor Speed In Figure 13.4 it is assumed that the helicopter is flying straight and level at a speed V. Subsequently, the speed is increased by a small amount, AV. The flapping motion of the blades therefore increases (see Section 4.9 of Chapter 4). Helicopter Flight Control Systems Figure 13.4 by the dashed line.) Such a tilt of the coning axis leads to the development of a force F, which is in an opposite sense to the direction of flight. As a result of this force, the velocity of the main rotor falls, and hence the helicopter reduced by an amount AV, the cone axis would then have been deflected forward, and the force, Fx, would have developed in the same sense as the direction of flight, thereby causing an increase in the forward speed. It can be concluded that with respect to changes in speed, the main rotor is statically stable. Angle of Attack In Figure 13.5 the helicopter is once more assumed to be flying straight and level with its main rotor at an angle of attack of (YMR*. The thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence
any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thrust delivered by the main rotor passed through the helicopter's c.g. and hence any moment of the thelicopter's c.g. and hence any moment of th rotor is reduced by an amount AaA. (See the dashed line in Figure 13.5.) The vector of thrust is now deflected forward. Static Stability Figure 13.5 Static stability of main rotor with angle of attack. A moment, MT, given in eq. (13.72), is established which causes the value of the angle of attack of the main rotor to decrease: This moment is destabilizing. If the angle of attack of the main rotor is increased, however, the thrust vector will tilt aftwards and a nose-up moment, MT, will be established causing the angle of attack. Provided that no translation occurs, a helicopter in hovering motion has neutral stability with respect to any change in attitude. Fuselage to static stability is not negligible, however. For a single rotor helicopter, for example, the fuselage is statically unstable in all three axes of motion. A small tailplane is sometimes installed at the aft end of the fuselage to improve the static stability of longitudinal motion in straight and level flight. Its influence is practically nil at low speeds and at hover. However, the degree of instability in longitudinal motion can be reduced from the value at hover by increasing forward speed and by reducing the angle of attack until, at negative angles of attack, the fuselage plus tailplane possesses some static stability. This principle can be seen by referring to Figure 13.6. The longitudinal static stability is denoted by Ma. Curve 2 represents the change in M, with forward speed for a helicopter with an articulated rotor. The same characteristic is represented in curves 1 and 3 for a helicopter with a hinge- Helicopter Flight Control Systems Figure 13.6 Fuselage stability characteristics. less rotor and for the fuselage only of a helicopter, respectively. Note that in each of these curves dM, ldV is positive. The changes in static stability with forward speed corresponding to a tailplane is shown as curve 4; curve 5 represents the same characteristic for another tailplane, the same as that corresponding to curve 4, but with twice its area. Note that for these curves dMddV is negative. The effect of having a helicopter with a hingeless rotor, the fuselage of curve 3 and the tailplane of curve 4, is shown in curve A. Curve B represents the results of combining curves 2, 3 and 4 and curve C is the result of the combination 1, 3 and 5. It is obvious that providing adequate static stability throughout the speed range of a helicopter is fitted with a tail rotor it has a profound effect on the fuselage's static stability for, if the directional equilibrium is disrupted and the helicopter turns to the right, say, the angle of attack of the blade elements of the tail rotor gives the fuselage and, consequently, the thrust must also increase by some amount, AT. Therefore, the moment of this thrust must also increase thereby restoring equilibrium. In this manner the tail rotor gives the fuselage and, consequently, the thrust must also increase and, consequently and the helicopter turns to the right. directional static stability. If the hub of the main rotor has offset horizontal (lagging) hinges (see Figure 13.2), the hinge moments associated with that offset of the hinge and the rotational speed of the rotor, the greater is the static stability possessed by the helicopter. These same factors also contribute to the increase in damping moment contributed by the main rotor. Dynamic Stability 13.4 465 DYNAMIC STABILITY Since the flying qualities of a helicopter are markedly different in forward flight and in hovering motion, these two flight regimes are dealt with separately. The subject of dynamic stability is further discussed in Johnson (1980), Nikolsky (1951) and Mil et al. (1967). 13.4.1 Longitudinal Motion Stick-Jxed, forward flight The pilot's stick being assumed fixed, there are no control inputs, ijB, SeO, ijA or ST: the dynamic stability properties are determined solely from the coefficient matrix. For straight and level flight, Y = A 0 (13.79) V (13.80) 2 z, Hence, the corresponding coefficient matrix, A;Sng,can be expressed as: and it corresponding characteristic polynomial can be found by evaluating: 1 A Z - ALng 1, i.e.: I XI - A&,, I = a4A4+ a3A3 + a2A2 + alA + a. (13.82) where: For complete dynamic stability it is necessary that every real root, or every real part of any complex root, shall be negative. With the wide availability of computers it is now a simple matter to assess helicopter stability: simply read in the coefficient matrix, and check the real eigenvalues and the real parts of the complex eigenvalues to determine if they are negative. Otherwise polynomial routines or the algebraic checks of Routh and Hurwitz (see Chapter 7) should be used. Hovering Motion When a helicopter hovers, V is zero and, usually, x,, x, m, and m+ are negligible, i.e the equations of motion given by eqs (13.48)-(13.52) now become: + x8BSB + xg8 2000 + x 0 w = Z, U + Z, W + zs SB + zs So B 80 O q = muu + mqq + msBSB + m8 6, Oo O u = X, U - mgeF (13.89) (13.90) Hence, the characteristic polynomial can be shown to be: Ahover = h3 - (xu + mq)h2 + x, mqA + mgm, (13.91) which is usually factored into the form: The factor (A + pl) corresponds to a stable, subsidence mode, whereas the quadratic factor corresponds to an unstable, oscillatory mode since 5 invariably lies in the range 0 to - 1.0. Consequently, the longitudinal. It is easy to show (from eqs (13.88)-(13.90) that: i.e. the vertical motion of a helicopter at hover is described by a first order linear differential equation, with a time constant given by: The time-to-half amplitude is typically about 2 s since the value of z, typically lies within the range - 0.01 to - 0.02. In many ways, this simplified representation of the vertical motion in response to collective input is misleading. The vertical damping, z,, is not a simple aerodynamic term but is composed of contributions from the fuselage and from the inflow created by the rotor. In hovering motion, the inflow contribution is predominant. The value of z,, however, which is speed dependent, does have a marked effect on the thrust-to-weight ratio required for a particular height response is considerably affected by the response time of the engine(s) driving the rotor. Of considerable importance to any control in helicopters is the nature of the engine response. In terms of system design and analysis it is imperative to Dynamic Stability have a reasonable mathematical representation of the dynamics of the engine and the transmission system. Unfortunately, such reasonable and simple models are not easily found in the open literature: representation by a low order linear model is frequently misleading. Since no adequate engineltransmission model was available to the author, it has been decided to omit such dynamic effects from the analyses presented in this chapter, but the reader who finds himself concerned with the practicality of helicopter flight control systems is reminded that such dynamics is as a result of the coupling of the motion via the pitching moments which come about as a result of the change in longitudinal velocity, i.e. Mu (the so-called speed stability), and the longitudinal component of the gravititational force. For static stability, the requirement is that the constant term of the characteristic polynomial shall be positive, i.e. mgm, > 0 (13.95) The inequality (13.95) can be satisfied with a positive value of mu. The oscillation associated with the longitudinal dynamics is only mildly unstable with a typical period of 10-20 s. Time-to-double amplitude, td, is about 3-4 s. Both the period and the time-to-double amplitude are sufficiently long for the motion to be controllable by a pilot. Although the hub moments available in helicopters with hingeless rotors, or articulated rotors with offset hinges, are very much greater than in other types, thereby greatly increasing the capability of the rotor to produce moments about the helicopter's c.g., the character of the helicopter's c.g., the character of the helicopter's c.g., the character of the helicopter's dynamics are not radically altered, although there is a real improvement in the controllability. influence the dynamics. The moment derivatives, Mu, M., Ma, and Ma, are increased by as much as three or four times. 4 The pitch damping deritative, M., is increased even more which results in an increase in the value of the real root, pl, of the vertical mode; it also increases somewhat the period and time-to-double amplitude of the oscillatory mode) than a corresponding helicopter with an articulated rotor, and because it also has greater control power, the task of controlling such a helicopter is easier. In summary, for a hovering helicopter, the longitudinal dynamics are described by a stable, subsidence mode (a large negative real root due to pitch damping) and a mildly unstable, oscillatory mode (due to the speed stability Mu). A pilot will have good control over the angular acceleration of the helicopter, but poor direct control over translation. Because of the low damping, in hover the control sensitivity is high. This combination of high sensitivity and only indirect control of translational velocity makes a hovering helicopter Flight Control Systems to the ground drifts considerably: this makes the task of
station-keeping, for which helicopters are universally employed, particularly taxing. Forward Flight (with a Tailplane) In forward flight the unstable, oscillatory mode is made worse with an increase of speed. However, the addition of a tailplane can provide sufficient extra damping to result in the oscillatory mode being stabilized. The longitudinal dynamic stability of a helicopter with a hingeless rotor is particularly bad at high speed and is generally inferior to that of a comparable helicopter with flapping hinges of small offset. Of course, its control power is generally increased and, therefore, a suitable SAS may be used to recover the required degree of stability. From Figure 13.7 it can be seen that, for a helicopter with an articulated rotor, the period of the unstable oscillatory mode increases with forward speed, and becomes stable at about 85 knots. This value of speed is influenced by the size of tailplane used. In Figure 13.7(b) it will be seen that for a hingeless rotor, the period does not change much with forward speed; if anything, there is a tendency to a much greater degree of instability at high speed, as a result of the tailplane, which can be inferred from Figure 13.6). This tendency can be somewhat abated by increasing the area of the tailplane, which can be inferred from Figure 13.7(b). 13.4.2 Lateral Motion Assume straight and level flight, i.e. : Y = A o then the coefficient matrix, Al,, can be re-expressed as: Dynamic Stability 0 (a) 20 40 60 Forward flightstabilityparameters. (a)Articulatedrotor. (b)Hingeless rotor. The characteristic polynomial is given by (XI - A;, A,, = h(b4h4 + A) + (b2h4) + $b^{3h^3} + b^{2X^2} + b^{X}I$ which can be expanded to: + bO) (13.98) where: (13.99) b4 = 1 - (i ~ zlixx)p) + lpnr + lvV(izzlizz) + nvV bl = yv(lpnr - lrnp) + Ev(npV - mg) - nvl, V lrnp (13.100) (13.101) (13.102) 470 Helicopter Flight Control Systems The single A term implies that A = 0 is a solution of the characteristic equation, and, consequently, a helicopter has neutral stability in heading. Hovering Motion In hover is considered it is found that a number of stability derivatives are either zero, or negligible, which leads to a substantial simplification of the equations of motion. However, such simplifications do not occur in lateral motion studies, because the yawing (r) and rolling (p) motions are coupled by virtue of the tail rotor is on the roll axis, then I can be considered negligible. Then the characteristic polynomial becomes: The root (A = n,) means that the yawing motion is stable (since nr is invariably negative) and independent of sideways and rolling motion. The cubic can be factored into: The first factor corresponds to a stable rolling, subsidence mode; the quadratic represents an unstable, oscillatory mode. Typically, for the rolling subsidence mode, t, is less than 0.5 s; the period of the oscillation is about 15-20 s, whereas the time-to-double amplitude is about 5 s. Forward Flight The quartic of eq. (13.98) has been solved for a range of values of the advance ratio, p, and the values of the real and imaginary parts of the corresponding eigenvalues have been displayed as a root locus diagram in Figure 13.8. It is evident from this figure that as the forward speed of the helicopter increases the complex roots become stable. There is now a spiral mode, a rolling subsidence mode (still rapid) and a stable oscillatory mode corresponding to dutch roll oscillation, i.e. : Hence, this helicopter will 'weathercock' with very little translation sideways. If a hingeless rotor is employed it can increase the hub moment by about a factor of five, in relation to an articulated rotor with a hinge offset by 4 per cent. Such an increase the stability derivatives, I and I,, but not the stability derivative, n,. The quartic then becomes: (A + n,)(A + P~)(A" +?) (13.107) Stability Augmentation Systems Articulated rotor I 0.2 t0 -0 --3 -jw Figure 13.8 Root locus diagram. The root of the rolling subsidence mode, p2, has typically a value of about 10.0-15.0; t, reduces. The oscillatory mode, which was unstable, is now neutrally stable, with a period of about 15-20 s. 13.5 STABILITY AUGMENTATION SYSTEMS From earlier chapters it can be learned how the application of feedback control laws are: How such feedback stable. For helicopter longitudinal dynamics the most common feedback control laws are: How such feedback control stable. control laws are implemented can depend on whether passive or active methods are to be used. There are a few passive techniques in current use; the best known is the stabilizer bar to be found on some Bell helicopters. (See, for example, the illustration of the Bell 212 in Jane's All the World's Aircraft (1983-1984).) 472 Helicopter Flight Control Systems 13.5.1 Stabilizing Bar This simple mechanical device is essentially a gyroscope: it is a bar pivoted to the rotor shaft and has a viscous damper provided. The bar is caused to tilt relative to the shaft a change in the pitch of the rotor blade will be caused to tilt relative to the shaft and has a viscous damper provided. the bar is denoted by 6. The equation motion of the bar is given by: where q is the angular pitching velocity of the rotor hub, and 4 the azimuthal angle of the blade, i.e. the angle between the blade span and the rear centre-line of the blade is arranged to be proportional to the bar displacement, i.e. : If the constant, k, is selected to be 1.0 then a tilt of the rotor. Therefore, although the pitch angle of the rotor can be defined as : 0 = O0 - SA cos + - SB sin + (13.113) where SA represents the of the rotor. amplitude of the lateral cyclic deflection, and SB the amplitude of the longitudinal cyclic deflection. 0 can be written as: I Figure 13.9 Schematic representation of Bell stabilizing bar. Stability Augmentation Systems 473 If eq. (13.114) is substituted in eq. (13.114) is substituted in eq. (13.114). following equations result: + + The prime denotes d/d+. The characteristic equation of these simultaneous differential equations is a cubic which is of little practical use. The first order factor can be shown (with a little manipulation) to be: which is of the form of eq. (13.109). Thus, any change in the rotor's speed, or the pitch rate of the rotor hub, will cause a change in the longitudinal cyclic deflection which tends to oppose the causative change. Note that the settling time of the response of the system represented by eq. (13.117) depends upon K, the damper coefficient. Flight tests of this stabilizing bar have shown that it provides a lagged feedback control and has been observed to increase the stabilizing devices are the Hiller bar and the gyro bar which is a feature of the Lockheed rigid rotor. The Hiller bar has a close resemblance to the Bell stabilizing bar except that damping is provided aerodynamically by means of small aerofoils mounted on the bar, rather than by using a viscous damper. Unlike the Bell stabilizer, a Hiller pilot controls the bar directly. Both the Bell and the Hiller bars are best suited to a two-bladed see-saw rotor. The Lockheed rigid rotor is a three-bladed rotor. The most serious disadvantage of stabilizer bars, apart from mechanical complexity, is that they add to the total drag of the rotor. 13.5.2 AFCSs for Helicopters SAS In helicopters, the basis instability is such that the AFCS has to provide both restoring and damping moments. The control laws in general use tend to be either eq. (13.108) or (13.109). However, only limited control authority can ever be allowed, since control is through the rotor which provides the helicopter's sustaining lift and forward propulsion. 'Hands-off' operation of any helicopter's sustaining lift and forward provides the helicopter's sustaining lift and forward provides thelicopter's sustaining lift and forward provide helicopter flight. A representative block diagram of such a stick feel system is shown in Figure 13.10. Note the presence of an input signal, I;,~,, provided from trim actuator) 1 - I Acceleration Velocity Displacement (From pilot) Spring Fsprmg u Figure 13.10 Block diagram of a stick feel system. to zero the force required to be produced by a pilot for a constant manoeuvre demand. The feedback spring force is often dispensed with. It should be appreciated, nevertheless, that the dynamics of the stick feel system act as a pre-filter, in the manner outlined in Section 10.8 of Chapter 10. The simple SAS represented by Figure 13.11 is robust and requires no electrical trim signal for varying c.g. margins, because there will be no input to the servomechanism when there is no rate of change of displacement from the datum. The system is an excellent regulator which maintains its helicopter at the datum to which it has been trimmed. Although the block diagram shows a 'leaky integrator' path in parallel with the output from the rate gyroscope, the effect of these parallel paths is identical to that of a phase lag network. The feedback control law is: Tilt angle Helicopter Figure 13.11 Block diagram of an SAS. 4 Stability Augmentation Systems Hence: -- 4(s) when: - KqK (1 + 7'1s) (1 + T s) In response to a disturbance, the 'leaky integrator' path produces a signal proportional to the angle through which the helicopter has been displaced from its equilibrium value at the time of the disturbance. The control action applied through the swash plate tends to reduce this angular disturbance to zero. If the helicopter does not respond to this corrective control action, or it is held in this new angular disturbance to zero. If the helicopter does not respond to this corrective control action, or it is held in this new angular disturbance to zero. If the helicopter does not respond to this corrective control action, or it is held in this new angular disturbance to zero. If the helicopter does not respond to this corrective control action, or it is held in this new angular disturbance to zero. If the
helicopter does not respond to this corrective control action, or it is held in this new angular disturbance. position is considered as the equilibrium. ASE ASE is an attitude control system, represented in Figure 13.12. The rate signal required for the inner loop SAS is obtained by differentiating the output signal from the attitude gyroscope, the use of which implies a real attitude datum. Since a helicopter has to be flown at any speed or attitude, this datum must be variable. The input from the c.g. trim system centres the gyro for any given flight condition. However, since the attitude control system tends to hold the datum and therefore opposes even manoeuvre demands, a signal from the stick is trimmed to some new datum position, this signal establishes a new datum for the gyro. When the ASE is being used to control bank angle, it is not centred once the angle has been reached; in any emergency, release of the stick re-establishes straight and level flight. An ASE can hold attitude indefinitely. Although it is more complicated than a SAS it performs the same function. However, it does offer the means for providing automatic trim, automatic flight control and, of course, of changing the control characteristics. On helicopters the failure of SAS functions must not be critical, hence it is rare to provide a provide a state of the means for p redundant channels. CSAS When a helicopter has poor handling qualities, the performance of a specified mission without an AFCS can lead to levels of workload for the pilot which are unacceptably high, particularly if he is required continuously to monitor such Helicopter Flight Control Systems Stick displacement 1 Tilt angle of rotorswash plate I t I L---+u Helicopter dynamics r h - *a, *0 Pick off I C.g. trim signal Figure 13.12 Differentiating network Block diagram of pitch attitude control system. quantities as rotor speed and control system. The fight is being carried out within the limits of safe operation. A CSAS is usually provided in such cases. It is similar to the ASE but involves the use of a Kalman filter, requiring input signals from the gyroscopes, an accelerometer and the inertial navigation system (INS) velocity sensor. The resulting estimated signal is compared with a reference value generated by a set of model dynamics (see Figure 13.13). A signal proportional to any error between these values is filtered and then added to the feedback signals from the attitude and rate gyros. Decoupling signals from other axes are added at the same point. Such an AFCS is larger and more complicated than any discussed earlier, a booster servo is usually added to amplify the stick displacement. Station-keeping System This AFCS is used to enable a helicopter to maintain its position fixed in space to keep its station - for quite long periods of time. (See Hall and Bryson (1973) for further discussion of this system.) Obviously, the situation requires that the flight is carried out at hover, or near hover, i.e. with forward speeds not greater than 1m s-l. For a Sikorsky S-61 helicopter, in which the blade dynamics are also taken into account, the state and control vectors are defined as: where OR denotes the pitch tilt angle of the rotor, +R the roll tilt angle of the rotor, qR the rate of pitch tilt angle, OF the pitch Stability Augmentation Systems 477 Velocity signal from INS Trim actuator displacement - Decoupling signal (from other axis) Figure 13.13 Block diagram of CSAS. attitude of fuselage, v the velocity along the y-axis of the fuselage, ?iAthe pitch angle of longitudinal cyclic, and aB the pitch angle of lateral cyclic. The corresponding coefficient matrix, A, and driving matrix, B, are: r o 0 - 0 I 0 1 ! Rigid body (fuselage) I 41.3 - 600.0 - 30.3 - 42.6 0 0 0 5.0 3.5 [Rotor dynamics 1 0 0 0 1 0 22.0 0 0 0 I - 0.92 - 0.04 0.004 j f Rigid body (fuselage) 0.17 0 j j - 1.0 0 I 18.4 - 0.01 15.0 - 0.05 Rotor coupling effects coupling effects (shown on page 478). dynamics (shown on page 478) Helicopter Flight Control Systems Rigid body (fuselage) coupling effects 0 0 0 / o Rotor dynamics (shown on page 477) [i 0 0 0 30.4 - 50.6 0 - 0 - 0.05 0.03 0 - 0.1 0 0 0.14 - 0.26 I Rotor coupling effects (shown on page 477) r j I - 0.2 0.001 0.0008 - 0.002 - 0.06 / 32.2 1.4 - 4.4 - 0.007 - 0.017 Rigid body (fuselage) dynamics It is obvious from an inspection of A that there are profound coupling motion. If it is assumed that the rotor disc can be tilted instantaneously the appropriate model - 0.02 The eigenvalues corresponding to these two models are shown in Table 13.1. The response of the basic helicopter, with and without rotor dynamics, is shown are the pitch and roll attitude of the fuselage and the change in forward and side velocities, corresponding to an impulse control input, in the longitudinal cyclic. Similar impulse responses can also be found for the lateral cyclic. It should be noted from Table 13.1 and Figure 13.14 that when it is assumed that the rotor dynamics can be ignored) one of the rigid body modes of the fuselage is more lightly damped and slower. A feedback control law can easily be found using the LQP method outlined in Section 8.3 of Chapter 8. For the modes including the rotor dynamics Without rotor dynamics Without rotor dynamics Without rotor dynamics of uncontrolled helicopter Flight Control Systems -6001 0 I I I 1 2 3 I 4 I 5 Time and the section 8.3 of Chapter 8. For the modes including the rotor dynamics Without rotor dy (s) (a) Time (s) (b) Figure 13.14 Response of uncontrolled helicopter. (a) With rotor dynamics. (b) Without rotor dynamics. (b) Without rotor dynamics. (c) Without rotor penalize excessive controlled deflections of the longitudinal and lateral cyclic, results in an optimal gain matrix, K, which is found to be: Stability Augmentation Systems The eigenvalues corresponding to the closed loop optimal system are given in Table 13.2. Table 13 dynamics Another optimal feedback control law may be obtained in a like manner for the helicopter with instantaneous tilting of the rotor disc. The choice of weighting factors on the state and control variables were identical to those used earlier, i.e. The resulting optimal gain matrix was found to be: The corresponding closed loop eigenvalues are also given in Table 13.2. The responses of the pitch and roll attitudes of the helicopter fuselage, and the changes in the forward and side velocities, to a unit impulse, in first the longitudinal cyclic and then the lateral cyclic, are shown in Figure 13.15 for the helicopter withlwithout rotor dynamics. It should be noted from these responses how effectively the controlled helicopter keeps its station: compare particularly the responses of the translational velocities with those which arose in the uncontrolled case. Further discussion on stability augmentation systems, v /. t i t -5 .-. i i i u Attitude (deg) Velocity (ft s-') i: lo: %.: 5.'; ;\; -1s-; -20 -25; ..: 1:'..../, ii, j 1:.0 IIIII 2345 Time (s) (a) Attitude (deg) Velocity (ft s-') -810 II 1 I 2 345 Time (s) (b) Figure 13.15 13.6 Optimal control response of helicopter. (a) With rotor dynamics. (b) Without rotor dynamics. CONCLUSIONS This chapter opens with a brief introduction to the helicopter and its rotor systems. Some distinctive features of helicopter flight which give rise to particular control problems were indicated before proceeding to a development of small perturbation motion for both longitudinal and lateral motion. Next, the particular qualities of static stability which obtain in Exercises 483 helicopters are dealt with, before turning to the special problems of dynamic stability for which the most effective SASs. The objectives of such SASs are discussed before a description and analysis of a rotor stabilizing bar, a very common helicopter SAS, are presented. Active SAS and CSASs are then discussed before dealing finally with optimal control of a station-keeping helicopter. A special feature of this analysis is the impact which the inclusion of the dynamics of the main rotor in the equations of motion can make to the response obtained from the controlled helicopter. 13.7 EXERCISES 13.1 The state vector of the Black Hawk helicopter (UH-60) can be defined as: and the control vector as: u' = [SB &A ST soo iht] where iht represents the change in incidence of the helicopter at forward speed: (i) 1ft s-' (ii) 150 ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability
of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment on the stability of the helicopter at forward speed: (i) 1ft s-' (comment each of these flight conditions, and compare. (b) Find the following transfer functions for the hovering condition: p(s) (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration of lateral cyclic control; (ii) lateral acceleration of the longitudinal cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions for the hovering condition: p(s) (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions for the hovering condition: p(s) (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft s-' relating: (i) normal acceleration at the c.g. to a deflection of lateral cyclic control; (ii) SA(S (c) 46) (Iv) ~B(s> Find the transfer functions at Uo = 150ft control; (d) What is the peak normal acceleration to a unit step deflection of the longitudinal cyclic, and how long does it take after the application response concave downwards within 2.0s of the application of the control? 13.2 A helicopter, which is perturbed by a wind gust, is to be maintained at zero ground speed by means of controlling its motion about the pitch axis. If the deflection angle of the rotor swash plate is denoted by q and a feedback control law, q = K,O + K,u + K,q is used, find values of KO, K,, and K , which will optimize the helicopter's hovering flight, in the sense that its use minimizes the ISE where u is taken to be the error, since u represents any small variation in horizontal speed from the derived ground speed of 0.0. The helicopter being used is a Sikorsky S-58 and the appropriate, but approximate, equations of motion are given by: 13.3 The Iroquois (UH-1H) helicopter has its state vector defined as: x'=[uwqOvp+r] and its control vector as: u' = [SB Seo SA 81-1 The appropriate matrices A and B for the hovering case, i.e. Uo A 0 m s-', are: Exercises (a) Find the eigenvalues of the uncontrolled helicopter. (b) Determine, by any appropriate means, a state feedback control law which will completely stabilize the helicopter. (c) Does the controlled helicopter have acceptable flying qualities? 13.4 (a) Show that, for hovering motion, the vertical velocity of a helicopter can be expressed as a simple, first order differential equation: (b) How significant was your assumption in part (a) that the rotor speed is not fixed, design a height control system to allow the helicopter to hover without the pilot's attention. (d) If we are a simple, first order differential equation: (b) How significant was your assumption in part (a) that the rotor speed is not fixed, design a height control system to allow the helicopter to hover without the pilot's attention. is represented by a (1 - cos) gust (see Chapter 5), what would be the corresponding height change if Z, = - 0.015? The scale length of the gust is 10.0 m. 13.5 A helicopter has the capability of making a landing by means of maintained, even though there is a loss of power due to engine failure, and the helicopter descends at a steady rate. (Warning: even for forward flight, this autorotation descent rate is quite large, so autorotation descents are used only in emergencies.) 486 Helicopter Flight Control Systems Assume that the collective pitch is unchanged. The equation of motion for the vertical acceleration of a helicopter is given by: (~ 1 g) h= W - T where h is the helicopter above the ground, W is the gross weight and T the thrust developed by the rotor. If Qo and Clo represent the torque needed in level flight and the initial rotor speed, respectively, show that: (a) The descent velocity (sinking speed) of the helicopter can be expressed as: h = @4(t + 7) where the time constant T (b) The rotor speed becomes: = NZbfidQO aOT/(t + 7) If NZb = 3 000 kg m2, Qo = 7 500 N-m, and Clo = 36 rad s-l, sketch the response of h and Cl with time. (d) If the aircraft is at a height of 100 ft when autorotation begins, calculate the aircraft's sinking rate and its rotor speed at the moment of ground contact. (c) 13.6 A helicopter at near hover flight condition, with constant rotor speed at the moment of ground contact. pitch tilt angle of the rotor, \$R the roll tilt angle of the rotor, pR the roll tilt angle of the rotor, q~ the rate of pitch tilt angle of the rotor, q~ the pitch rate attitude of the fuselage, pF the roll attitude of the fuselage, a small change in the velocity of the c.g. along the X-axis of the rotor, a small change in the velocity of the rotor, a small change in the velocity of the c.g. along the X-axis of the rotor, a small change in the velocity of the c.g. along the X-axis of the rotor, a small change in the velocity of the rotor, b the the fuselage, v a small change in the velocity of the c.g. along the Y-axis of the fuselage, SB the longitudinal cyclic pitch. The corresponding matrices A and B are shown below. Rotor dynamics 1 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 0 0 0 - 41.3 - 600.0 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuselage) coupling effects 0 - 40.4 - 50.6 Rigid body (fuse Helicopter Flight Control Systems Note that the rotor dynamics being represented in the aircraft equations of motion. When the helicopter is represented as a solid disc, the state vector is then defined as: xi = [OF + F PF 9~ U V] and the control vector becomes: u' = [- + R OR1 where +R denotes the lateral tilt angle of the rotor disc, and OR denotes the longitudinal tilt angle of the rotor disc. The corresponding matrices are Al and B1: Al = -0010000 - 0.0420.32000 - 0.0420.32000 - 0.0420.3200 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.0420.32000 - 0.042000 - 0.042000
- 0.042000 - 0.042000 - 0.042000 - 0.042000 - 0.042000 - 0.042000 - 0.042000 - 0.04200 - 0.042000 - 0.04 with blade flapping motion and then with the solid rotor disc. (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion: (a) Ql = diag[10.0 2.0 2.0 5.0 1.0 10.01 (d) Using the feedback control law for the helicopter without blade flapping motion: (a) Ql = diag[10.0 2.0 2.0 5.0 1.0 10.01 (d) Using the feedback control law for the helicopter without blade flapping motion: (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion and then with the solid rotor disc. (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion: (c) Por the following weighting matrices, Q1 and GI, find an optimal feedback control law for the helicopter without blade flapping motion and then with the solid rotor disc. (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion and then with the solid rotor disc. (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion and then with the solid rotor disc. (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion: (a) Ql = diag[10.0 2.0 2.0 5.0 1.0 10.01 (d) Using the feedback control law for the helicopter without blade flapping motion: (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion about the eigenvalues obtained for the helicopter without blade flapping motion: (b) What is significant about the eigenvalues obtained for the helicopter without blade flapping motion about the eigenvalues obtained for the helicopter without blade flapping motion about the eigenvalues obtained for the helicopter without blade flapping motion about the eigenvalues obtained for the helicopter without blade flapping motion about the eigenvalues obtained for the helicopter without blade flapping motion about the eigenvalue about the helicopter without blade flapping motion about the eige obtained in part (c), determine the pitch rate response to an initial value of the fuselage pitch rate of - 3" s-l of the solid rotor disc helicopter. (e) Find the same response for the helicopter with blade flapping motion, still using the control law found in part (c). Using weighting matrices Q and G References 489 determine the pitch rate vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the control law for the helicopter with state vector, x. (g) Use the 13.7 A small experimental helicopter, with a single main rotor and a NOTAR system, is flying at a near hover condition i.e. Uo = 0.3 m s-I. In the course of the flight condition are: (a) If an impulse of 0.035rad-s is applied to the lateral cyclic, what is the resulting steady state value of the helicopter to reach its new heading? (c) Is the helicopter stable? Can it be maintained in its near hover state with the NOTAR system failure? Could you design an AFCS to permit automatic hovering without using the NOTAR system? 13.8 NOTES 1. Some newer types of helicopter are called NOTARs (no tail rotors), yet even though the method of generating the countering moment has been changed in these types, the same function as the tail rotor performance is maintained. That is OX parallel to the flight path, O Z pointing vertically pointing starboard. This assumes that the direction of rotation of the main rotor is counter-clockwise (viewed from above), the usual direction for USA and UK manufactured helicopters. These comparisons are made in relation to an articulated rotor of the same type, but with no flap downwards, and OY i nge offset. This supposes that xi is g rate. The same system operates on p, the roll rate, provided that a properly oriented rate gyroscope is fitted. 2. 3. 4. 5. 13.9 REFERENCES URAMWELL, A.R.S. 1976. Helicopter. Ungar. HALL, W.E. and A.E. BRYSON. 1973. Inclusion of rotor dynamics in control helicopters. J. Air 10(4): 200-206. design for 490 Helicopter Flight Control Systems JOHNSON, W. 1980. Helicopter Theory. Princeton University Press. LEFORT, P. and R. MENTHE. 1963. L'Helicopter Theory. Princeton University Press. LEFORT, P. and R. MENTHE. 1963. L'Helicopter Flight Control Systems JOHNSON, W. 1980. Helicopter Theory. Princeton University Press. LEFORT, P. and R. MENTHE. 1963. L'Helicopter Theory. Princeton University Press. LEFORT, P. and R. MENTHE. 1963. L'Helicopter Theory. Princeton University Press. LEFORT, P. and R. MENTHE. 1963. L'Helicopter Theory. Princeton University Press. LEFORT, P. and R. MENTHE. 1963. L'Helicopter Theory. Princeton University Press. MIL, M.L. et al. 1966. Helicopter Calculation and Design Vol. 1. Aerodynamics. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Design. Vol. 2. Vibration and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. 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Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA TTF-519. NIKOLSKY, A.A. 1951. Helicopter Calculation and Dynamic Stability. NASA T Systems 14.1 INTRODUCTION In the preceding chapters it is assumed that any AFCS being considered had a feedback control law which was synthesized as a continuous function of time. Most current and new types of AFCS now depend (and all future types of AFCS now depend) upon digital synthesized as a continuous function of time. is now, or is going to be, digital, The objectives of this chapter are, therefore: 1. To introduce several essential concepts of digital control, 2. To discuss a few effects upon the dynamic performance of an AFCS of using such a means of control. The distinguishing feature of digital control is that some of the signals within the system cannot be known at every instant of time, but are known only at particular instants. For example, imagine that a switch be closed at some instant of time, say t,, its output will be as shown
in Figure 14.2(b). (x(t) is shown in Figure 14.2(c). If some longer period of time, T, passes, and then the switch is once again rapidly closed and opened, the resulting output is as represented in Figure 14.2(c). Figure 14.2(d). If it is arranged to rapidly close and open the switch in a regular cycle, of period T, the output which results has the form represented in Figure 14.3. Whenever the duration of switch closure is extremely brief, the period At + 0; the output from the switch in a regular cycle, of period T, the output which results has the form represented in Figure 14.3. Figure 14.4. This signal has been denoted as y*, and it is said to be the sampled version of the input signal, x(t). The discrete signal, y*, is known accurately only at the sampling intervals (ktl + T), where k = 0, 1, 2, When the sampling period, T, is constant, the sampling is uniform; much of the work in this chapter assumes uniform sampling. When At -+ 0 and the switch is closed periodically every T seconds, the switch is referred to as a sampler (or sampling switch. t (4 O tI Time Figure 14.2 Switch output signals. Figure 14.3 Periodically sampled signal. A Simple Discrete Control System Figure 14.4 Sampled signal. Figure 14.5 Idealized sampler, in Figure 14.5. A sampler, then, is a device for transforming a continuous into a discrete signal is regarded as being digital.' 14.2 A SIMPLE DISCRETE CONTROL SYSTEM A simple system, in which the variables are continuous, is represented by Figure 14.6. The system is merely an integrator whose output is also used as a negative feedback signal. The transfer function of this simple system can easily be shown to be: Y(s)/R(s) = Kl(s + K) (14.1) Figure 14.6 Simple first order system. If the input, r(t), is a unit step function, then: y(t) = 1 - ePk' (14.2) The output is also used as a negative feedback signal. response for this system is shown in Figure 14.7. Now if it is supposed that the error signal, e, is sampled in the manner shown in Figure 14.7 Step response of simple first order system. device Figure 14.8 Sampled first order system. instant kT be denoted by ek. The rate of change of the output signal can then be expressed as: dy ldt = Kek (14.3) For any arbitrary input, r(t), the situation may be represented by Figure 14.9. Error signals at sampling instants. Let: A ~ K T A Simple Discrete Control System 495 (sometimes referred to as the specific control step), then: $y_k + I = (1 - A) Y \sim (14.7) 1.e.$: $Y_k = -k - 1 h) Yl (14.8)$ The output response is heavily dependent upon the particular value of A. For example, suppose both A and yl are unity. Then y2, y3, y4, etc. are all zero. If A is 0.5, then for y, = 1.0, the output sequence becomes: y2 = -0.5, y3 = 0.25, y4 = -0.125, y5 = 0.0625, etc. If A is increased to 1.5 the output sequence becomes: y2 = -0.5, y3 = 0.25, y4 = -0.125, y5 = 0.0625, etc. If A is increased to 1.5 the output sequence becomes: y2 = -0.5, y3 = 0.25, y4 = -0.125, y5 = 0.0625, etc. If A is increased to 1.5 the output sequence becomes: y2 = -0.5, y3 = 0.25, y4 = -0.125, y5 = 0.0625, etc. If A is increased to 1.5 the output sequence becomes: y2 = -0.5, y3 = 0.25, y4 = -0.125, y5 = 0.0625, y6 = -0.03126, etc. When A is increased to 2.0, however, the output sequence alternates between - 1.0 and 1.0. The responses corresponding to these sequences are shown in Figure 14.8. The dynamic situation can be summarized thus: If 0 < A < 1, y(t) tends to zero without oscillation. If A = 1, y(t) reaches zero in a single sampling interval. If 1 < A < 2, y(t) is oscillatory and unstable. If A < 0, y(t) is oscillatory and unstable. If A < 0, y(t) is oscillatory and unstable. If A < 0, y(t) is oscillatory and unstable. If A < 0, y(t) is oscillatory and unstable. If A < 0, y(t) is oscillatory and unstable. If A < 0, y(t) is unstable. If A < 0, y(t) is oscillatory and unstable. If A < 0, y(t) is unstable. If A Systems Yk+ 1=r (14.9) It should be noted that, whereas the continuous system of Figure 14.6 can never be unstable merely as a consequence of the value chosen for the sampling period, T, in relation to K. Such an oscillatory response is referred to as a 'hidden oscillation' since it occurs solely as a result of having used digital control with a particular value of sampling rate. The condition which will represent the transfer function of a phase advance network: + G(s) = U(s)IE(s) = (1 + ~ s) / (1 a m) 0 5 a5 1 The corresponding differential equation is given by: e - vt = u + a - u. Let en - u, -en and u, be the value of the continuous signal at the (n - 1)th and nth sampling instants respectively. Now: t b (en - en - 1)T is the sampling period. Then: $U_{+} = en = + en - Ten clen - 1 a7 7 - (u_{+} - U_{+} + c^{2}a_{+}) T - where: CI = 1 a7 7 - (u_{+} - U_{+} + c^{2}a_{+}) T + (u_{+} - u_{+}) T + (u$ $(T + 7)/(T + a \sim Appendix A 554$ It is evident from inspection of (A.62) that some stability has to be provided by the control amplifier and, consequently, it has a transfer function of the form: (A.63) (A.64) Usually T2 is negligible, therefore: (A. 6.5) where: (A.66) (A.67) (A.68) A.6 ANGLE OF ATTACK SENSOR There are a number of methods of sensing angle of attack, but essentially only two types are in general use. In low speed flight, the moving vane type is preferred. It is a small vane protruding into the airstream, close to the body of the airstream, close to the body of the airstream. the angle of attack. The use of this type in AFCS work is restricted since its accuracy is much affected by local flow conditions. The other method employs a stationary pressure-ratio sensing probe, a sketch of which is shown in Figure A.18. The angle of attack is usually obtained from the pressures measured at two (often more) suitable positioned orifices, i.e.: (A.69) where K is a constant, M is the Mach number of the moving air, P is the sideslip angle, (PT - P,) is a measure of the dynamic pressure and Mach number change slowly compared to change sin the angle of attack, which is governed by the short Actuators and Sensors PT W in d y f p2 / Probe Figure A.18 Pressure-ratio angle of attack sensor. period motion of the aircraft. The sideslip effect cannot be easily ignored. With the pressures from the probe being fed to bellows, which results from volume changes in pressure, is used to drive position transducers to provide an electrical output signal. There is, however, an unavoidable time delay, TL the time constant of the interconnecting pneumatic lines, and TB the time constant of the bellows. A.7 REFERENCES AHRENDT, W.R. and C.J. SAVANT. 1960. Servomechanism Practice. New York: McGraw-Hill. COLLEWE, J.G.R. 1970. Sensor and actuator dynamics. In A.L. Greensite, Analysis and GREEN, Design of Space Vehicle Flight Control Systems, Vol. 11, Chapter 3. New York: Spartan Books. W.L. 1985. Aircraft Hydraulic System. Chichester: Wiley. Appendix B Stability Derivatives for Several Represented here. These aircraft are generic types and are referred to as follows: ALPHA BRAVO CHARLIE DELTA ECHO FOXTROT GOLF a a a a a four-engined, jet fighter aircraft twin-engined, jet fight and its particular flight condition, the aircraft name is given first followed by a number corresponding to the flight condition. For example, FOXTROT. B.2 AIRCRAFT DATA B.2.1 ALPHA - A four-engined, executive jet aircraft General Parameters Wing area (m2) Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) I, : I, P: P 50.4 5.325 3.33 59.2 0.25 E - 6.77 0.73 Stability Derivatives 557 Weight (kg): Approach 10 635 All other flight conditions 17 000 57 000 171 500 218 500 7 500 162 000 185 000 330 000 6 900 Inertias (kg m2) Zxx ZYY : Zzz : 1x2 1 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N m2) a. (degrees) Yo (degrees) 1 2 S.L. 0.2 67.7 2 844.0 6.5 0 6 100 0.35 110.6 4 000 + 9.9 0 + Flight condition 3 4 6 100 0.75 237.1 18 338 2.6 0 12 200 0.8 236.0 8 475 4.2 0 + + Stability derivatives Longitudinal Motion Stability derivatives Longitudinal Motion Stability derivative 1 B.2.2 - A twinengined, jet fighter aircraft BRAVO 2 Flight condition 3 4 General Parameters Wing area (m2): Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 56.5 3.0 4.86 210 (no reheat) 0.255 L or 0.311 L Weight (kg) : Approach 15 x lo3 Inertias (kg m2): Ixx : All other flight conditions 16 x lo3 559 Stability Derivatives Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) q (N m-2) a. (degrees) Yo (degrees) c.g. Flight condition 3 1 2 S.L. 0.4 136 11348 3.5 0 0.311 + 4 9 150 0.8 240 10 700 2.5 0 0.311 + 4 9 150 0.8 240 10 700 2.5 0 0.311 + 5 tability Derivatives Longitudinal Motion only Stability derivative B.2.3 1 CHARLIE 2 Flight condition 3 4 - A very large, four-engined, passenger jet aircraft General Parameters Wing area (m2): Aspect ratio: Chord, C (m): Total related thrust (kN): C.g.: 510 7.0 8.3 900 0.25 E Appendix B Pilot's location (m) (relative to c.g.) Weight (kg): Approach 250 000 All other flight conditions 290 000 Inertias (kg m2): zxx Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) q (N mP2) a0 (degrees) Yo (degrees) Yo (degrees) Flight condition 3 1 2 S.L. 0.198 67 2 810 8.5 0 6 100 0.5 158 8 667 6.8 0 4 6 100 0.8 250 24 420 0 0 12 200 0.8 250 9 911 4.6 0 Stability Derivatives Longitudinal Motion Cont'd Stability derivative 4 1 Flight condition 3 2 - 0.357 - 0.378 0.7 x 1 0 - ~ 4 - 0.421 - 0.668 - 0.339 1.09 - 2.08 - 1.16 0.67 x 1 0 - ~ 0.67 related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1. : 1. P : P Weight (kg): 576 7.75 9.17 730 0.3C 25.0 + 2.5 Approach 264 000 All other flight conditions 300 000 Appendix B Inertias (kg m2) zXx 2.6 4.25 6.37 3.4 IYY: IZZ: 1x2 : x 107 3.77 4.31 7.62 3.35 x lo7 x l no. Uo (m s-l) (N m2) a. (degrees) Yo (degrees) a Flight condition 3 1 2 S.L. 0.22 75 3 460 2.7 0 6 100 0.6 190 11730 2.2 0 + + 4 6 100 0.8 253 20 900 0.1 0 12 200 0.875
260 10100 + 4.9 0 + Stability Derivatives Longitudinal Motion Stability derivatives Longitudinal Motion Stability derivatives 1 2 Flight condition 3 4 Stability Derivatives 563 Lateral Motion Stability derivatives 8.2.5 1 ECHO Flight condition 3 2 - A single-engined, CCV, jet fighter aircraft General Parameters Wing area (m2): Aspect ratio: Chord, t (m): Total related thrust (kn): C.g.: Pilot's location (m) (relative to c.g.) 1, : 1, P : P Weight (kg) : Inertias (kg m2): zxx: IYY: IZZ: I ~~: 4 26 3.0 3.33 11 0.35 E - 3.9 0.326 84.52 11 x 6.38 x 7.24 x 4.7 x 103 lo4 lo4 104 Appendix B 564 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) q (N mP2) cro (degrees) Yo (degrees) Flight condition 3 1 2 S.L. 0.6 207 26 245 1.92 0 4 600 0.8 258 25 860 2.17 0 + + 9 100 0.95 288 17 362 4.25 0 + 4 15 250 1.7 502 23 400 + 1.6 0 Stability Derivatives Longitudinal Motion only Stability derivative B.2.6 1 FOXTROT 2 4 - A twinengined, jet fighterlbomber aircraft General Parameters Wing area (m2): Aspect ratio: Chord, C (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1, : zz;: Flight conditions 173 32 100 16 000 181 400 2 100 33 900 166 000 190 000 3 000 Inertias (kg m2): L X : IYY: Zzz : 1x2 : Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Flight conditions Parameter Height (m) Ach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (degrees) Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N mP2) a. (degrees) Yo (deg derivative 1 2 Flight condition 3 4 - A twin-piston engined, general aviation aircraft General Parameters Wing area (m2) Aspect ratio: Chord, E (m): Total related thrust (kN): C.g.: Pilot's location (m) (relative to c.g.) 1, : 1, P : P Weight (kg): Approach 20 Inertias (kg m2) zxx: IYY: Zzz : 1x2: 13 470 20 450 27 200 2 150 All other flight conditions 27.75 Stability Derivatives 567 Flight Conditions Parameter Height (m) Mach no. Uo (m s-l) 4 (N m2) a. (degrees) Flight condition 3 1 2 S.L. 0.143 50.0 1530 S.L. 0.19 65 2 590 1600 0.207 70 1960 6 500 0.345 105 3 440 - - - 4 Stability Derivatives Longitudinal Motion Stability derivative 1 2 Flight condition 3 4 Appendix B 568 Lateral Motion Stability derivative 1 2 Flight condition 3 4 Appendix C Mathematical Models of Human Pilots C.1 INTRODUCTION Notwithstanding the extent to which flight control is being made automatic, it remains essential for the designers of flight control systems to remember that a human pilot acts as the 'outer loop' of a complete flight control system. As AFCSs have been improved and developed, the need for such representation has been recognized for a considerable time. It has been the cause of a great amount of research which is recorded in a most extensive literature. Chief among the workers researching in this field have been McRuer, Krendel and Graham, and it is their work (see the various references at the end of this appendix) which provides the basis for those models dealt with briefly below. More extensive models exist, such as Paper Pilot (Dillow, 1971), but they are beyond the scope of an introductory textbook such as this. There are several reasons for using a mathematical model in studies relating to the performance of closed loop flight control systems being operated by a human pilot; the include the following: 1. The prediction of what may be possible from some given arrangement. 2. The evolution and, perhaps, development of critical flight or simulator experimental results. From examining the nature of a pilot's behaviour when flying it becomes clear that he normally demonstrates those

characteristics commonly described as adaptive and multimodal. Even when carrying out familiar tasks, the pilot is also capable of learning. This knowledge suggests that the construction of any appropriate mathematical model may incorporate some of the following features: 1. The differential equations involved should be invariant, or time-varying 2. The model may be multi- or single-variable. 3. The equations may be linear or non-linear. 4. The data may be continuous or sampled. Appendix C 570 The model should represent adequately the pilot's actions when carrying out a pursuit task or controlling the aircraft using a compensatory display. From extensive experiments on human operators it has been learned that one appropriate form of model was a describing function which represents the linear response of the operator whose actual response of the operator whose actual response of the describing function model does depend upon the addition of a remnant term, but, for simplicity, only the linear models represented by describing functions are used here. A remnant term can be included in the model is given in paragraph 4 below. C.2 CLASSICAL MODELS 1. The pilot's response is denoted by v,; his command is taken as pcomm. Basically, the model assumes that the response is linear and proportional to the command is taken as pcomm. Basically, the model assumes that the response is denoted by v,; his command is taken as pcomm. Basically, the model assumes that the response is denoted by the finite reaction, but with a pure time delay caused by the finite reaction time of the pilot. C. I from which it can be deduced that Figure C. Block diagram of pilot model - lead term and pure time delay. The transfer function and can only be completely represented by an infinite series. Consequently, a suitable approximation is needed. One of the most accepted is the first order Pad6 approximation: Let: R1 = d p + d Mathematical Models of Human Pilots then: fl=dp+3= 2 2 7 7 -v--vp - 4 v 2 - -XI 7 7 However, V = .'. KpT~Ijcomm + Kpcomm - K p T ~ Ijc o m m Refer to Figure C.2. Figure C. pure time delay. (C. 10) K T ... v. = - 1 v + K Pcomm + & pcomm (C.11) Tl Tl Using the first order Pad6 approximation of eq. (C.3) and choosing the state variables for this model to be: XI = v, X2 = v + v (C. 12) (C.13) the following equations are obtained: (C. 15) Appendix 572 3. C Refer to Figure C.3. The term: $o_{i}/(s^2 + 250, s + w_i)$ represents the addition of a neuromuscular lag to the model. The transfer can easily be represented by the following state function V(s)lpComm(s) equation: [:I = [- 21n] Finally, if we define x3 as v (C.3) then: - + 0 [I] [f 1 + + v,, and use the Pad6 approximation of (C. 19) - 1 (C. 17) pcomm Pcomm 0 - 4. a Refer to Figure C.4. Using a more comprehensive model relating to hovering motion in which a remnant term and phase advance compensation are added, the following equations are obtained: 1 el=-- Kpl KPTL1 el - -pcomm Tl Tl Tl (c.21) Mathematical Models of Human Pilots Figure C.4 Block diagram of pilot model - phase advance, pure time delay, lead term neuromuscular lag and remnant term. (C.22) $e^3 = e^2 + e^4$ (C. 23) $i^4 = -m \sim e' + , uRv$ (2 w (C. 24) R) ~ Also: (C.26) N o w let: XI x 2 = e^4 = e^1 x 5 = v + vp (C.27) then the following state equation can be obtained: -ir + E i = A x + B z + M n (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.28) v, = Cx (C.29) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.38) v, = Cx (C.39) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.38) v, = Cx (C.39) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.38) v, = Cx (C.39) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 - K ~ ~ T11 ~ (C.38) v, = Cx (C.39) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - Kp2 C.3 02 ~ (C.39) where: x1 = [XI x 2 z 1 = [~ c o m mY X g X',] x5] Appendix C 0 UR M = (C.35) 0 0 0 C = [0 0 - K ~ w ~ (C.36) REFERENCES 1971. 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